Revised Edition

Heat and Mass Transfer

SI UNITS

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NOMENCLATURE

A	Area
c	Specific heat
c_p	specific heat at constant pressure
c_v	Specific heat at constant volume
C	Mass concentration
C_{fx}	Local skin friction coefficient
$\overline{C}_f$	Average skin friction coefficient
D	Diffusion coefficient
D, d	Diameter
E	Total emissive power
E_b	Emissive power of a black body
E_λ	Monochromatic emissive power
f	Interchange factor
F	Correction factor
G	Irradiation
G	Universal gas constant
$\overline{h}$	Average heat transfer coefficient
h_{mc}	Mass transfer coefficient based upon concentration
h_{mp}	Mass transfer coefficient based upon pressure
J	Radiosity
k	Thermal conductivity
L	Length
m	Mass
$\dot{m}$	Mass flow rate
M	Molecular weight
m^*	Mass fraction
N	Mass flux of species
p	Pressure
P	Perimeter
q	Heat transfer per unit area per unit time
q_g	Heat generated per unit volume per unit time
Q	Heat transfer rate per unit time
Q'	Total heat transfer
Q_g	Heat generated per unit time
Q_g'	Total heat generated
R	Characteristic gas constant
R_{th}	Thermal resistance
$(R_{th})_{cond.}$	Conductive thermal resistance
$(R_{th})_{conv.}$	Convective thermal resistance
$(R_{th})_{rad.}$	Radiative thermal resistance
t	temperature
T	Absolute temperature
U	Overall heat transfer coefficient, velocity of fluid
x	Mole fraction

Greek Notations

$\alpha\left(=\dfrac{k}{\rho c}\right)$	Thermal diffusivity; Absorptivity
β	Coefficient of volume expansion; Temperature coefficient of thermal conductivity
τ	Time; Transmittivity
τ_{th}	Thermal time constant
θ	Temperature difference; Momentum thickness
θ_m	Log-mean temperature difference
δ	Hydrodynamic boundary layer thickness
δ^*	Displacement thickness
δ_e	Energy thickness
δ_{th}	Thermal boundary layer thickness
ψ	Stream function
ϕ	Velocity potential
σ	Stefan-Boltzmann constant ($=5.67 \times 10^{-8}$ W/m^{2}K^4)

ρ	Reflectivity; Mass density
ε	Emissivity
λ	Wavelength
Ω	Collision integral

Dimensionless Groups

$Bi\left(=\frac{hL}{k}\right)$	Biot number
$Gr\left(=\frac{L^3 g\beta\Delta t}{v^2}\right)$	Grashof number
$Gz\left(=\frac{mc_p}{Lk}\right)$	Graetz number
$Le\left(=\frac{\alpha}{D}\right)$	Lewis number
$Nu\left(=\frac{hx}{h}\right)$	Nusselt number
$\overline{Nu}\left(=\frac{\bar{h}x}{k}\right)$	Average Nusselt number
$Pe\left(=Re.Pr=\frac{LU}{\alpha}\right)$	Peclet number
$Pr\left(=\frac{c_p\mu}{k}=\frac{v}{\alpha}\right)$	Prandtl number
$Re\left(=\frac{\rho Ux}{\mu}\right)$	Reynolds number
$Sh=\frac{h_m.x}{D}$	Sherwood number
$Sc\left(=\frac{v}{D}\right)$	Schmidt number
$St\left(=\frac{h}{\rho Uc_p}\right)$	Stanton number

Subscripts

i, 1	Inner or inlet conditions
0, 2	Outer or outlet conditions
hf	Hot fluid
cf	Cold fluid
w	Wall conditions

CHAPTER

1

Basic Concepts

1.1. Heat transfer – general aspects – Heat – Importance of heat transfer – Difference between thermodynamics and heat transfer – Modes of heat transfer.

1.2. Heat transfer by conduction: Fourier's law of heat conduction – Thermal conductivity of materials – Thermal resistance (R_{th}).

1.3. Heat transfer by convection.

1.4. Heat transfer by radiation. Highlights – Theoretical Questions – Unsolved Examples.

1.1. HEAT TRANSFER–GENERAL ASPECTS

1.1.1. HEAT

"*The energy in transit is termed* **heat**"

While Aristotle was of the opinion that fire was one of the four primary elements, Plato thought that the heat was sort of motion of particles; accordingly there are two theories of heat. Any theory should be able to explain the facts given below :

(*i*) Whenever there is an exchange of heat, heat is consumed (heat lost by the hot body is always equal to heat gained by the cold body).

(*ii*) The heat flow takes place from higher to lower temperature.

(*iii*) The substances expand on heating.

(*iv*) In order to change the state of a body from solid to liquid or liquid to gas without rise in temperature, certain amount of heat is required.

(*v*) When a body is heated or cooled its weight does not change.

According to the **modern or dynamical theory of heat:** "*Heat is a form of energy. The molecules of a substance are in parallel motion. The mean kinetic energy per molecule of the substance is proportional to its absolute temperature*".

A molecule may consist of one or two or many atoms depending upon the nature of the gas. The force of attraction between the molecules of a perfect gas is negligible. The atoms in a molecule vibrate with respect to one another, consequently a molecule has '*vibrational energy*'. The whole molecule may rotate about one or more axes, so it can have '*rotational energy*'. A molecule has '*translational energy*' due to its motion. Thus kinetic energy of a molecule is the sum of its translational, rotational and vibrational energies. Summarily *heat energy given to a substance is used in increasing its internal energy*. Increase in internal energy causes increase in kinetic energy or potential energy or increase in both the energies. Due to increase in kinetic energy of a molecule, its translational, vibrational or rotational energy may increase.

1.1.2. IMPORTANCE OF HEAT TRANSFER

Heat transfer may be defined as :

"*The transmission of energy from one region to another as a result of temperature gradient*".

In *heat transfer* the driving potential is *temperature difference* whereas in *mass transfer* the driving potential is *concentration difference*. In *mass transfer* we concentrate upon *mass motion which result in changes in composition*, and are *caused by the variations in concentrations of the various constituent species*. This transfer, in literature, is also known as "**diffusion**".

The *study of heat transfer* is carried out for the follows *purposes* :

1. To estimate the rate of flow of energy as heat through the boundary of a system under study (both under steady and transient conditions).
2. To determine the temperature field under steady and transient conditions.

In almost every branch of engineering, heat transfer (and mass transfer) problems are encountered which cannot be solved by thermodynamic reasoning alone but require an analysis based on heat transfer principles. The *areas covered* under the discipline of *heat transfer* are :

- Design of thermal and nuclear power plants including heat engines, steam generators, condensers and other heat exchange equipments, catalytic converters, heat shields for space vehicles, furnaces, electronic equipments etc.
- Internal combustion engines.
- Refrigeration and air conditioning units.
- Design of cooling systems for electric motors, generators and transformers.
- Heating and cooling of fluids etc. in chemical operations.
- Construction of dams and structures; minimisation of building-heat losses using improved insulation techniques.
- Thermal control of space vehicles.
- Heat treatment of metals.
- Dispersion of atmospheric pollutants.

1.1.3. THERMODYNAMICS

1.1.3.1. Definition

Thermodynamics may be defined as follows:

"*Thermodynamics is an axiomatic science which deals with the relations among heat, work and properties of system which are in equilibrium. It describes state and changes in state of physical systems.*"

Thermodynamic, basically entails four laws or axioms known as Zeroth, First, Second and Third law of thermodynamics.

- *First law* throws light on *concept of internal energy.*
- Zeroth law deals with *thermal equilibrium* and establishes a *concept of temperature.*

- Second law indicates the limit of *converting heat into work* and introduces the *principle of increase of entropy*.
- Third law defines *absolute zero of entropy*.

These laws are based on experimental observations and have no *mathematical proof*. Like all physical laws, these laws are based on *logical reasoning*.

1.1.3.2. Thermodynamic systems

System, boundary and surroundings :

System. A system is a *finite quantity of matter or a prescribed region of space* (Refer Fig. 1.1).

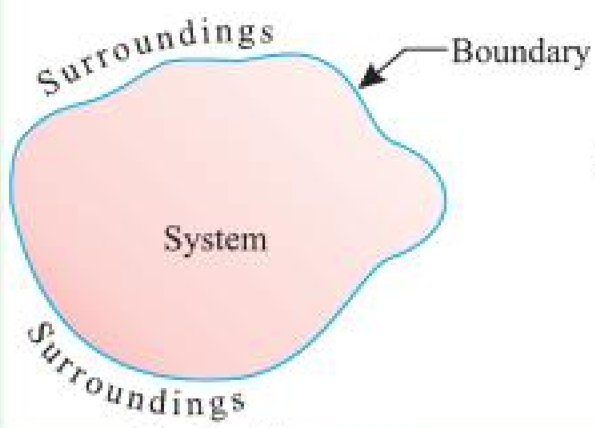


Fig. 1.1. The system.

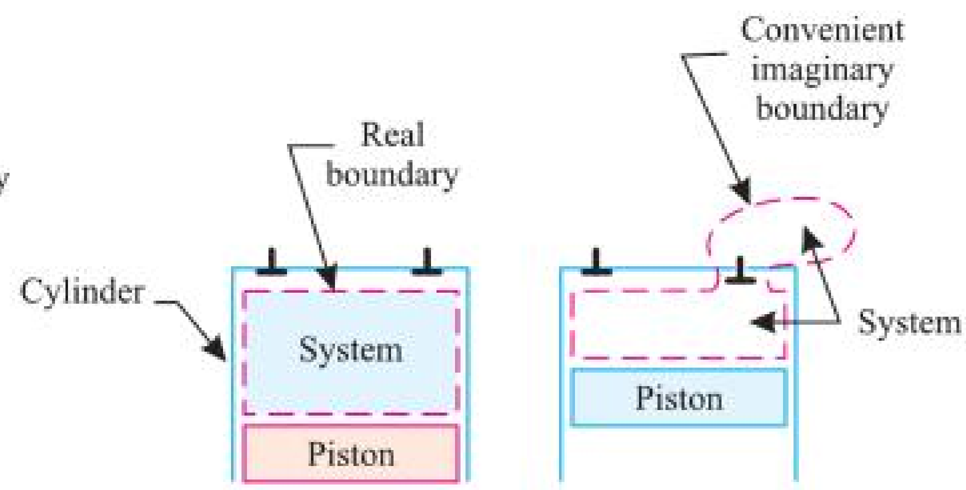


Fig. 1.2. The real and imaginary boundary.

Boundary. The *actual or hypothetical envelop enclosing the system* is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded. The boundary may be *real or imaginary*. It is not difficult to envtisage a real boundary but an example of imaginary boundary would be one drawn around a system consisting of a fresh mixture about to enter the cylinder of an I.C engine together with remanants of the last cylinder charge after the exhaust process (Fig 1.2).

Refer to Fig. 1.3. If the boundary of the system is impervious to the flow of matter, it is called a *closed system*. An example of this system is mass of gas or vapour contained in an engine cylinder, the boundary of which is drawn by the cylinder walls, the cylinder head and piston crown. Here the *boundary is continuous and no matter may enter or leave*.

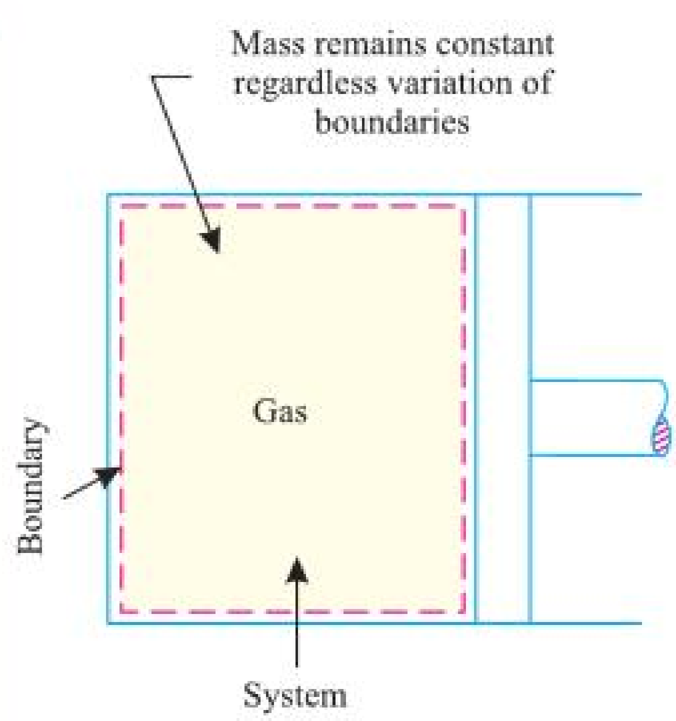


Fig. 1.3. Closed system.

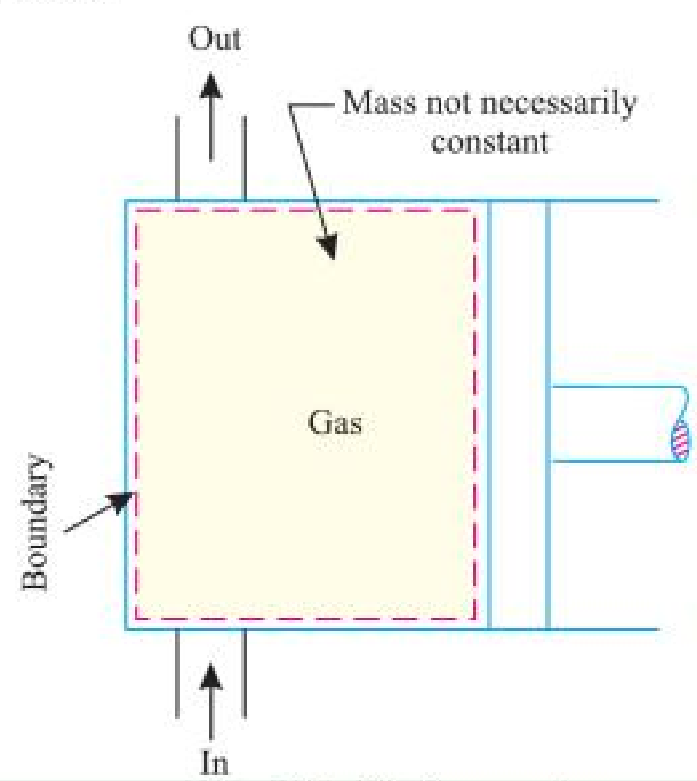


Fig. 1.4. Open system.

Open system :

Refer figure 1.4. An open system is one in which *matter flows into or out of the system*. Most of the engineering systems are open.

Isolated system :

An isolated system is that system *which exchanges neither energy nor matter with any other system or with environment.*

Adiabatic system :

An adiabatic system is one *which is thermally insulated from its surroundings*. It can, however, *exchange work with its surroundings*. If it does not, it becomes an isolated system.

Phase :

A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

Homogeneous system :

A system which consists of a single phase is termed as homogeneous system. *Examples* : Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

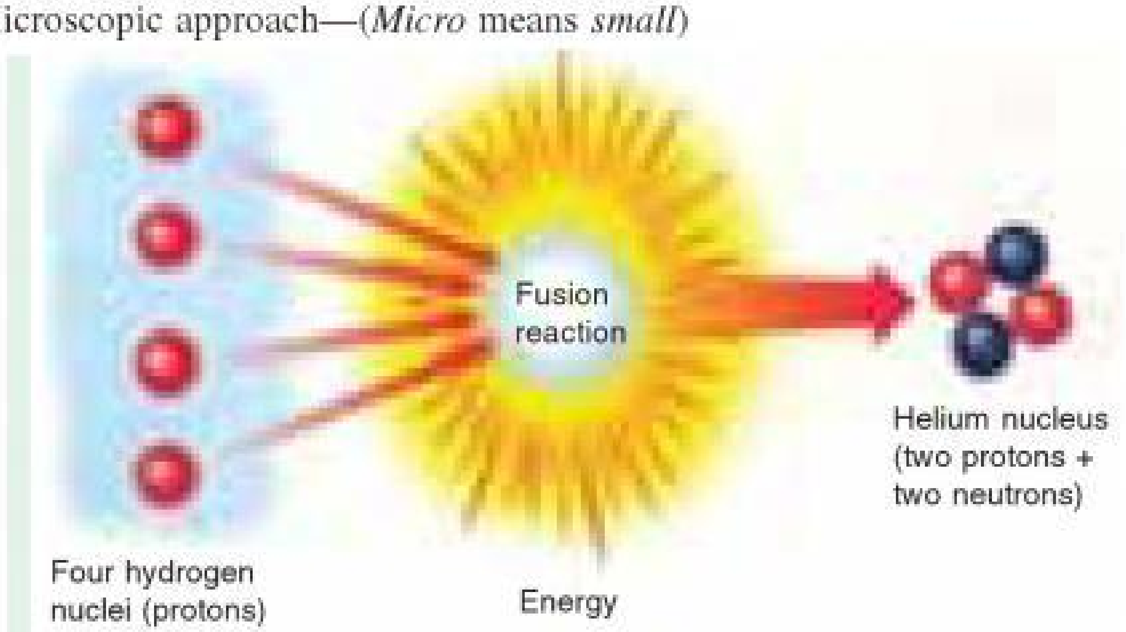
Heterogeneous system :

A system which consists of two or more phases is called a heterogeneous system. *Examples* : Water plus steam, ice plus water and water plus oil.

1.1.3.3. Macroscopic and microscopic points of view

Thermodynamic studies are undertaken by the following two different approaches :

1. Macroscopic approach—(*Macro* mean *big or total*)
2. Microscopic approach—(*Micro* means *small*)



Sun is the major source of energy for the earth.

These approaches are discussed (in a comparative way) below :

> **Note.** Although the macroscopic approach seems to be different from microscopic one, there exists relation between them. Hence when both the methods are applied to a particular system, they give the *same result*.

1.1.3.4. Pure substance

A "*pure substance*" *is one that has a homogeneous and invariable chemical composition even though there is a change of phase*. In other words, it is a system which is (*a*) homogeneous in composition, (*b*) homogeneous in chemical aggregation. *Examples* : Liquid, water, mixture of liquid water and steam, mixture of ice and water. The mixture of liquid air and gaseous air is *not* a pure substance.

S.No.	Macroscopic approach	Microscopic approach
1.	In this approach a certain quantity of matter is considered without taking into account the events occuring at molecular level. In other words this approach to thermodynamics is concerned with *gross or overall behaviour.* This is known as "*Classical thermodynamics*".	The approach considers that the system is matter of a very large number of discrete particles known as *molecules*. These molecule have different velocities and energies. The values of these energies are constantly changing with time. This approach to thermodynamics which is concerned directly with the *structure of the matter* is known as "*Statistical thermodynamics*".
2.	The analysis of macroscopic system requires simple mathematical formulae.	The behaviour of the system is found by using statistical methods as the number of molecules is very large. So advanced statistical and mathematical methods are needed to explain the change in the system.
3.	The values of the properties of the system are their average values. For example, consider a sample of a gas in a closed container. The *pressure* of the gas is the average value of the pressure exerted by millions of individual molecules. Similarly the *temperature* of this gas is the average value of translational kinetic energies of millions of individual molecules. These properties like *pressure* and *temperature* can be measured very easily. The *changes in properties can be felt by our senses.*	The properties like *velocity*, *momentum*, impulse kinetic energy, force of impact etc. which describe the molecule *cannot be easily measured by instruments. Our senses cannot feel them.*
4.	In order to describe a system only a few properties are needed.	Large number of variables are needed to describe a system. So the approach is complicated.

1.1.3.5. Thermodynamic equilibrium

A system is in *thermodynamic equilibrium* if the temperature and pressure at all points are same; there should be no velocity gradient; the chemical equilibrium is also necessary. Systems under temperature and pressure equilibrium but not under chemical equilibrium are sometimes said to be in metastable equilibrium conditions. *It is only under thermodynamic equilibrium conditions that the properties of a system can be fixed.*

Thus for attaining a state of *thermodynamic equilibrium* the following three types of equilibrium states must be achieved :

1. Thermal equilibrium. The temperature of the system does not change with time and has same value at all points of the system.

2. Mechanical equilibrium. There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.

3. Chemical equilibrium. No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

1.1.3.6. Properties of systems

A "*property of a system*" is *a characteristic of the system which depends upon its state, but not upon how the state is reached*. There are two sorts of property :

1. Intensive properties. These properties *do not depend on the mass of the system. Example* : Temperature and pressure.

2. Extensive properties. These properties *depend on the mass of the system. Example* : Volume. Extensive properties are often divided by mass associated with them to obtain the intensive poroperties. For example, if the volume of a system of mass m is V, then the specific volume of matter within the system is $\frac{V}{m} = v$ which is an intensive property.

1.1.3.7. State

"State" *is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a "state".*

It follows from the definition of state that each property has a single value at each state. Stated differently, all properties are *state* or *point functions*. Therefore, all properties are identical for identical states.

On the basis of the above discussion, we can determine if a given variable is *property* or not by applying the following tests :

- *A* **variable** *is a property, if and only if, it has a single value at each equilibrium state.*
- A *variable* is a property, if and only if, the change in its value between any two prescribed equilibrium states is single-valued.

Therefore, *any variable whose change is fixed by the end states is a "property".*

1.1.3.8. Process

A process occurs when the system undergoes a change in a state or an energy transfer at a steady state. A process may be *non-flow* in which a fixed mass within the defined boundary is undergoing a change of state. *Example* : A substance which is being heated in a closed cylinder undergoes a non-flow process (Fig. 1.3.). *Closed systems undergo non-flow processes*. A process may be a flow process

Petroleum, Coal, etc. are energy sources which originally obtained the energy from the Sun.

in which mass is entering and leaving through the boundary of an open system. In a steady flow process (Fig. 1.4.) mass is crossing the boundary from surroundings at entry, and an equal mass is crossing the boundary at the exit so that the total mass of the system remains constant. In an open system it is necessary to take account of the work delivered from the surroundings to the system at entry to cause the mass to enter, and also, of the work delivered from the system at surroundings to cause the mass to leave, as well as any heat or work crossing the boundary of the system.

Quasi-static process. Quasi means 'almost'. A quasi-static process is also called a *reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.*

1.1.3.9. Cycle

Any process or series of processes whose end states are identical is termed a **cycle**. The processes through which the system has passed can be shown on a state diagram, but a complete section of the path requires in addition a statement of the heat and work crossing the boundary of the system. Fig.1.5 shows such a cycle in which a system commencing at condition '1' changes in pressure and volume through a path 1-2-3 and returns to its initial condition '1'.

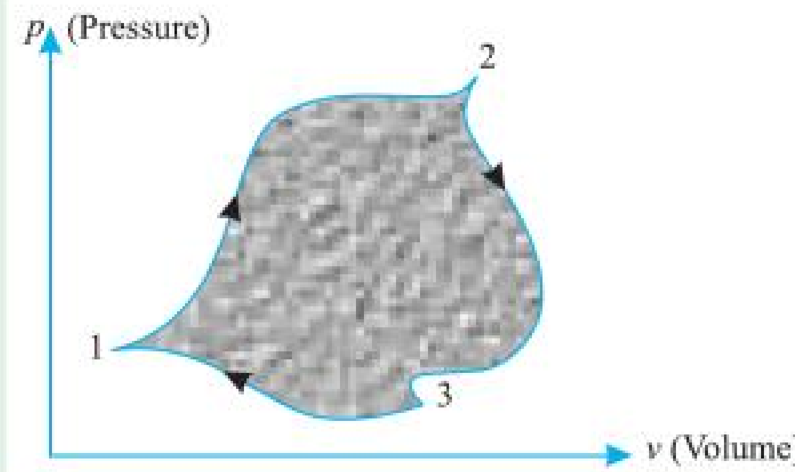


Fig. 1.5. Cycle of operations.

1.1.3.10. Point function

When two properties locate a point on the graph (coordinate axes) then those properties are called as *point function*.

Examples. Pressure, temperature, volume etc.

$$\int_1^2 dV = V_2 - V_1 \text{ (an } \textit{exact differential}\text{)}$$

1.1.3.11. Path function

There are certain quantities which cannot be located on a graph by a *point* but are given by the *area* or so, on that graph. In that case, the area on the graph, pertaining to the particular process, is a function of the path of the process. Such quantities are called *path functions*.

Examples : Heat, work etc.

Heat and work are *inexact differentials*. Their change cannot be written as differences between their end states.

Thus $\int_1^2 \delta Q \neq Q_2 - Q_1$ and is shown as ${}_1Q_2$ or Q_{1-2}

Similarly $\int_1^2 \delta W \neq W_2 - W_1$, and is shown as ${}_1W_2$ or W_{1-2}

Note. The operator δ is used to denote inexact differentials and operator *d* is used to denote exact differentials.

1.1.3.12. Temperature

The temperature is a thermal state of a body which distinguishes a hot body from a cold body. The temperature of a body is *proportional to the stored molecular energy i.e.*, the average molecular kinetic energy of the molecules in a system. (A particular molecule does not have a temperature, it has energy; the gas as a system has temperature).

Instruments for measuring ordinary temperatures are known as thermometers and those for measuring high temperatures are known as *pyrometers*.

It has been found that a gas will not occupy any volume at a certain temperature. This temperature is known as *absolute zero temperature*. The temperatures measured with absolute zero as basis are called *absolute temperatures*. Absolute temperature is stated in degree centigrade. The point of absolute temperature is found to occur at 273ºC (app.) below the freezing point of water.

Then, Absolute temperature = Thermometer reading in ºC + 273.

Absolute temperature in degree centigrade is known as degree kelvin, denoted by K (SI units).

1.1.3.13. Pressure

The **pressure** *of a system is the force exerted by the system on unit area of boundaries.*

Units of pressure are :

SI Units. N/m^2 (sometimes called *pascal*, Pa) or bar

$$1 \text{ bar} = 10^5 \text{ N/m}^2 = 10^5 \text{ Pa}$$

$$\text{Standard atmospheric pressure} = 1.01325 \text{ bar} = 0.76 \text{ m Hg.}$$

MKS System. kgf/cm^2, mm of mercury, metre or mm of water column.

$$\text{Standard atmospheric pressure} = 1.033 \text{ kgf/cm}^2 \text{ (= 760 mm of Hg)}$$

$$\text{Technical pressure} = 1 \text{ kgf/cm}^2.$$

The pressure gauges, vacuum gauges or manometers are used for measuring fluid pressure. These devices indicate pressure relative to atmospheric pressure and this pressure is known as *gauge pressure*. To get absolute pressure, atmospheric pressure is *added* to the gauge pressure. In other words,

Absolute pressure = Gauge pressure + atmospheric pressure.

Vaccum is defined as the *absence of pressure*. A *perfect vacuum* is obtained when *absolute pressure is zero*, at this instant *molecular momentum is zero*.

Energy, Work and Heat

1.1.3.14. Energy

"*Energy*" is a general term embracing *energy in transition and stored energy*. The stored energy of a substance may be in the forms of *mechanical energy* and *internal energy* (other forms of stored energy may be chemical energy and electrical energy). Part of the stored energy may take the form of either potential energy (which is the gravitational energy due to height above a chosen datum line) or kinetic energy due to velocity. The balance part of the energy is known as *internal energy*. In a *non-flow process* usually there is no change of potential or kinetic energy and hence change of mechanical energy will not enter the calculations. In a *flow process*, however, there may be changes in both potential and kinetic energy and these must be taken into account while considering the changes of stored energy. *Heat and work* are the forms of energy in transition. These are the only forms in which energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

Piston and piston rod.

1.1.3.15. Work

Work is said to be done when a *force moves through a distance*. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done *W* is the product of the force (pressure × area), and the distance it moves in the direction of the force. Fig. 1.6 (*a*) illustrates this with the conventional piston and cylinder arrangement, the heavy line defining the boundary of the system. Fig. 1.6 (*b*) illustrates another way in which work might be applied to a system. A force is exerted by the paddle as it changes the momentum of the fluid, and since this force moves during rotation of the paddle room work is done.

Work *is a transient quantity which only appears at the boundary while a change of state is taking place within a system. Work is 'something' which appears at the boundary when a system changes its state due to the movement of a part of the boundary under the action of a force.*

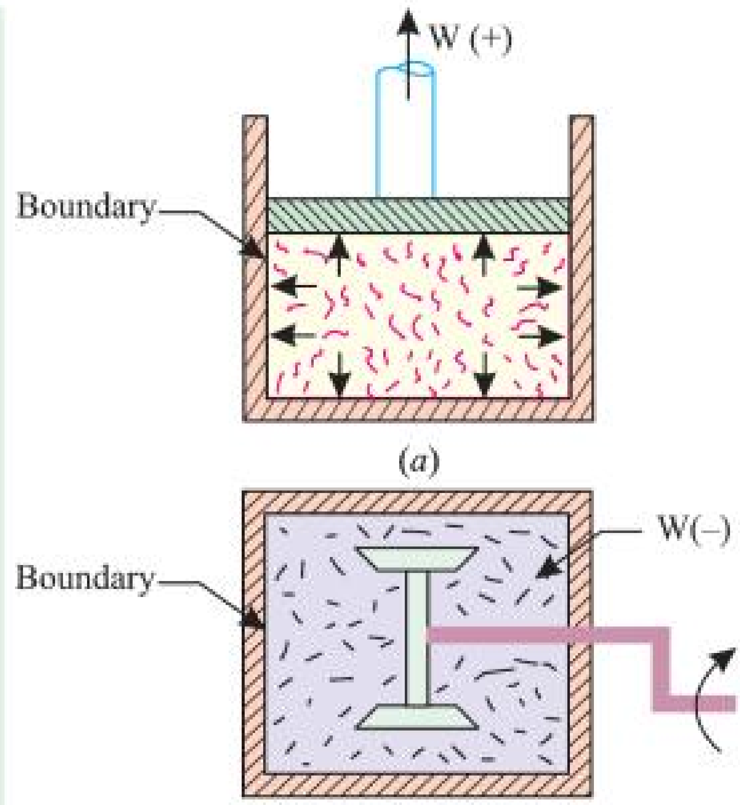


Fig. 1.6.

Sign convention :

- If the work is done *by* the system *on* the surroundings, *e.g.*, when a fluid expands pushing a piston outwards, the work is said to be *positive*.

i.e., *Work output of the system* $= + W$

- If the work is done *on* the system by the surroundings, *e.g.*, when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be *negative*.

i.e., *Work input to system* $= - W$

1.1.3.16. Heat

Heat (denoted by the symbol Q), may be, defined in an analogous way to work as follows :

"**Heat** *is 'something' which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings*".

Heat, like work, is a transient quantity which only appears at the boundary while a change is taking place within the system.

It is apparent that neither δW or δQ are exact differentials and, therefore, any integration of elemental quantities of work or heat which appear during a change from state 1 to state 2 must be written as

$$\int_1^2 \delta W = W_{1-2} \text{ or } {}_1W_2 \text{ (or } W\text{), and}$$

$$\int_1^2 \delta Q = Q_{1-2} \text{ or } {}_1Q_2 \text{ (or } Q\text{)}$$

Sign convention :

If the heat flows *into* a system *from* the surroundings, the quantity is said to be *positive* and conversely, if heat flows *from* the system to the surroundings it is said to be *negative*.

In other words :

Heat received by the system $= + Q$

Heat rejected or given up by the system $= - Q.$

1.1.3.17. Comparison of work and heat

Similarities :

(*i*) Both are *path functions and inexact differentials*.

(*ii*) Both are boundary phenomenon *i.e.*, both are recognized at the boundaries of the system as they cross them.

(*iii*) Both are associated with a process, not a state. Unlike properties, work or heat has no meaning at a state.

(*iv*) Systems possess energy, but not work or heat.

Dissimilarities :

(*i*) In heat transfer temperature difference is required.

(*ii*) In a stable system there cannot be work transfer, however, there is no restriction for the transfer of heat.

(*iii*) The sole effect external to the system could be reduced to rise of a weight but in the case of a heat transfer other effects are also observed.

1.1.4. DIFFERENCES BETWEEN THERMODYNAMICS AND HEAT TRANSFER

The fundamental differences between thermodynamics and heat transfer are given below :

To understand the difference between thermodynamics and heat transfer, let us consider the cooling of a hot steel bar which is placed in a water bath. *Thermodynamics* may be used to predict the final equilibrium temperature of the steel bar-water combination; however, it will not help us to find out how long it takes to reach this equilibrium condition or what the temperature of the bar will be after a certain length of time before the equilibrium condition is attained. *Heat transfer* on the other hand, may be used to *predict* the *temperatures of both the bar and the water as a function of time*.

Heat transfer theory *combines thermodynamics and rate equations together* (to quantify the rate at which heat transfer occurs in terms of the degree of non-equilibrium).

	Thermodynamics	Heat transfer
1.	It deals with the *equilibrium states* of matter, and precludes the existence of a temperature gradient.	It is inherently a *non-equlibrium process* (since a temperature gradient must exist for exchange of heat to take place).
2.	When a system changes from one equilibrium state to another, thermodynamics helps to determine the quantity of work and heat interactions. It describes how much heat is to be exchanged during a process but does not hint how the same could be achieved.	It helps to *predict the distribution of temperature* and to *determine the rate at which energy is transferred* across a surface of interest due to temperature gradients at the surface, and difference of temperature between different surfaces.

1.1.5. BASIC LAWS GOVERNING HEAT TRANSFER

The following are the *basic laws* which govern heat transfer :

1. First law of thermodynamics :

In the early part of nineteenth century the scientists developed the concept of energy and hypothesis that it can neither be created nor destroyed; this came to be known as the '*law of the conservation of energy*'. *The first law of thermodynamics* is merely one statement of this general law/principle with particular reference to heat energy and mechanical work *i.e.*, work, and is *stated* as follows :

"*When a system undergoes a thermodynamic cycle then the net heat supplied to the system from the surroundings is equal to the net work done by the system on its surroundings.*

$$\oint dQ = \oint dW$$

where $\oint$ represents the sum for the complete cycle".

The first law of thermodynamics applies to *reversible as well as irresversible transformations*. For non-cyclic process, a more general formulation of first law of thermodynamics is required. A new concept which involves a term called *internal energy* fulfills this need. As per this law, the relationships for closed and open systems are as follows :

Closed system : The net flow of heat across the system boundary + heat generated inside the system = change in the internal energy of system.

Open system : The net energy transported through a control volume + energy generated within the control volume = change in the internal energy in the control volume.

2. Second law of thermodynamics:

It states that "*heat will flow naturally from one reservoir to another at a lower temperature, but not in opposite direction without assistance*".

This law establishes the direction of energy transport as heat and postulates that the flow of energy as heat through a system boundary will always be in the direction of lower temperature (or along negative temperature gradient).

3. *Law of conservation of mass*. This law is used to determine the parameters of flow.

4. *Newton's laws of motion*. These laws are used to determine fluid flow parameters.

In a steam engine coal burns producing heat, heat generates steam which is finally converted into mechanical energy. Steam engine is an external combustion engine.

5. *The rate equations*. These equations are made applicable depending upon the mode of heat transfer being considered.

1.1.6. MODES OF HEAT TRANSFER

Heat transfer which is defined as the *transmission of energy from one region to another as a result of temperature gradient* takes place by the following three modes :

(*i*) Conduction; (*ii*) Convection; (*iii*) Radiation.

Heat transmission, in majority of real situations, occurs as a result of combinations of these modes of heat transfer. *Example* : The water in a boiler shell receives its heat from the fire-bed by conducted, convected and radiated heat from the fire to the shell, conducted heat through the shell and conducted and convected heat from the inner shell wall, to the water. *Heat always flows in the direction of lower temperature.*

The above three modes are similar in that a temperature differential must exist and the heat exchange is in the direction of decreasing temperature; each method, however, has different controlling laws.

Conduction :

"*Conduction*" is the *transfer of heat from one part of a substance to another part of the same* *substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.*

In *solids*, the heat is conducted by the following *two mechanisms* :

(*i*) *By lattice vibration* (the faster moving molecules or atoms in the hottest part of a body transfer heat by impacts some of their energy to adjacent molecules).

(*ii*) *By transport of free electrons* (Free electrons provide an energy flux in the direction of decreasing temperature — For metals, especially good electrical conductors, the electrtonic mechanism is responsible for the major portion of the heat flux except at low temperature).

In case of *gases*, the mechanism of heat conduction is simple. The kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion exchanging energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it loses energy by collisions.

In liquids, the mechanism of heat is nearer to that of gases. However, the molecules are more closely spaced and intermolecular forces come into play.

Convection :

"*Convection*" *is the transfer of heat within a fluid by mixing of one portion of the fluid with another.*

- Convection is possible only in a fluid medium and is *directly linked with the transport of medium itself.*
- Convection constitutes the *macroform* of the heat transfer since macroscopic particles of a fluid moving in space cause the heat exchange.
- The effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid.

This mode of heat transfer is met with in situations where energy is transferred as heat to a flowing fluid at any surface over which flow occurs. This mode is *basically conduction in a very thin fluid layer at the surface and then mixing caused by the flow.* The heat flow depends on the properties of fluid and is independent of the properties of the material of the surface. However, the shape of the surface will influence the flow and hence the heat transfer.

Free or natural convection. Free or natural convection occurs when the fluid circulates by virtue of the natural differences in densities of hot and cold fluids; the denser portions of the fluid move downward because of the greater force of gravity, as compared with the force on the less dense.

Forced convection. When the work is done to blow or pump the fluid, it is said to be *forced convection.*

Radiation :

"*Radiation*" *is the transfer of heat through space or matter by means other than conduction or convection.*

Radiation heat is thought of as *electromagnetic waves or quanta* (as convenient) an emanation of the same nature as light and radio waves. *All bodies radiate heat; so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits.* Radiant *energy* (being electromagnetic radiation) *requires no medium for propagation and will pass through vaccum.*

Note: The rapidly oscillating molecules of the hot body produce electromagnetic waves in hypothetical medium called *ether*. These waves are identical with light waves, radio waves and X-rays, differ from them only in *wavelength* and travel with an approximate velocity of 3×10^8 m/s. These waves carry energy with them and *transfer it to the relatively slow- moving molecules of the cold body* on which they happen to fall. The molecular energy of the later increases and results in a rise of its temperature. Heat travelling by radiation is known as *radiant heat*.

The properties of radiant heat in general, are similar to those of light. Some of the properties are :

(*i*) It does not require the presence of a material medium for its transmission.

(*ii*) Radiant heat can be reflected from the surfaces and obeys the ordinary laws of reflection.

(*iii*) It travels with velocity of light.

(*iv*) Like light, it shows intereference, diffraction and polarisation etc.

(*v*) It follows the law of inverse square.

The wavelength of heat radiations is longer than that of light waves, hence they are invisible to the eye.

1.2. HEAT TRANSFER BY CONDUCTION

1.2.1. FOURIER'S LAWS OF HEAT CONDUCTION

Fourier's law of heat conduction is an emperical law based on observation and states as follows :

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow".

Mathematically, it can be represented by the equation :

$$Q \propto A \cdot \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows, °C or K, and

dx = Thickness of body in the direction of flow, m.

Thus, $$Q = -k \cdot A \frac{dt}{dx} \qquad ...(1.1)$$

where, k = Constant of proportionality and is known as *thermal conductivity of the body.*

The – ve sign of k [eqn. (1.1)] is to take care of the decreasing temperature alongwith the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative along positive x direction and, therefore, the value as Q becomes* + ve.

Assumptions :

The following are the assumptions on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions.*
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear.*
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).

Some essential features of Fourier's law :

Following are some essential features of Fourier's law :

1. It is applicable to all matter (may be solid, liquid or gas).
2. It is based on experimental evidence and cannot be derived from first principle.

3. It is a vector expression indicating that heat flow rate is in the direction of decreasing temperature and is normal to an isotherm.
4. It helps to define thermal conductivity 'k' (transport property) of the medium through which heat is conducted.

1.2.2. THERMAL CONDUCTIVITY OF MATERIALS

From eqn. (1.1), we have

$$k = \frac{Q}{A} \cdot \frac{dx}{dt}$$

The value of $k = 1$ when $Q = 1$, $A = 1$ and $\frac{dt}{dx} = 1$

Now $k = \frac{Q}{1} \cdot \frac{dx}{dt}$ (unit of k : $\text{W} \times \frac{1}{\text{m}^2} \times \frac{\text{m}}{\text{K (or °C)}}$ = W/mK. or W/m°C)

Thus, the **thermal conductivity** *of a material is defined* as follows :

"*The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference*".

It follows from eqn. (1.1) that materials with high thermal conductivities are good conductors of heat, whereas materials with low thermal conductivities are good thermal insulator. *Conduction of heat occurs most readily in pure metals, less so in alloys, and much less readily in non-metals*. The very low thermal conductivities of certain thermal insulators *e.g.*, cork is due to their porosity, the air trapped within the material acting as an insulator.

Thermal conductivity (a property of material) depends essentially upon the following *factors* :

(*i*) Material structure
(*ii*) Moisture content
(*ii*) Density of the material
(*iv*) Pressure and temperature (operating conditions).

Thermal conductivities (average values at normal pressure and temperature) of some common materials are as under :

Material	Thermal conductivity (k) (W/mK)	Material	Thermal conductivity (k) (W/mK)
1. *Silver*	410	8. *Asbestos sheet*	0.17
2. *Copper*	385	9. *Ash*	0.12
3. *Aluminium*	225	10. *Cork, felt*	0.05 – 0.10
4. *Cast iron*	55–65	11. *Saw dust*	0.07
5. *Steel*	20–45	12. *Glass wool*	0.03
6. *Concrete*	1.20	13. *Water*	0.55 – 0.7
7. *Glass (window)*	0.75	14. *Freon*	0.0083

Following points *regarding thermal conductivity – its variation for different materials and under different conditions are worth noting* :

1. Thermal conductivity of a material is due to flow of free electrons (in case of *metals*) and lattice vibrational waves (in case of *fluids*).
2. Thermal conductivity in case of *pure metals* is the highest (k = 10 to 400 W/m°C). It decreases with increase in impurity.

   The range of k for other materials is as follows :

Alloys : $k = 12$ to 120 W/m °C
Heat insulating and building materials : $k = 0.023$ to 2.9 W/m°C
Liquids : $k = 0.2$ to 0.5 W/m°C
Gases and vapours : $k = 0.006$ to 0.05 W/m°C

3. Thermal conductivity of a metal varies considerably when it (metal) is heat treated or mechanically processed / formed.
4. Thermal conductivity of *most metals decreases with the increase in temperature* (*aluminium and uranium being the exceptions*).
   - In most of *liquids* the value of thermal conductivity tends to decrease with temperature (water being an exception) due to decrease in density with increase in temperature.
   - In case of gases the value of thermal conductivity *increases with temperature.*

   Gases with higher molecular weights have smaller thermal conductivities than with lower molecular weights. This is because the mean molecular path of gas molecules decreases with increase in density and k is directly proportional to the mean free path of the molecule.
5. The dependence of thermal conductivity (k) on temperature, for most materials is almost linear;

$$k = k_0 (1 + \beta t) \qquad ...(1.2)$$

where, k_0 = Thermal conductivity at 0°C, and

β = Temperature coefficient of thermal conductivity, 1/°C (It is usually *posi- tive for non-metals and insulating materials* (magnesite bricks being the exception) and *negative for metallic conductors* (aluminium and certain non-ferrous alloys are the exceptions).

6. In case of solids and liquids, thermal conductivity (k) is only very weakly dependent on pressure; in case of gases the value of k is independent of pressure (near standard atmospheric).
7. In case of non-metallic solids :
   - Thermal conductivity of porous materials depends upon the type of gas or liquid present in the voids.
   - Thermal conductivity of a damp material is considerably higher than that of the dry material and water taken individually.
   - Thermal conductivity increases with in- crease in density.
8. The Wiedemann and Franz law (based on experiment results), regarding thermal and electrical conductivi- ties of a material, states as follows :

   "The ratio of the thermal and electrical conductivi-

Diesel engine is an Internal Combustion (IC) engine where fuel burns inside the cylinder.

ties is the same for all metals at the same temperature; and that the ratio is directly proportional to the absolute temperature of the metal."

Mathematically, $$\frac{k}{\sigma} \propto T$$

or, $$\frac{k}{\sigma T} = C \qquad ...(1.3)$$

where, k = Thermal conductivity of metal at temperature T(K),

σ = Electrical conductivity of metal at temperature T (K), and

C = Constant (for all metals), referred to as Lorenz number ($= 2.45 \times 10^{-8}$ WΩ/K^2; Ω stands for ohms).

This law conveys that the materials which are *good conductors of electrticity are also good conductors of heat.*

1.2.3. THERMAL RESITANCE (R_{th})

When two physical systems are described by similar equations and have similar boundary conditions, these are said to be *analogous*. The heat transfer processes may be compared by *analogy* with the flow of electricity in an electrical resistance. As the flow of electric current in the electrical resistance is directly proportional to potential difference (dV); similarly heat flow rate, Q, is directly proportional to temperature difference (dt), the driving force for heat conduction through a medium.

As per Ohm's law (in elctric-circuit theory), we have

$$\text{Current } (I) = \frac{\text{Potential difference } (dV)}{\text{Electrical resistance } (R)} \qquad ...(1.4)$$

By analogy, the heat flow equation (Fourier's equation) may be written as

$$\text{Heat flow rate } (Q) = \frac{\text{Temperature difference } (dt)}{\left(\frac{dx}{kA}\right)} \qquad ...(1.5)$$

By comparing eqns. (1.4) and (1.5), we find that I is analogous to, Q, dV is analogous to dt and R is analogous to the quantity $\left(\frac{dx}{kA}\right)$. The quantity $\frac{dx}{kA}$ is called **thermal conduction resistance** $(R_{th})_{cond.}$ *i.e.*,

$$(R_{th})_{cond.} = \frac{dx}{kA}$$

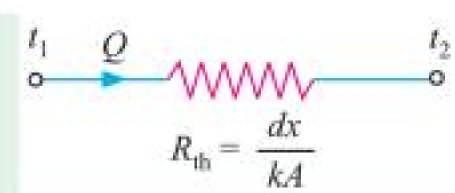


Fig. 1.7.

- The reciprocal of the thermal resistance is called *thermal conductance*.
- It may be noted that *rules for combining electrical resistances in series and parallel apply equally well to thermal resistances.*

The concept of thermal resistance is quite helpful while making calculations for flow of heat.

Example 1.1. *Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C and the other face at 50°C. Take thermal conductivity of copper as 370 W/m°C.*

Solution. Temperature difference, $dt (= t_2 - t_1) = (50 - 350)$

Thickness of copper plate, L = 45 mm = 0.045 m

Thermal conductivity of copper, k = 370 W/m°C

Rate of heat-transfer per unit area, q :

From Fourier's law

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

...(Eqn. 1.1)

or,

$$q = \frac{Q}{A} = -k\frac{dt}{dx}$$

$$= -370 \times \frac{(50 - 350)}{0.045}$$

$$= 2.466 \times 10^6 \text{ W/m}^2 \text{ or}$$

2.466 MW/m² (Ans.)

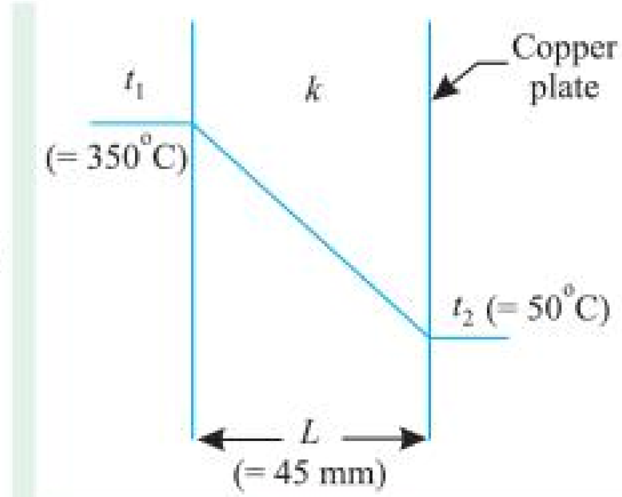


Fig. 1.8.

Example 1.2. *A plane wall is 150 mm thick and its wall area is 4.5 m². If its conductivity is 9.35 W/m°C and surface temperatures are steady at 150°C and 45°C, determine :*

(*i*) *Heat flow across the plane wall;*

(*ii*) *Temperature gradient in the flow direction.*

Solution. Thickness of the plane wall,

$$L = 150 \text{ mm} = 0.15 \text{ m}$$

Area of the wall, $A = 4.5 \text{ m}^2$

Temperature difference, $dt = t_2 - t_1 = 45 - 150 = -105°\text{C}$

Thermal conductivity of wall material,

$$k = 9.35 \text{ W/m°C}$$

(*i*) Heat flow across the plane wall, Q :

As per Fourier's law,

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15} = \mathbf{29452.5 \text{ W}}$$

(*ii*) Temperature gradient, $\frac{dt}{dx}$:

From Fourier's law, we have

$$\frac{dt}{dx} = -\frac{Q}{kA} = \frac{29452.5}{9.35 \times 4.5} = \mathbf{-700°C/m}$$

Example 1.3. *The following data relate to an oven :*

Thickness of side wall of the oven = 82.5 mm

Thermal conductivity of wall insulation = 0.044 W/m°C

Temperature on inside of the wall = 175°C

Energy dissipated by the electrical coil within the oven = 40.5 W

Determine the area of wall surface, perpendicular to heat flow, so that temperature on the other side of the wall does not exceed 75°C.

Solution. *Given* : $x = 82.5$ mm $= 0.0825$ *m*; $k = 0.044$ W/m°C; $t_1 = 175°\text{C}$; $t_2 = 75°\text{C}$; $Q = 40.5\text{W}$

Area of the wall surface, A :

Assuming one-dimentional steady state heat conduction, Rate of electrical energy dissipation in the oven.

= Rate of heat transfer (conduction) across the wall

i.e. $$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{x} = \frac{kA(t_1 - t_2)}{x}$$

or, $$40.5 = \frac{0.044\ A\ (175 - 75)}{0.0825}$$

or, $$A = \frac{40.5 \times 0.0825}{0.044\ (175 - 75)} = \mathbf{0.759\ m^2}$$

1.3. HEAT TRANSFER BY CONVECTION

The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed by *Newton's law of cooling* (*Refer Fig. 1.9*)

$$Q = hA\,(t_s - t_f) \qquad ...(1.6)$$

where, Q = Rate of conductive heat transfer,

A = Area exposed to heat transfer,

t_s = Surface temperature,

t_f = Fluid temperature, and

h = Co-efficient of convective heat transfer.

The units of h are,

$$h = \frac{Q}{A\,(t_s - t_f)} = \frac{\text{W}}{\text{m}^2\,°\text{C}} \quad \text{or} \quad \text{W/m}^2\,°\text{C}$$

or, $\text{W/m}^2\text{K}$

The coefficient of convective heat transfer 'h' (also known as *film heat transfer coefficient*) may be defined as "*the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time*."

The value of 'h' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.* viscosity, density, specific heat etc.).

(*ii*) Nature of fluid flow.

(*iii*) Geometry of the surface.

(*iv*) Prevailing thermal conditions.

Since 'h' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however, by dimensional analysis an equation for the purpose can be obtained.

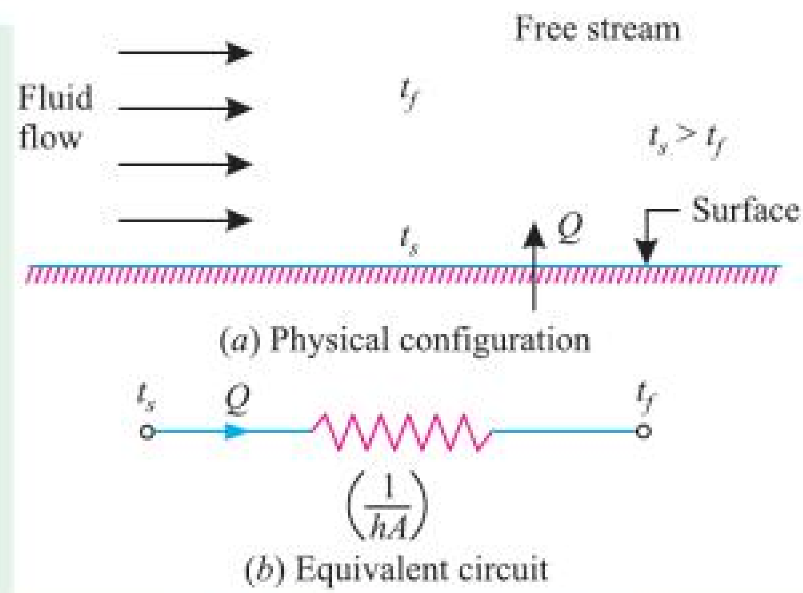


Fig. 1.9. Convective heat-transfer

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation*. Refer Fig. 1.9 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \text{...Eqn (1.6)}\right]$ is called **convection thermal resistance** $[(R_{th})_{conv.}]$ to heat flow.

Example 1.4. *A hot plate 1m × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5$ m^2

Plate surface temperature, t_s = 300°C

Temperature of air (fluid), t_f = 20°C

Connvective heat-transfer coefficient, h = 20 W/m^2 °C

Rate of heat transfer, Q :

From Newton's law of cooling,

$$Q = hA\,(t_s - t_f)$$
$$= 20 \times 1.5\,(300 - 20) = 8400 \text{ W or } \mathbf{8.4\ kW}$$

Example 1.5. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, $d = 1.5$ mm $= 0.0015$ m

Length of the wire, $L = 150$ mm $= 0.15$ m

∴ Surface area of the wire (exposed to heat transfer),

$A = \pi\, d\, L = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4}$ m^2

Wire surface temperature, t_s = 120°C

Water temperature, t_f = 100°C

Convective heat transfer coefficient, h = 4500 W/m^2 °C

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

∴ $Q = hA\,(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\,(120 - 100) = \mathbf{63.6\ W}$

1.4. HEAT TRANFER BY RADIATION

Laws of Radiation :

1. **Wien's law.** It states that *the wavelength λ_m corresponding to the maximum energy is inversely proportional to the absolute temperature T of the hot body.*

   i.e., $\lambda_m \propto \dfrac{1}{T}$ or, $\lambda_m T$ = constant ...(1.7)

2. **Kirchhoff's law.** It states that the *emissivity of the body at a particular temperature is numerically equal to its absorptivity for radiant energy from body at the same temperature.*

3. **The Stefan-Boltzmann law.** The law states that *the emissive power of a black body is directly proportional to fourth power of its absolute temperature.*

   i.e., $Q \propto T^4$ ...(1.8)

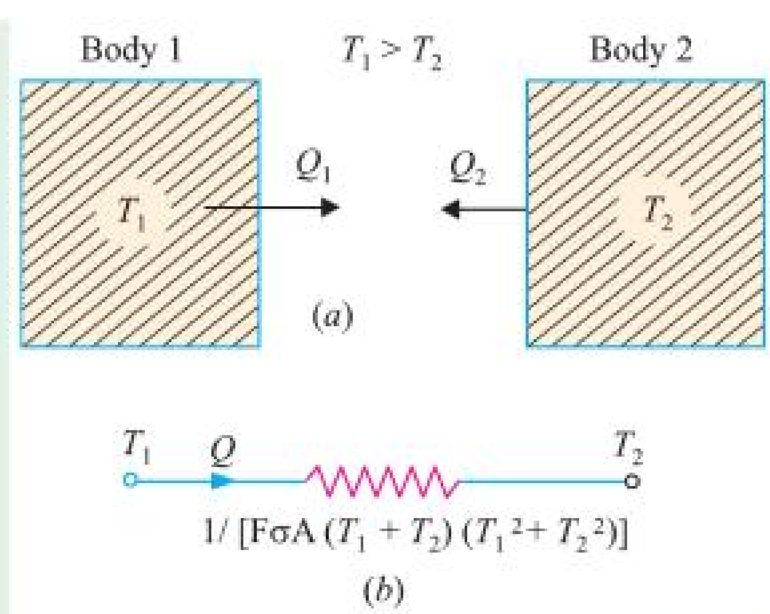


Fig. 1.10. Heat transfer by radiation.

Refer Fig. 1.10 (*a*)

$$Q = F\,\sigma\,A\,(T_1^4 - T_2^4) \quad ...(1.9)$$

where, F = A factor depending on geometry and surface properties,

$$\sigma = \text{Stefan-Boltzmann constant}$$
$$= 5.67 \times 10^{-8}\ \text{W/m}^2\text{K}^4,$$
$$A = \text{Area, m}^2\text{, and}$$
$$T_1, T_2 = \text{Temperatures, degrees kelvin (K).}$$

This equation can also be rewritten as :

$$Q = \frac{T_1 - T_2}{1/[F\,\sigma\,A\,(T_1 + T_2)\,(T_1^2 + T_2^2)]} \qquad ...(1.10)$$

where denomenator is **radiation thermal resistance**, $(R_{th})_{rad}$. [Fig. 1.10 (*b*)]

i.e., $$(R_{th})_{rad} = 1/[F\,\sigma\,A\,(T_1 + T_2)\,(T_1^2 + T_2^2)]$$

The values of F are available for simple configurations in the form of charts and tables.

$F = 1$ ... for simple cases of black surface enclosed by other surface

F = emissivity (ε) ... for non-black surface enclosed by other surface.

[*Emissivity* (ε) is defined as the ratio of heat radiated by a surface to that of an ideal surface.]

Example 1.6. *A surface having an area of 1.5 m² and maintained at 300°C exchanges heat by radiation with another surface at 40°C. The value of factor due to the geometric location and emissivity is 0.52. Determine :*

(*i*) *Heat lost by radiation,*

(*ii*) *The value of thermal resistance, and*

(*iii*) *The value of equivalent convection coefficient.*

Solution. *Given* : $A = 1.5$ m²; $T_1 = t_1 + 273 = 300 + 273 = 573$K; $T_2 = t_2 + 273 = 40 + 273 = 313$K; $F = 0.52$.

(*i*) Heat lost by radiation, Q :

$$Q = F\,\sigma\,A\,(T_1^4 - T_2^4) \qquad ...[\text{Eqn. (1.9)}]$$

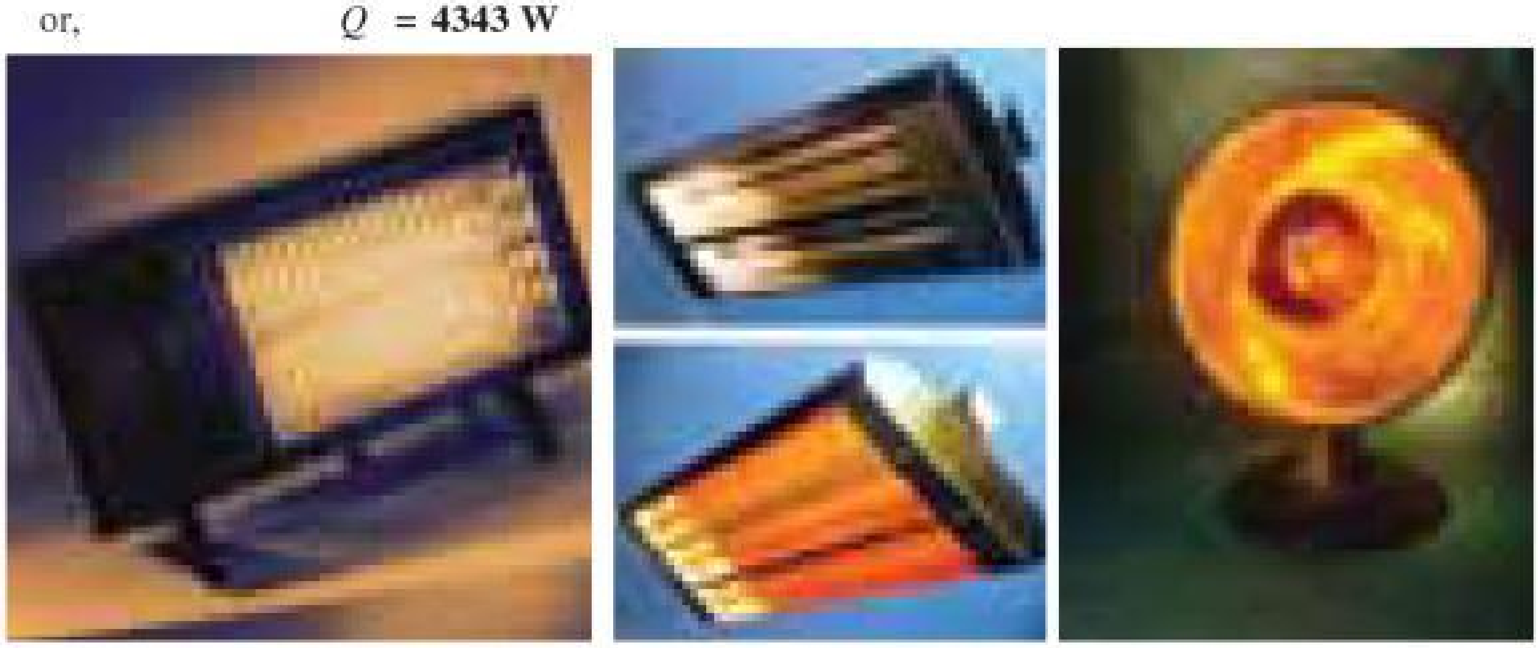
(where $\sigma = 5.67 \times 10^{-8}$ W/m² K⁴)

or, $$Q = 0.52 \times 5.67 \times 10^{-8} \times 1.5\,[(573)^4 - (313)^4]$$

$$= 0.52 \times 5.67 \times 1.5 \left[\left(\frac{573}{100}\right)^4 - \left(\frac{313}{100}\right)^4\right]$$

(Please note this step)

or, $$Q = \mathbf{4343\ W}$$



These appliances are used to radiate heat in the winter.

(*ii*) The value of thermal resistance, (Rth)rad :

We know that, $$Q = \frac{(T_1 - T_2)}{(R_{th})_{rad}}$$ ...[Eqn. (1.10)]

$$\therefore \quad (R_{th})_{rad.} = \frac{(T_1 - T_2)}{Q} = \frac{(573 - 313)}{4343} = \mathbf{0.0598\ °C/W}$$

(*iii*) The value of equivalent convection coefficient, hr :

$$Q = h_r A\,(t_1 - t_2)$$

or, $$h_r = \frac{Q}{A\,(t_1 - t_2)} = \frac{4343}{1.5\,(300 - 40)} = \mathbf{11.13\ W/m^2\ °C}$$

[Alternatively, $$h_r = F\sigma\,(T_1 + T_2)\,(T_1^2 + T_2^2)$$ ...From eqn. (1.10)

$$= 0.52 \times 5.67 \times 10^{-8}\,(573 + 313)\,(573^2 + 313^2)$$

$$= \mathbf{11.13\ W/m^2\ °C}$$]

Example 1.7. *A carbon steel plate (thermal conductivity = 45 W/m°C) 600 mm × 900 mm × 25 mm is maintained at 310°C. Air at 15°C blows over the hot plate. If convection heat transfer coefficient is 22 W/m² °C and 250 W is lost from the plate surface by radiation, calculate the inside plate temperature.*

Solution. Area of the plate exposed to heat transfer,

$$A = 600\ \text{mm} \times 900\ \text{mm} = 0.6 \times 0.9 = 0.54\ \text{m}^2$$

Thickness of the plate, $L = 25\ \text{mm} = 0.025\ \text{m}$

Surface temperature of the plate, $t_s = 310°\text{C}$

Temperature of air (fluid), $t_f = 15°\text{C}$

Convective heat transfer coefficient,

$$h = 22\ \text{W/m}^2\,°\text{C}$$

Heat lost from the plate surface by radiation,

$$Q_{rad.} = 250\text{W}$$

Thermal conductivity, $k = 45\ \text{W/m °C}$

Inside plate temperature, t_i :

In this case the heat conducted through the plate is removed from the plate surface by a *combination of convection and radiation.*

Heat conducted through the plate = Convection heat losses + radiation heat losses.

or, $$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$-kA\,\frac{dt}{dx} = hA\,(t_s - t_f) + F\sigma A\,(T_s^4 - T_f^4)$$

or, $$-45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54\,(310 - 15) + 250\ \text{(given)}$$

or, $$-45 \times 0.54 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

or, $$972\,(t_i - 310) = 3754.6$$

or, $$t_i = \frac{3754.6}{972} + 310 = \mathbf{313.86°C}$$

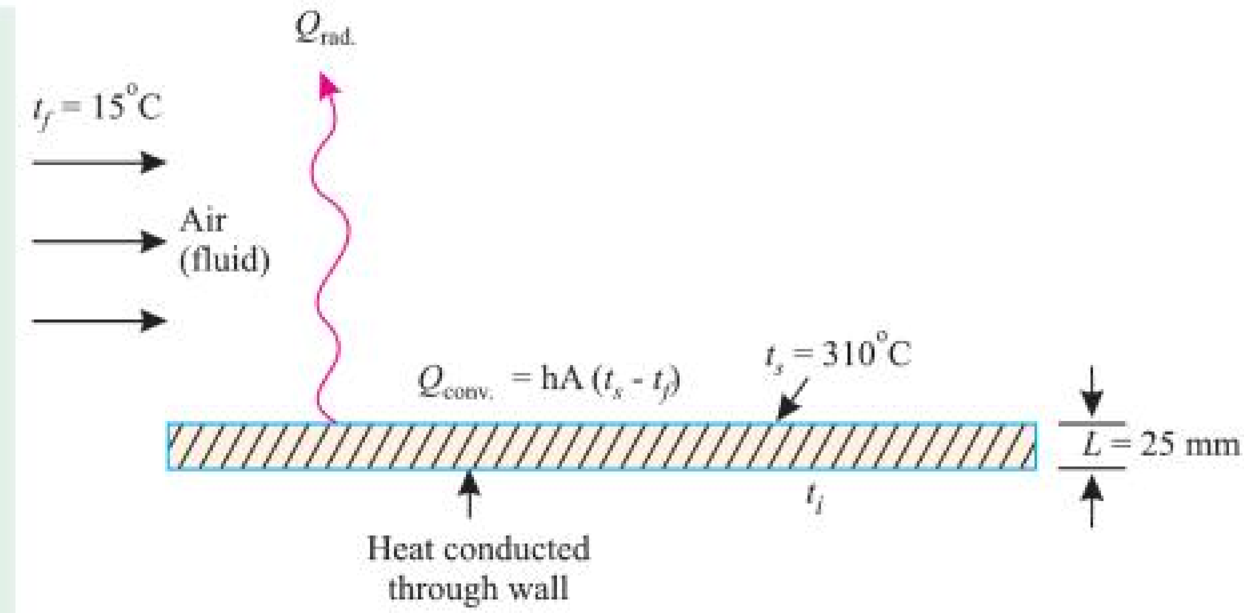


Fig. 1.11. Combination of conduction, convection and radiation heat transfer.

Example 1.8. *A surface at 250°C exposed to the surroundings at 110°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 75W/m²°C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 10W/m°C, what is the temperature gradient at the surface in the solid ?*

Solution. Temperature of the surface, $t_s = 250°C$

Temperature of the surroundings, $t_{sur} = 110°C$

The convection co-efficient, $h = 5 W/m^2°C$

Radiation factor, $F = 1$

Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

Conductivity of the solid, $k = 10 W/m°C$

Temperature gradient, $\frac{dt}{dx}$:

Heat conducted through the plate = Convection heat losses + radiation heat losses

i.e., $Q_{cond.} = Q_{conv.} + Q_{rad.} \quad -kA\frac{dt}{dx} = hA(t_s - t_{sur}) + F\sigma A(T_s^4 - T_{sur}^4)$

Substituting the values, we have

$$-10 \times \frac{dt}{dx} = 75(250 - 110) + 1 \times 5.67 \times 10^{-8}[(250 + 273)^4 - (110 + 273)^4]$$

$$-10 \times \frac{dt}{dx} = 10500 + 5.67\left[\left(\frac{523}{100}\right)^4 - \left(\frac{383}{100}\right)^4\right]$$

$$= 10500 + 3022.1 = 13522.1$$

$$\therefore \quad \frac{dt}{dx} = -\frac{13522.1}{10} = \mathbf{-1352.21\ °C/m}$$

HIGHLIGHTS

1. The energy in transit is termed *heat*.
2. *Heat transfer* may be defined as "T*he transmission of energy from one region to another as a result of temperature gradient*."
3. The study of heat transfer is carried out for the following *purposes* :
   (*i*) To estimate the rate of flow of energy as heat through the boundary of a system under study (both steady and transient conditions).
   (*ii*) To determine the temperature field under steady and transient conditions.

4. *Thermodynamics* is an axiomatic science which deals with the relations among heat, work and properties of system which are in *equilibrium*. It describes state and changes in state of physical systems. It basically entails four laws or axioms known as Zeroth, First, Second and third law of thermodynamics.
5. Heat transfer theory combines thermodynamics and rate equations together (to quantify the rate at which heat transfer occurs in terms of the degree of non-equilibrium).
6. Basic laws which govern the heat transfer are :
   (*i*) First laws of thermodynamics
   (*ii*) Second law of thermodynamics
   (*iii*) Law of conservation of mass
   (*iv*) Newton's laws of motion
   (*v*) The rate equations.
7. Heat transfer takes place by the following three modes :
   (*i*) Conduction (*ii*) Convection (*iii*) Radiation.

   '*Conduction*' is the transfer of heat from one part of a substance to another part of the same substance, or from one substance to another in physical contact with it, without appreciable displacement of molecules forming the substance.

   '*Convection*' is the transfer of heat within a fluid by mixing of one portion of the fluid with another. Convection is possible only in a fluid medium and is directly linked with the transport of medium itself.

   '*Radiation*' is the transfer of heat through space or matter by means other than conduction or convection. Radiant energy (being electromagnetic radiation) requires no medium for propagation and will pass through a vaccum.
8. *Fourier's law of heat conduction states* : "The rate of flow of heat through a single homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of path of the heat flow".

   Mathematically, $$Q = -kA\frac{dt}{dx}$$

   where, Q = Heat flow through a body per unit time, W,
   A = Surface area of heat flow (perpendicular to the direction of flow), m^2,
   dt = Temperature difference of the faces of the block,
   dx = Thickness of the body, and
   k = Thermal conductivity of the body.

   The –ve sign of k is to take care of the decreasing temperature along with the direction of increasing thickness or the direction of heat flow.
9. The *thermal conductivity* (k) of a material is defined as : "The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference."
10. The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed Newton's law of cooling :

   $$Q = hA\,(t_s - t_f)$$

   where, Q = Rate of convective heat transfer,
   A = Area exposed to heat transfer,
   t_s = Surface temperature,
   t_f = Fluid temperature, and
   h = Coefficient of convective heat transfer.

   Units of h : $W/m^2{}^\circ C$ or W/m^2K
11. The *Stefan-Boltzmann law states* : "The emissive power of a black body is directly proportional to fourth power of its absolute temperature."

12. *Thermal resistance (R_{th}) :*

Conduction thermal resistance, $(R_{th})_{cond.} = \dfrac{dx}{kA}$

Convection thermal resistance, $(R_{th})_{conv.} = \dfrac{1}{hA}$

Radiation thermal resistance, $(R_{th})_{rad.} = \dfrac{1}{1/[F\sigma A\,(T_1 + T_2)\,(T_1^2 + T_2^2)]}$

THEORETICAL QUESTIONS

1. Define the following terms :
   (*i*) Heat (*ii*) Heat transfer (*iii*) Thermodynamics.
2. What is the difference between thermodynamics and heat transfer ?
3. Enumerate the basic laws which govern the heat transfer.
4. Name and explain briefly the various modes of heat transfer.
5. What is conduction heat transfer ? How does it differ from convective heat transfer ?
6. What is the significance of heat transfer ?
7. Enumerate some important areas which are covered under the discipline of heat transfer.
8. What is the difference between the 'natural' and 'forced' convection ?
9. What is 'Fourier's law of conduction'? State also the assumptions on which this law is based.
10. State some essential features of Fourier's law.
11. How is thermal conductivity of a material defined ? What are its units ?
12. What is thermal resistance ?
13. What is 'Newton's law of cooling ?
14. What is Stefan's Boltzmann law ?

UNSOLVED EXAMPLES

1. The inner surface of a plane brick wall is at 40°C and the outer surface is at 20°C. Calculate the rate of heat transfer per m^2 of surface area of the wall, which is 250 mm thick. The thermal conductivity of the brick is 0.52 W/m°C. (**Ans.** 41.6 W/m^2)
2. A plane wall (thermal conductivity = 10.2 W/m°C) of 100 mm thickness and area $3m^2$ has steady surface temperature of 170°C and 100°C. Determine :
   (*i*) The rate of heat flow across the plane wall;
   (*ii*) The temperature gradient in the flow direction. [**Ans.** (*i*) 21.42 kW; (*ii*) – 700°C/m]
3. Determine the heat transfer by convection over a surface of 0.75 m^2 if the surface is at 200°C and the fluid is at 80°C. The value of convective heat transfer is 25 W/m^2 °C. [**Ans.** 2.25 kW]
4. A surface of area $3m^2$ and at 200°C exchanges heat with another surface at 30°C by radiation. If the value of factor due to the geometric location and emissivity is 0.69, determine :
   (*i*) The rate of heat transfer,
   (*ii*) The value of thermal resistance, and
   (*iii*) The equivalent convection coefficient. [**Ans.** (*i*) 4885.6W; (*ii*) 0.0348 °C/W; (*iii*) 9.58 W/m^2 °C]
5. A surface at 200°C exposed to the surroundings at 60°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 80W/m^2 °C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 15W/m °C what is the temperature gradient at the surface ? (**Ans.** – 889.4 °C/m)

PART I

HEAT TRANSFER BY CONDUCTION

CHAPTER

Conduction–Steady–State One Dimension

2

2.1. INTRODUCTION

In this chapter an attempt will be made to derive general heat conduction equation and examine the applications of Fourier's law of heat conduction to the calculation of heat flow in some simple one-dimensional systems. Under the category of one-dimensional systems several different physical shapes may fall; *when the temperature of the body is a function only of radial distance and is independent of azimuth angle or axial distance cylindrical and spherical systems are treated as one-dimensional*. In case of problems of two-dimensional nature the effect of a second-space coordinate may be so small that it may be neglected and the heat-flow problems of multi-dimensional type may be approximated with a one-dimensional analysis; in such cases the differential equations are simplified and as a consequence of this simplification much easier solution in available.

2.2. GENERAL HEAT CONDUCTION EQUATION IN CARTESIAN COORDINATES

Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 2.1.

Let, t = Temperature at the left face $ABCD$; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small*, and

$\frac{dt}{dx}$ = Temperature changes and rate of change along X-direction.

Then, $\left(\frac{\partial t}{\partial x}\right)dx$ = Change of temperature through distance dx, and

$t + \left(\frac{\partial t}{\partial x}\right)dx$ = temperature on the right face $EFGH$ (at a distance dx from the left face $ABCD$).

Further, let, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along X, Y and Z axes.

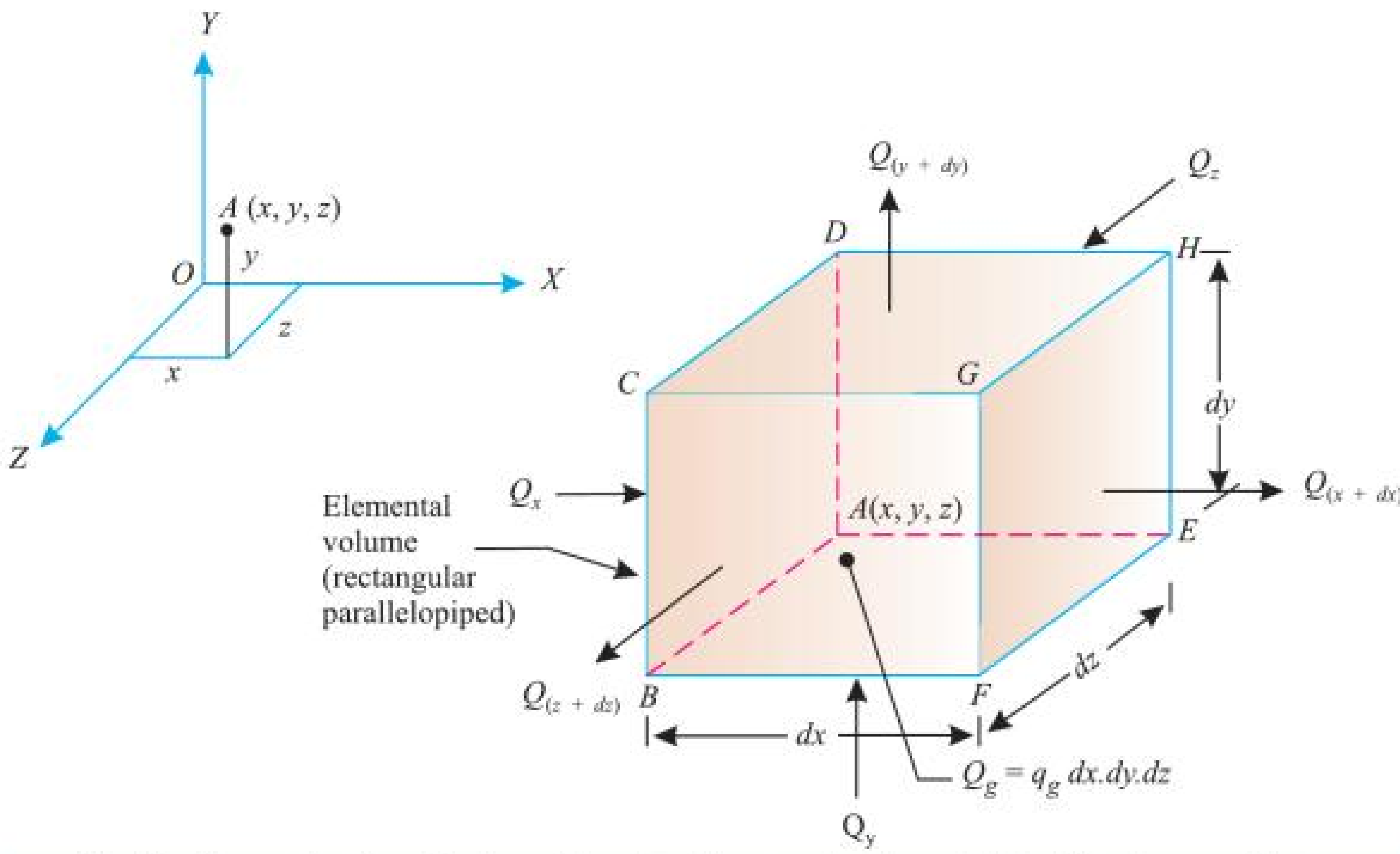


Fig. 2.1. Elemental volume for three-dimensional heat conduction analysis - Cartesian coordinates.

[If the directional characterisitics of a material are equal/same, it is called an "*Isotropic material*" and if unequal/different "*Anisotropic material*".]

q_g = Heat generated per unit volume per unit time.

[Inside the control volume there may be heat sources due to flow of electric current in electric motors and generators, nuclear fission etc.]

(**Note :** q_g may be function of position or time, or both).

ρ = Mass density of material, and

c = Specific heat of the material.

Energy balance/equation for volume element :

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (A) + heat generated within the element (B) = Energy stored in the element (C). ...(1)

Let, Q = Rate of heat flow in a direction, and

$Q' = (Q.d\tau)$ = Total heat flow (flux) in that direction (in time $d\tau$).

A. *Net heat accumulated in the element due to conduction of heat from all the directions considered*:

Quantity of heat flowing into the element from the left face $ABCD$ during the time interval $d\tau$ in X-direction is given by :

Heat influx, $$Q'_x = -\,k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\cdot d\tau \qquad ...(i)$$

During the same time interval $d\tau$ the heat flowing out of the right face of control volume ($EFGH$) will be :

Heat efflux, $$Q'_{(x+dx)} = Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx \qquad ...(ii)$$

∴ Heat accumulation in the element due to heat flow in X-direction,

$$dQ'_x = Q'_x - \left[Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx\right] \qquad \text{[Subtracting (ii) from (i)]}$$

$$= -\frac{\partial}{\partial x}(Q'_x)\,dx$$

$$= -\frac{\partial}{\partial x}\left[-k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\,.\,d\tau\right]dx$$

$$= \frac{\partial}{\partial x}\left[k_x\,\frac{\partial t}{\partial x}\right]dx.dy.dz.d\tau \qquad ...(2.1)$$

Similarly the heat accumulated due to heat flow by conduction along Y and Z directions in time $d\tau$ will be :

$$dQ'_y = \frac{\partial}{\partial y}\left[k_y\,\frac{\partial t}{\partial y}\right]dx.dy.dz.d\tau \qquad ...(2.2)$$

$$dQ'_z = \frac{\partial}{\partial z}\left[k_z\,\frac{\partial t}{\partial z}\right]dx.dy.dz.d\tau \qquad ...(2.3)$$

Boilers in a plant. A good boiler should efficiently take the heat from the fuel and minimise loss of heat from inside to outside.

∴ *Net heat accumulated* in the element due to conduction of heat from all the coordinate directions considered

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau \qquad ...(2.4)$$

B. *Total heat generated within the element* (Q_g') :

The total heat generated in the element is given by

$$Q_g' = q_g\,(dx.dy.dz)\;d\tau \qquad ...(2.5)$$

C. *Energy stored in the element* :

The total heat accumulated in the element due to heat flow along coordinate axes (Eqn. 2.4) and the heat generated within the element (Eqn. 2.5) together serve to increase the thermal energy of the element/lattice. This increase in thermal energy is given by

$$\rho\,(dx.dy.dz)\,c.\frac{\partial t}{\partial \tau}\,.\,d\tau \qquad ...(2.6)$$

[∵ Heat stored in the body = Mass of the body × specific heat of the body material × rise in the temperature of body].

Now, substituting eqns. (2.4), (2.5), (2.6), in the eqn. (1), we have

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau + q_g\;(dx.dy.dz.)d\tau = \rho\,(dx.dy.dz)\;c.\frac{\partial t}{\partial \tau}\,.\,d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \qquad ...(2.7)$$

or, using the vector operator ∇, we get

$$\nabla\,.(k\nabla t) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \qquad ...[2.7\,(a)]$$

This is known as the **general heat conduction equation for 'non-homogeneous material', 'self heat generating'** and **'unsteady three-dimensional heat flow'**. This equation establishes in *differential form the relationship between the time and space variation of temperature at any point of solid through which heat flow by conduction takes place.*

General heat conduction equation for constant thermal conductivity :

In case of homogeneous (in which properties *e.g.*, specific heat, density, thermal conductivity etc. are same everywhere in the material) and isotropic (in which properties are independent of surface orientation) material, $k_x = k_y = k_z = k$ and diffusion equation Eqn. (2.7) becomes

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho.c}{k}\cdot\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau} \qquad ...(2.8)$$

where, $$\alpha = \frac{k}{\rho.c} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity}}$$

The quantity, $\alpha = \dfrac{k}{\rho.c}$ is known as **thermal diffusivity**.

– The larger the value of α, the faster will the heat diffuse through the material and its temperature will change with time. This will result either due to a high value of thermal conductivity k or a low value of heat capacity $\rho.c$. A low value of heat capacity means the less amount of heat entering the element, would be absorbed and used to raise its temperature and more would be available for onward transmission. Metals and gases have relatively high value of α and their response to temperature changes is quite rapid. The non-metallic solids and liquids respond slowly to temperature changes because of their relatively small value of thermal diffusivity.

– Thermal diffusivity is an important characteristic quantity for *unsteady conduction situations*.

Eqn. (2.8) by using Laplacian ∇^2, may be written as :

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...[2.8 (a)]}$$

Eqn. (2.8), governs the temperature distribution under unsteady heat flow through a material which is homogeneous and isotropic.

Other simplified forms of heat conduction equation in cartesian coordinates :

(*i*) For the case when *no internal source of heat generation is present*, Eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

[Unsteady state $\left(\frac{\partial t}{\partial \tau} \neq 0\right)$ heat flow with no internal heat generation]

or,

$$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{(Fourier's equation)} \quad \text{...(2.9)}$$

(*ii*) Under the situations when temperature does not depend on time, the conduction then takes place in the steady state $\left(i.e., \frac{\partial t}{\partial \tau} = 0\right)$ and the eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or,

$$\nabla^2 t + \frac{q_g}{k} = 0 \quad \text{(Poisson's equation)} \quad \text{...(2.10)}$$

In the absence of internal heat generation, Eqn. (2.10) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or,

$$\nabla^2 t = 0 \quad \text{(Laplace equation)} \quad \text{...(2.11)}$$

(*iii*) *Steady state and one-dimensional heat transfer*:

$$\frac{\partial^2 t}{\partial x^2} + \frac{q_g}{k} = 0 \quad \text{...(2.12)}$$

(*iv*) *Steady state, one-dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = 0 \quad \text{...(2.13)}$$

(*v*) *Steady state, two dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \quad \text{...(2.14)}$$

(*vi*) *Unsteady state, one dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...(2.15)}$$

2.3. GENERAL HEAT CONDUCTION EQUATION IN CYLINDRICAL COORDINATES

While dealing with problems of conduction of heat through systems having cylindrical geometries (*e.g.*, rods and pipes) it is convenient to use cylindrical coordinates.

Consider an elemental volume having the coordinates (r, ϕ, z), for three-dimensional heat conduction analysis, as shown in Fig. 2.2.

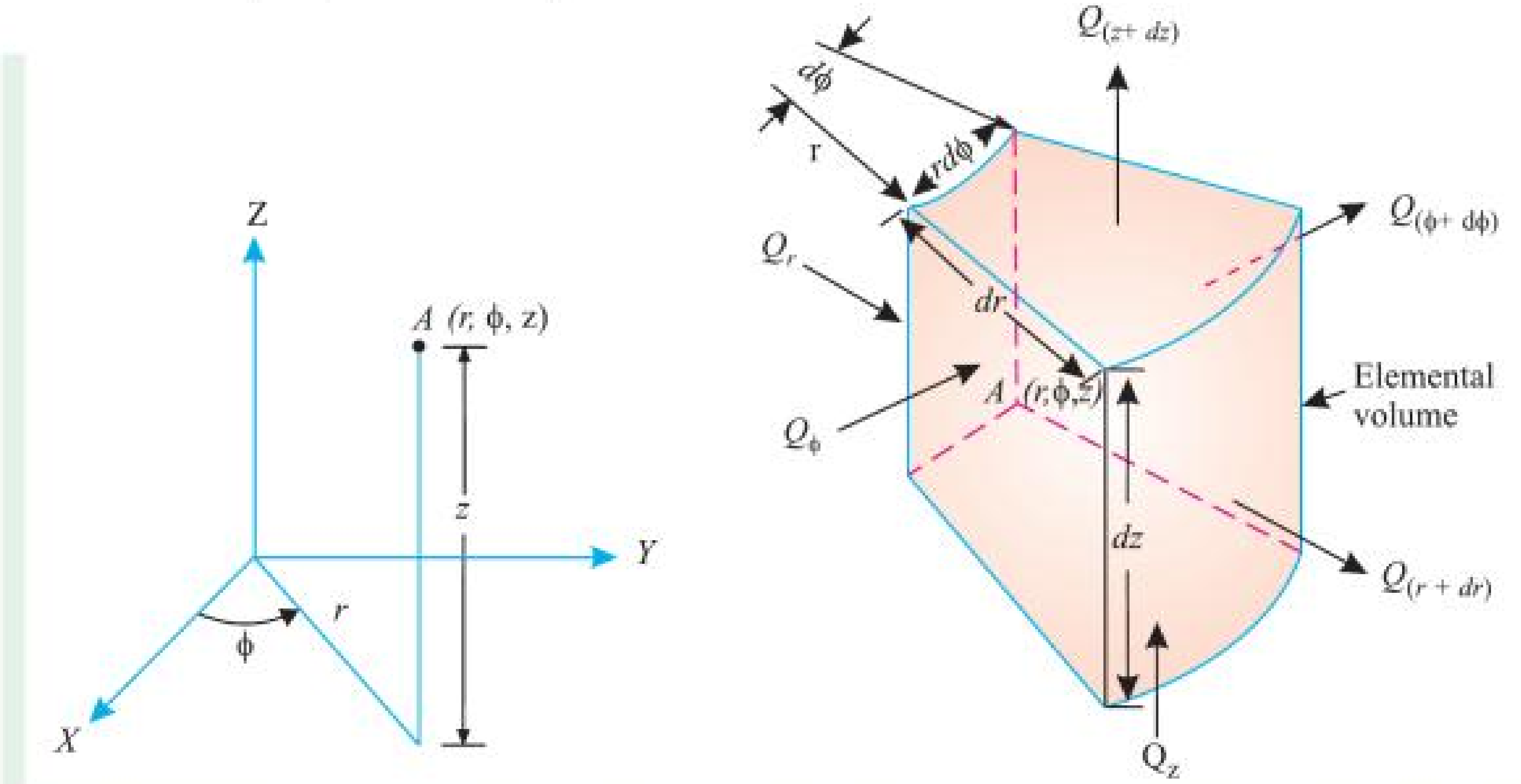


Fig. 2.2. Elemental volume for three-dimensional heat conduction analysis - Cylindrical coordinates.

The volume of the element = $rd\phi.dr.dz$

Let, q_g = Heat generation (uniform) per unit volume per unit time.

Further, let us assume that k (thermal conductivity), ρ (density), c (specific heat) do not alter with position.

A. *Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered* :

Heat flow in radial direction (x–ϕ) plane :

Heat influx, $$Q'_r = -k\ (rd\phi.dz)\ \frac{\partial t}{\partial r}\ .\ d\tau \qquad ...(i)$$

Heat efflux, $$Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}(Q_r)\,dr \qquad ...(ii)$$

$\therefore$ Heat accumulation in the element due to heat flow in *radial direction*,

$$dQ'_r = Q'_r - Q'_{(r+dr)} \qquad \text{[subtracting (ii) from (i)]}$$

$$= -\frac{\partial}{\partial r}(Q'_r)\,dr$$

$$= -\frac{\partial}{\partial r}\left[-k\ (rd\phi.dz)\ \frac{\partial t}{\partial r}\ .\ d\tau\right]dr$$

$$= k\ (dr.d\phi.dz)\ \frac{\partial}{\partial r}\left(r\cdot\frac{\partial t}{\partial r}\right)d\tau$$

$$= k\ (dr.d\phi.dz)\left(r\,\frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r}\right)d\tau$$

$$= k\ (dr.rd\phi.dz)\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right] d\tau \qquad ...(2.16)$$

Heat flow in *tangential direction* (*r–z*) *plane* :

Heat influx, $$Q'_\phi = -\ k\ (dr.dz)\ \frac{\partial t}{r.\partial\phi}\ d\tau \qquad ...(iii)$$

Heat efflux, $$Q'_{(\phi + d\phi)} = Q'_\phi + \frac{\partial}{r.\partial\phi}(Q'_\phi)\ rd\phi \qquad ...(iv)$$

Heat accumulated in the element due to heat flow in *tangential direction*,

$$dQ'_\phi = Q'_\phi - Q'_{(\phi + d\phi)} \qquad \text{[subtracting (iv) from (iii)]}$$

$$= -\frac{\partial}{r.\partial\phi}(Q_\phi)\ r.d\phi$$

$$= -\frac{\partial}{r.\partial\phi}\left[-\ k\ (dr.dz)\ \frac{\partial t}{r.\partial\phi}.\ d\tau\right] r.d\phi$$

$$= k\ (dr.d\phi.dz)\ \frac{\partial}{\partial\phi}\left(\frac{1}{r}\cdot\frac{\partial t}{\partial\phi}\right) d\tau$$

$$= k\ (dr.rd\phi.dz)\ \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2}.\ d\tau$$

Piston assembly. The fins around the cylinder are meant to spread the heat and speed-up cooling.

Heat flow in *axial direction* (*r*-ϕ *plane*) :

Heat influx, $$Q'_z = -\ k\ (r.d\phi.dr)\ \frac{\partial t}{\partial z}\ d\tau \qquad ...(v)$$

Heat efflux, $$Q'_{(z + dz)} = Q'_z + \frac{\partial}{\partial z}(Q'_z)\ dz \qquad ...(vi)$$

Heat accumulated in the element due to heat flow in *axial direction*,

$$dQ'_z = Q'_z - Q'_{(z + dz)} \qquad \text{[subtracting (vi) from (v)]}$$

$$= -\frac{\partial}{\partial z}\left[-\ k\ (r.d\phi.dr)\ \frac{\partial t}{\partial z}.\ d\tau\right] dz$$

$$= k\ (dr.rd\phi.dz)\ \frac{\partial^2 t}{\partial z^2}.\ d\tau \qquad ...(2.18)$$

Net heat accumulated in the element

$$= k.dr.rd\phi.dz\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau \qquad ...(2.19)$$

B. *Heat generated within the element* (Q'_g) :

The total heat generated within the element is given by

$$Q'_g = q_g\ (dr.rd\phi.dz).d\tau \qquad ...(2.20)$$

C. *Energy stored in the element* :

The increase in thermal energy in the element is equal to

$$= \rho\,(dr.rd\phi.dz).c.\frac{\partial t}{\partial\tau}.\ d\tau \qquad ...(2.21)$$

Now, $(A) + (B) = (C)$ ... Energy balance/equation

$$\therefore \quad k.dr.rd\phi.dz\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau + q_g\ (dr.rd\phi.dz).d\tau$$

$$= \rho\,(dr.rd\phi.dz).c.\frac{\partial t}{\partial\tau}.\ d\tau$$

Dividing both sides by $dr.rd\phi.dz.d\tau$, we have

$$k\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2}+\frac{\partial^2 t}{\partial z^2}\right]+q_g=\rho.c.\frac{\partial t}{\partial \tau}$$

or,
$$\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2}+\frac{\partial^2 t}{\partial z^2}\right]+\frac{q_g}{k}=\frac{\rho c}{k}\cdot\frac{\partial t}{\partial \tau}=\frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau} \quad ...(2.22)$$

Equation (2.22) is the **general heat conduction equation in cylindrical coordinates**.

In case there are no *heat sources present* and the heat flow is *steady* and *one-dimensional*, then eqn. (2.22) reduces to

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}=0 \quad ...(2.23)$$

or,
$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{dt}{dr}=0$$

or,
$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right)=0$$

Since $\frac{1}{r}\neq 0$, therefore,

$$\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right) \text{ or } r\cdot\frac{dt}{dr}=\text{constant} \quad ...(2.24)$$

Equation (2.22) can also be derived by transformation of coordinates, as follows :

$$x=r\cos\phi,\ y=r\sin\phi \text{ and } z=z$$

Now, by chain rule :

$$\frac{\partial t}{\partial r}=\frac{\partial t}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial t}{\partial y}\cdot\frac{\partial y}{\partial r}=\frac{\partial t}{\partial x}\cos\phi+\frac{\partial t}{\partial y}\sin\phi$$

or,
$$\cos\phi\,\frac{\partial t}{\partial r}=\cos^2\phi\cdot\frac{\partial t}{\partial x}+\sin\phi\,.\cos\phi\,.\frac{\partial t}{\partial y} \quad ...(i)$$

(Multiplying both sides by $\cos\phi$)

Also,
$$\frac{\partial t}{\partial \phi}=\frac{\partial t}{\partial x}\cdot\frac{\partial x}{\partial \phi}+\frac{\partial t}{\partial y}\cdot\frac{\partial y}{\partial \phi}=\frac{\partial t}{\partial x}(-r\sin\phi)+\frac{\partial t}{\partial y}(r\cos\phi)$$

or,
$$\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}=-\sin^2\phi\,\frac{\partial t}{\partial x}+\sin\phi\,.\cos\phi\,.\frac{\partial t}{\partial y} \quad ...(ii)$$

(Multiplying both sides by $\frac{\sin\phi}{r}$)

From Eqns. (*i*) and (*ii*), we have

$$\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}=-\sin^2\phi\,\frac{\partial t}{\partial x}+\left[\cos\phi\,.\frac{\partial t}{\partial r}-\cos^2\phi\frac{\partial t}{\partial x}\right]$$

$$=-\frac{\partial t}{\partial x}+\cos\phi\,\frac{\partial t}{\partial r}$$

∴
$$\frac{\partial t}{\partial x}=\cos\phi\,\frac{\partial t}{\partial r}-\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi} \quad ...(iii)$$

Differentiating both sides with respect to x, we have

$$\frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)=\frac{\partial}{\partial x}\left[\cos\phi\cdot\frac{\partial t}{\partial r}-\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}\right]$$

or, $$\frac{\partial^2 t}{\partial x^2} = \cos\phi.\frac{\partial}{\partial r}\left(\frac{\partial t}{\partial x}\right) - \frac{\sin\phi}{r}\cdot\frac{\partial}{\partial\phi}\left(\frac{\partial t}{\partial x}\right)$$

$$= \cos\phi.\frac{\partial}{\partial r}\left(\cos\phi.\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial\phi}\right) - \frac{\sin\phi}{r}\cdot\frac{\partial}{\partial\phi}\left(\cos\phi\cdot\frac{\partial t}{\partial r} - \frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial\phi}\right)$$

[Substituting the value of $\frac{\partial t}{\partial x}$ from (*iii*)]

$$= \cos^2\phi.\frac{\partial^2 t}{\partial r^2} - \frac{\cos\phi\,.\sin\phi}{r^2}\cdot\frac{\partial t}{\partial\phi} + \frac{\sin^2\phi}{r}\cdot\frac{\partial t}{\partial r} + \frac{\sin^2\phi}{r^2}\cdot\frac{\partial^2 t}{d\phi^2} + \frac{\sin\phi.\cos\phi}{r^2}\cdot\frac{\partial t}{\partial\phi} \quad ...(iv)$$

Similarly, $$\frac{\partial^2 t}{\partial y^2} = \sin^2\phi.\frac{\partial^2 t}{\partial r^2} + \frac{\cos^2\phi}{r}\cdot\frac{\partial t}{\partial r} - \frac{\cos\phi\,.\sin\phi}{r^2}\cdot\frac{\partial t}{\partial\phi} + \frac{\cos^2\phi}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} - \frac{\cos\phi.\sin\phi}{r^2}\cdot\frac{\partial t}{\partial\phi} \quad ...(v)$$

By adding (*iii*) and (*iv*), we get

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2}$$

Substituting it in eqn (2.8), we get,

$$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial\tau}$$

which is the same as eqn. (2.22)

2.4. GENERAL HEAT CONDUCTION EQUATION IN SPHERICAL COORDINATES

Consider an elemental volume having the coordinates (r, ϕ, θ), for three dimensional heat conduction analysis, as shown in Fig. 2.3.

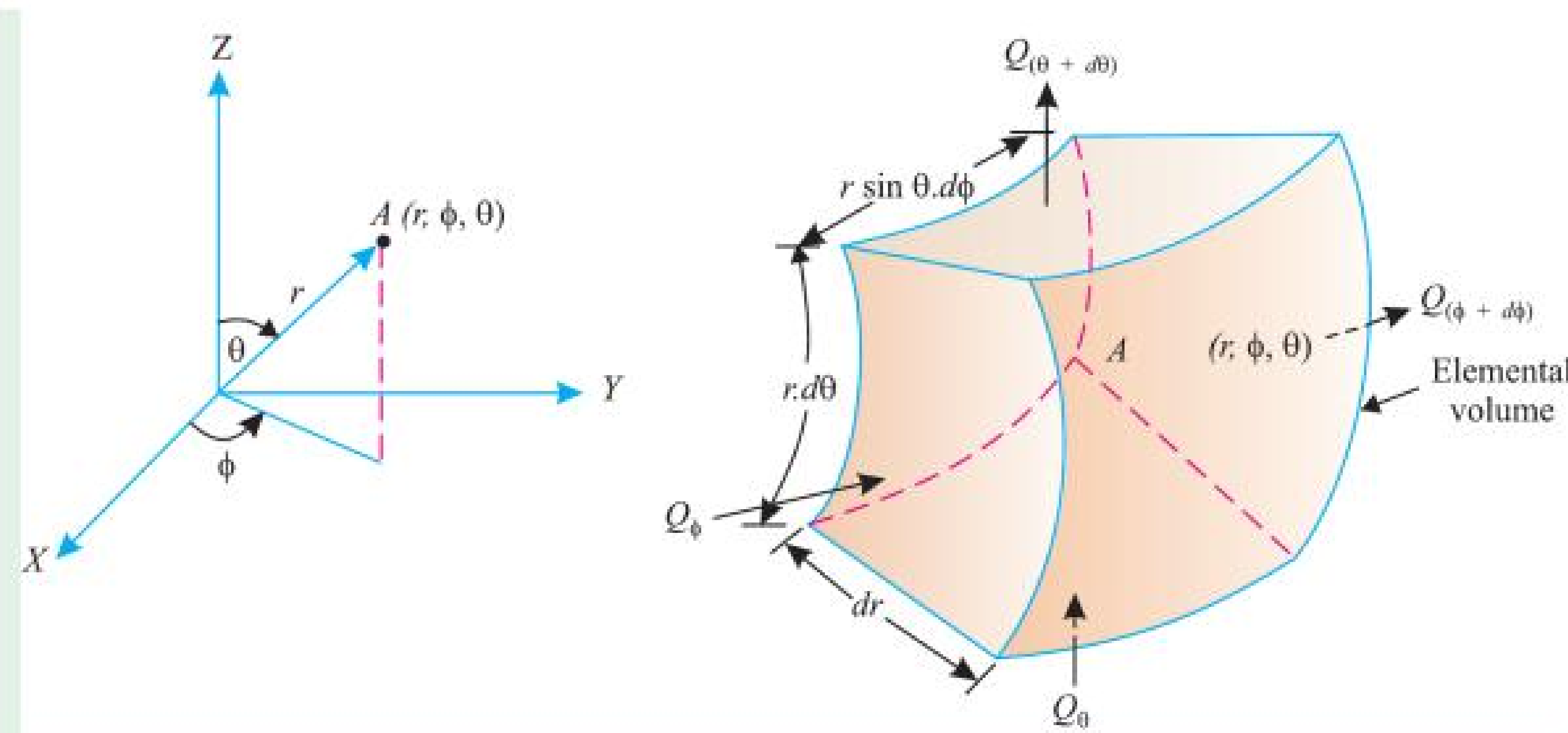


Fig. 2.3. Elemental volume for three-dimensional heat conduction analysis - Spherical coordinates.

The volume of the element = $dr.rd\theta.r\sin\theta\,d\phi$

Let, q_g = Heat generation (uniform) per unit volume per unit time.

Further let us assume that k (thermal conductivity), ρ (density), c (specific heat) do not alter with position.

A. *Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered* :

Heat flow through *r–θ plane; φ-direction* :

Heat influx, $$Q'_\phi = -k\,(dr.rd\theta)\,\frac{\partial t}{r.\sin\theta.\partial\phi}\,d\tau \qquad ...(i)$$

Heat efflux, $$Q'_{(\phi+d\phi)} = Q'_\phi + \frac{\partial}{r.\sin\theta.\partial\phi}(Q'_\phi)\;r\,\sin\theta.d\phi \qquad ...(ii)$$

∴ Heat accumulated in the element due to heat flow in the φ-direction,

$$dQ'_\phi = Q'_\phi - Q'_{(\phi+d\phi)} \qquad \text{[subtracting (ii) from (i)]}$$

$$= -\frac{1}{r\,\sin\theta}\cdot\frac{\partial}{\partial\phi}(Q'_\phi)\;r\,\sin\theta.d\phi$$

$$= -\frac{1}{r\,\sin\theta}\cdot\frac{\partial}{\partial\phi}\left[-k\,(dr.rd\theta)\,\frac{1}{r\,\sin\theta}\cdot\frac{\partial t}{\partial\phi}.\,d\tau\right]r\,\sin\theta.d\phi$$

$$= k\,(dr.rd\theta.r\,\sin\theta.d\phi)\,\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 t}{\partial\phi^2}\,d\tau \qquad ...(2.25)$$

Heat flow in *r–φ plane, θ-direction* :

Heat influx, $$Q'_\theta = -\,k\,(dr.\,r\,\sin\,\theta.\,d\phi)\,\frac{\partial t}{r\partial\theta}\cdot d\tau \qquad ...(iii)$$

Heat efflux, $$Q'_{(\theta+d\theta)} = Q'_\theta + \frac{\partial}{r\partial\theta}(Q'_\theta)\;rd\theta \qquad ...(iv)$$

∴ Heat accumulated in the element due to heat flow in the θ-direction,

$$dQ'_\theta = Q'_\theta - Q'_{(\theta+d\theta)} \qquad \text{[subtracting (iv) from (iii)]}$$

$$= -\frac{\partial}{r.\partial\theta}(Q'_\theta)\;r.d\theta$$

$$= -\frac{\partial}{r.\partial\theta}\left[-\,k\,(dr.r\,\sin\theta.d\phi)\,\frac{\partial t}{r.\partial\theta}.\,d\tau\right]r.d\theta$$

Spherical vessels.

$$= \frac{k}{r}\,\frac{dr.rd\phi.rd\theta}{r}\,\frac{\partial}{\partial\theta}\left[\sin\theta\cdot\frac{\partial t}{\partial\theta}\right]d\tau$$

$$= k\,(dr.rd\theta.r\sin\theta.d\phi)\,\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left[\sin\theta\cdot\frac{\partial t}{\partial\theta}\right]d\tau \qquad ...(2.26)$$

Heat flow in θ-ϕ *plane*, *r-direction* :

Heat influx, $$Q'_r = -\,k\,(rd\theta.r\sin\theta.d\phi)\,\frac{\partial t}{\partial r}\cdot\partial\tau \qquad ...(v)$$

Heat efflux, $$Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}(Q'_r)\,dr \qquad ...(vi)$$

∴ Heat accumulation in the element due to heat flow in the *r-direction*,

$$dQ'_r = Q'_r - Q'_{(r+dr)} \qquad \text{[subtracting } (vi) \text{ from } (v)]$$

$$= -\frac{\partial}{\partial r}(Q'_r)\,dr$$

$$= -\frac{\partial}{\partial r}\left[-\,k\,(rd\theta.r\sin\theta.d\phi)\,\frac{\partial t}{\partial r}\cdot d\tau\right]dr$$

$$= k\,d\theta.\sin\theta.d\phi\,dr\,\frac{\partial}{\partial r}\left[r^2\cdot\frac{\partial t}{\partial r}\right]d\tau$$

$$= k\,(dr.\,rd\theta.r\sin\theta.d\phi)\,\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left[r^2\cdot\frac{\partial t}{\partial r}\right]d\tau \qquad ...(2.27)$$

Net heat accumulated in the element

$$= k\,dr.rd\theta.r\sin\theta.d\phi\left[\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2.\frac{\partial t}{\partial r}\right)\right]d\tau \qquad ...(2.28)$$

B. *Heat generated within the element* (Q'_g) :

The total heat generated within the element is given by,

$$Q'_g = q_g\,(dr.rd\theta.r\sin\theta.d\phi)\,d\tau \qquad ...(2.29)$$

C. *Energy stored in the element* :

The increase in thermal energy in the element is equal to

$$\rho\,(dr.rd\theta\,.\,r\sin\theta.d\phi)\,c.\frac{\partial t}{\partial\tau}.\,d\tau \qquad ...(2.30)$$

Now, $(A) + (B) = (C)$ ...Energy balance/equation

$$\therefore\quad k\,dr.rd\theta.r\sin\theta.d\phi\left[\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta.\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2.\frac{\partial t}{\partial r}\right)\right].\,d\tau$$

$$+\,q_g\,(dr.rd\theta.r\sin\theta.d\phi)\,d\tau = \rho\,(dr.rd\theta.r\sin\theta.d\phi)\,c.\frac{\partial t}{\partial\tau}.d\tau$$

Dividing both sides by $k.(dr.rd\theta.\ r\sin\theta.d\phi)d\tau$, we get

$$\left[\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\sin\theta\,.\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\,.\frac{\partial t}{\partial r}\right)\right] + \frac{q_g}{k}$$

$$= \frac{\rho c}{k}\cdot\frac{\partial t}{\partial\tau} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial\tau} \qquad ...(2.31)$$

Equation (2.31) is the **general heat conduction equation in spherical coordinates**.

In case there are not *heat sources present* and the heat flow is *steady* and *one-dimensional*, then eqn. (2.31) reduces to

$$\frac{1}{r^2}\cdot\frac{d}{dr}\left(r^2\cdot\frac{dt}{dr}\right) = 0 \qquad ...(2.32)$$

Equation (2.31) can also be derived by transformation of coordinates as follows :

$$x = r \sin \theta \sin \phi \; ; \; y = r \sin \theta \cos \phi \; ; \; z = r \cos \theta$$

2.5. HEAT CONDUCTION THROUGH PLANE AND COMPOSITE WALLS

2.5.1. HEAT CONDUCTION THROUGH A PLANE WALL

Case I : Uniform thermal conductivity

Refer to Fig. 2.4 (*a*) Consider a plane wall of homogeneous material through which heat is flowing *only in x-direction*.

Let, L = Thickness of the plane wall,

A = Cross-sectional area of the wall,

k = Thermal conductivity of the wall material, and

t_1, t_2 = Temperatures maintained at the two faces 1 and 2 of the wall, respectively.

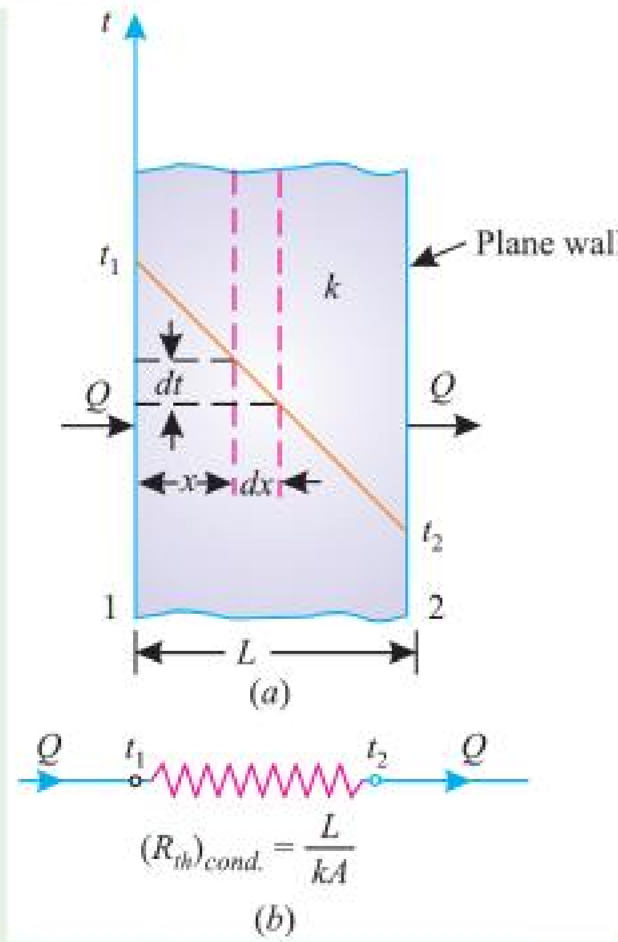


Fig. 2.4. Heat conduction through a plane wall.

The general heat conduction equation in cartesian coordinates is given by

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{...[Eqn. 2.8]}$$

If the heat conduction takes place under the conditions, steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, one-dimensional $\left[\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right]$ and with no internal heat generation $\left(\frac{q_g}{k} = 0\right)$ then the above equation is reduced to

$$\frac{\partial^2 t}{\partial x^2} = 0, \quad \text{or} \quad \frac{d^2 t}{dx^2} = 0 \qquad \text{...(2.33)}$$

By integrating the above differential twice, we have

$$\frac{dt}{dx} = C_1 \quad \text{and} \quad t = C_1 x + C_2 \qquad \text{...(2.34)}$$

where C_1 and C_2 are the arbitrary constants. The values of these constants may be calculated from the known boundary conditions as follows :

At $x = 0$ $\quad t = t_1$

At $x = L$ $\quad t = t_2$

Substituting the values in the eqn. (2.34), we get

$$t_1 = O + C_2 \quad \text{and} \quad t_2 = C_1 L + C_2$$

After simplification, we have, $C_2 = t_1$ and $C_1 = \frac{t_2 - t_1}{L}$

Thus, the eqn. (2.34) reduces to :

$$t = \left(\frac{t_2 - t_1}{L}\right) x + t_1 \qquad \text{...(2.35)}$$

The eqn. (2.35) indicates that *temperature distribution across a wall is linear* and is *independent of thermal conductivity*. Now heat through the plane wall can be found by using Fourier's equation as follows :

$$Q = -kA\frac{dt}{dx} \qquad \left(\text{where, } \frac{dt}{dx} = \text{Temperature gradient}\right) \quad \text{... [Eqn.(1.1)]}$$

But,
$$\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{L}\right)x + t_1\right] = \frac{t_2 - t_1}{L}$$

$\therefore$
$$Q = -kA\frac{(t_2 - t_1)}{L} = \frac{kA(t_1 - t_2)}{L} \quad \text{...(2.36)}$$

Eqn (2.36) can be written as :

$$Q = \frac{(t_1 - t_2)}{(L/kA)} = \frac{(t_1 - t_2)}{(R_{th})_{cond.}} \quad \text{...(2.37)}$$

where, $(R_{th})_{cond.}$ = Thermal resistance to heat conduction. Fig. 2.4 (*b*) shows the *equivalent thermal circuit* for heat flow through the plane wall.

Let us now find out the condition when instead of space, weight is the main criterion for selection of the insulation of a plane wall.

Thermal resistance (conduction) of the wall, $(R_{th})_{cond.} = \frac{L}{kA}$ ...(*i*)

Weight of the wall, $W = \rho A L$ ...(*ii*)

Eliminating L from (*i*) and (*ii*), we get

$$W = \rho A . (R_{th})_{cond.} kA = (\rho . k) A^2 . (R_{th})_{cond.} \quad \text{...(2.38)}$$

The eqn., (2.38) stipulates the condition that, for a specified thermal resistance, the *lightest insulation will be one which has the smallest product of density* (ρ) *and thermal conductivity* (k).

Case II. Variable thermal conductivity

A. *Temperature variation in terms of surface temperatures* (t_1, t_2) :

A diesel engine is more efficient due to internal combustion and better heat.

Let the thermal conductivity vary with temperature according to the relation

$$k = k_0 (1 + \beta t) \qquad ...(2.39)$$

[In most of the cases, the thermal conductivity is found to vary *linearly with temperature*]

where, k_0 = Thermal conductivity at zero temperature.

When the effect of temperature on thermal conductivity is considered, the Fourier's equation,

$$Q = -kA \frac{dt}{dx} \text{ is written as :}$$

$$Q = -k_0 (1 + \beta t) \frac{dt}{dx} . A \qquad ...(2.40)$$

or,
$$\frac{Q}{A} \cdot dx = -k_0 (1 + \beta t)\, dt$$

or,
$$\frac{Q}{A} \int_0^L dx = -k_0 \int_{t_1}^{t_2} (1 + \beta t)\, dt$$

or,
$$\frac{Q.L}{A} = -k_0 \left[t + \frac{\beta}{2} t^2 \right]_{t_1}^{t_2}$$

or,
$$\frac{Q.L}{A} = -k_0 \left[(t_2 - t_1) + \frac{\beta}{2} (t_2^2 - t_1^2) \right] \qquad ...(2.41)$$

$$= k_0 \left[(t_1 - t_2) + \frac{\beta}{2} (t_1 - t_2)(t_1 + t_2) \right]$$

$$= k_0 \left[1 + \frac{\beta}{2} (t_1 + t_2) \right] (t_1 - t_2)$$

$$= k_0 [1 + \beta\, t_m] (t_1 - t_2) \qquad \text{where } t_m = \frac{t_1 + t_2}{2}$$

$\therefore$
$$Q = k_0 (1 + \beta\, t_m). \frac{A (t_1 - t_2)}{L}$$

From eqn. (2.39) t is replaced by t_m, then

$$k_m = k_0 (1 + \beta t_m) \qquad ...(2.42)$$

$\therefore$
$$Q = k_m A \left[\frac{t_1 - t_2}{L} \right] \qquad ...(2.43)$$

where k_m is known as *mean thermal conductivity* of the wall material.

Further, if t is the temperature of the surface at a distance x from the left surface (Fig. 2.5), then eqn. (2.41) becomes

$$\frac{Qx}{A} = -k_0 \left[(t - t_1) + \frac{\beta}{2} (t^2 - t_1^2) \right] \qquad ...(2.44)$$

Form eqns. (2.41) and (2.44), we have

$$\left[(t_2 - t_1) + \frac{\beta}{2} (t_2^2 - t_1^2) \right] \frac{X}{L} = \left[(t - t_1) + \frac{\beta}{2} (t^2 - t_1^2) \right]$$

[Equating the values of Q and rearranging]

Solving the above equation for t, we get

$$t = \frac{1}{\beta} \left[(1 + \beta t_1)^2 - \{(1 + \beta t_1)^2 - (1 + \beta t_2)^2\} \frac{x}{L} \right]^{1/2} - \frac{1}{\beta} \qquad ...(2.45)$$

B. *Temperature variation in terms of heat flux* (Q) :

Fourier's equation for heat conduction is given by

$$Q = -\,kA.\frac{dt}{dx} = -\,k_0\ (1 + \beta t)\ A.\frac{dt}{dx}$$

or, $$Q.dx = -\,k_0\,(1 + \beta t)\,A.dt$$

Integrating both sides, we get

$$Q.x = -\,k_0 A\left(t + \frac{\beta}{2}t^2\right) + C \qquad ...(i)$$

(where, C = Constant of integration)

To evaluate C, applying the condition : At $x = 0$, $t = t_1$, we get

$$C = k_0 A\left(t_1 + \frac{\beta}{2}t_1^2\right)$$

Substituting the values of the constant C in (i), we get

$$Q.x = -\,k_0 A\left(t + \frac{\beta}{2}t^2\right) + k_0 A\left(t_1 + \frac{\beta}{2}\,t_1^2\right)$$

Dividing both sides by k_0A and rearranging, we obtain,

$$\frac{\beta}{2}\,t^2 + t + \left[\frac{Q.x}{k_0 A} - \left(t_1 + \frac{\beta}{2}\,t_1^2\right)\right] = 0$$

By solving the above quadratic equation, we have

$$\therefore \qquad t = \frac{-\,1 + \sqrt{1 - 4 \times \frac{\beta}{2}\left[\frac{Q.x}{k_0 A} - \left(t_1 + \frac{\beta}{2}\,t_1^2\right)\right]}}{2 \times \left(\frac{\beta}{2}\right)}$$

or, $$t = -\,\frac{1}{\beta} + \left[\frac{1}{\beta^2} - \frac{2}{\beta}\left(\frac{Q.x}{k_0 A} - t_1 - \frac{\beta}{2}\,t_1^2\right)\right]^{1/2}$$

$$= -\,\frac{1}{\beta} + \left[\frac{1}{\beta^2} - \frac{2}{\beta}\,t_1 + t_1^2 - \frac{2Q.x}{\beta k_0 A}\right]^{1/2}$$

$$= -\,\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{2Q.x}{\beta k_0 A}\right]^{1/2}$$

Hence,

$$t = -\,\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{2Q.x}{\beta k_0 A}\right]^{1/2} \qquad ...(2.46)$$

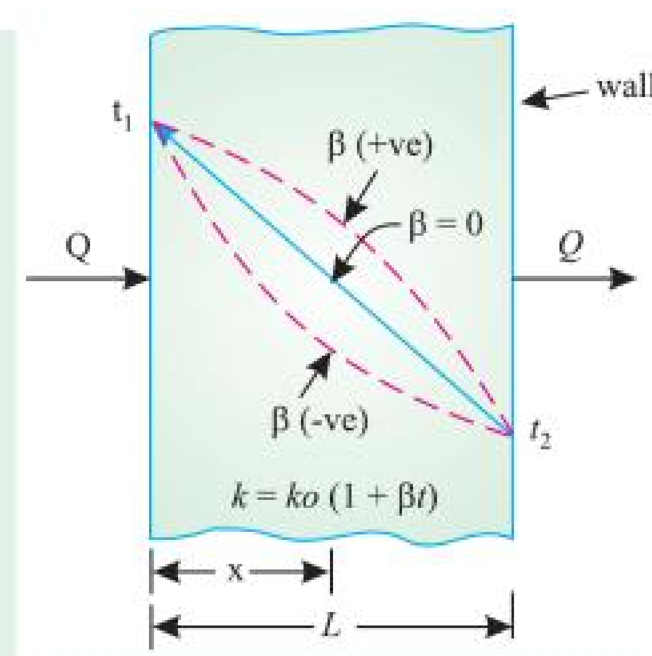


Fig. 2.5.

In most of the practical applications where the variation of temperature is small, the average value of k for the given temperature range is commonly used as given in eqn. (2.42).

If the variation of k with temperature is *not linear*, then

$$k = k_0 f(t), \text{ and}$$

$$\frac{Q}{A}\int_0^L dx = -\int_{t_1}^{t_2} [k_0\ f\,(t)\ dt$$

or, $$Q = \frac{A}{L}\left[-\int_{t_1}^{t_2} [k_0\ f\,(t)\ dt\right] \qquad ...(2.47)$$

But, $$Q = k_m A \left(\frac{t_1 - t_2}{L}\right) \qquad \text{...[Eqn. (2.43)]}$$

Equating these eqns. (2.47) and (2.43), we have

$$k_m = \frac{1}{(t_1 - t_2)}\left[\int_{t_1}^{t_2} [k_0 \, f(t) \, dt\right]$$

$$= \frac{1}{(t_1 - t_2)} \int_{t_2}^{t_1} [k_0 \, f(t) \, dt] \qquad \text{...(2.48)}$$

The effect of + β and – β on temperature is depicted in Fig. 2.5.

2.5.2. HEAT CONDUCTION THROUGH A COMPOSITE WALL

Refer to Fig. 2.6 (*a*). Consider the transmission of heat through a composite wall consisting of a number of slabs.

Let, L_A, L_B, L_C = Thicknesses of slabs *A*, *B* and *C* respectively (also called path lengths),

k_A, k_B, k_C = Thermal conductivities of the slabs *A*, *B*, and *C* respectively,

$t_1, t_4 \ (t_1 > t_4)$ = Temperatures at the wall surfaces 1 and 4 respectively, and

t_2, t_3 = Temperatures at the interfaces 2 and 3 respectively.

Since the quantity of heat transmitted per unit time through each slab/layer is same, we have,

$$Q = \frac{k_A . A\,(t_1 - t_2)}{L_A} = \frac{k_B \, . \, A\,(t_2 - t_3)}{L_B} = \frac{k_C \, . \, A\,(t_3 - t_4)}{L_C}$$

(Assuming that there is a perfect contact between the layers and no temperature drop occurs across the interface between the materials).

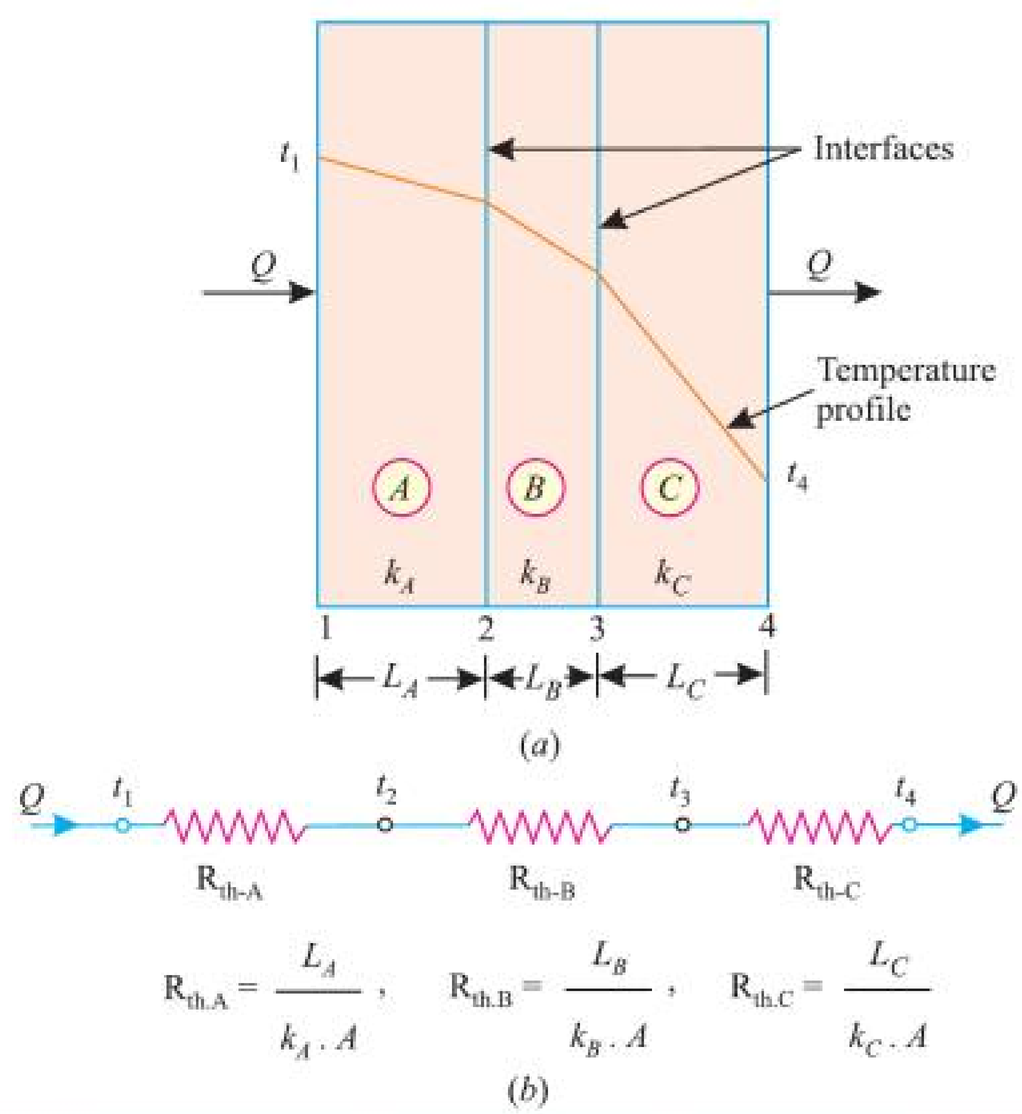


Fig. 2.6. Steady state conduction through a composite wall.

Rearranging the above expression, we get

$$t_1 - t_2 = \frac{Q \cdot L_A}{k_A \cdot A} \quad ...(i)$$

$$t_2 - t_3 = \frac{Q \cdot L_B}{k_B \cdot A} \quad ...(ii)$$

$$t_3 - t_4 = \frac{Q \cdot L_c}{k_C \cdot A} \quad ...(iii)$$

Adding (*i*), (*ii*) and (*iii*), we have

$$(t_1 - t_4) = Q\left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A}\right]$$

or,

$$Q = \frac{A\,(t_1 - t_4)}{\left[\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}\right]} \quad ...(2.49)$$

or,

$$Q = \frac{(t_1 - t_4)}{\left[\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A}\right]} = \frac{(t_1 - t_4)}{[R_{th-A} + R_{th-B} + R_{th-C}]} \quad ...[2.49\,(a)]$$

If the composite wall consists of n slabs/layers, then

$$Q = \frac{[t_1 - t_{(n+1)}]}{\sum_1^n \frac{L}{kA}} \quad ...(2.50)$$

In order to solve more complex problems involving both series and parallel thermal resistances, the electrical analogy may be used. A typical problem and its analogous electric circuit are shown in Fig. 2.7.

$$Q = \frac{\Delta\, t_{\text{overall}}}{\sum R_{th}} \quad ...(2.51)$$

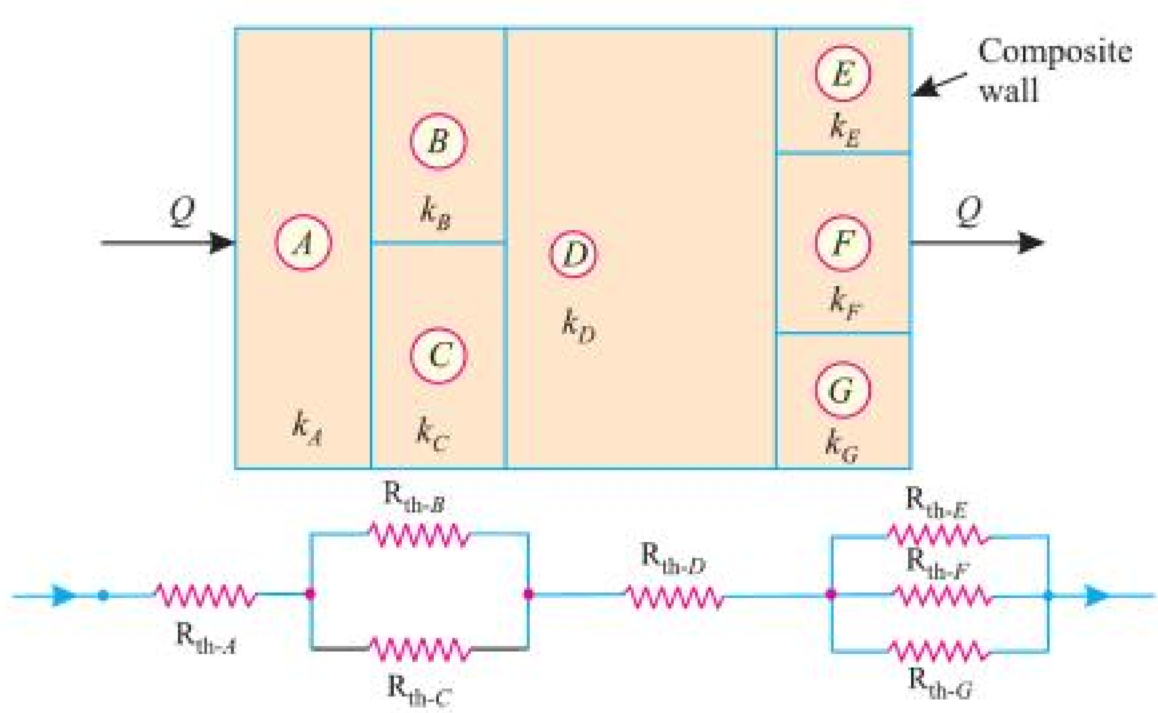


Fig. 2.7. Series and parallel one-dimensional heat transfer through a composite wall and electrical analog.

Thermal contact resistance. In a composite (multi-layer) wall, the calculations of heat flow are made on the assumptions : (*i*) The contact between the adjacent layers is perfect, (*ii*) At the interface there is no fall of temperature, and (*iii*) At the interface the temperature is continuous, although there is discontinuity in temperature gradient. In real systems, however, due to surface roughness and void spaces (usually filled with air) the contact surfaces *touch only at discrete locations*. Thus there is not a single plane of contact, which means that the area available for the flow of heat at the interface will be small compared to geometric face area. Due to this reduced area and presence of air voids, *a large resistance to heat flow at the interface occurs*. This resistance is known as *thermal contact resistance* and it causes temperature drop between two materials at the interface as shown in Fig. 2.8.

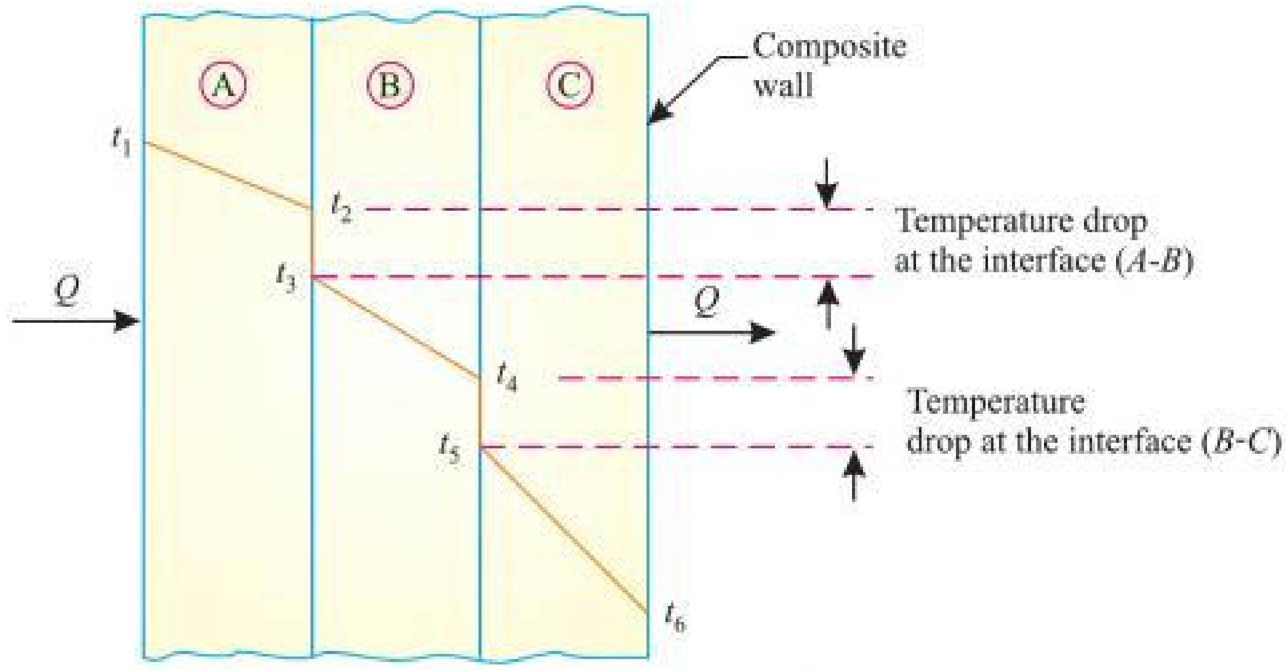


Fig. 2.8. Temperature drops at the interfaces.

Refer to Fig. 2.8. The contact resistances are given by

$$(R_{th-AB})_{\text{cont.}} = \frac{(t_2 - t_3)}{Q/A} \quad \text{and} \quad (R_{th-BC})_{\text{cont.}} = \frac{(t_4 - t_5)}{Q/A}$$

Boiler is being transported.

2.5.3. THE OVERALL HEAT-TRANSFER COEFFICIENT

While dealing with the problems of fluid to fluid heat transfer across a metal boundary, it is usual to adopt an overall heat transfer coefficient U which *gives the heat transmitted per unit area per unit time per degree temperature difference between the bulk fluids on each side of the metal.*

Refer to Fig. 2.9

Let,
L = Thickness of the metal wall,
k = Thermal conductivity of the wall material,
t_1 = Temperature of the surface–1,
t_2 = Temperature of the surface–2,
t_{hf} = Temperature of the hot fluid,
t_{cf} = Temperature of the cold fluid,
h_{hf} = Heat transfer coefficient from hot fluid to metal surface, and
h_{cf} = Heat transfer coefficient from metal surface to cold fluid.

(The suffices *hf* and *cf* stand for hot fluid and cold fluid respectively.)

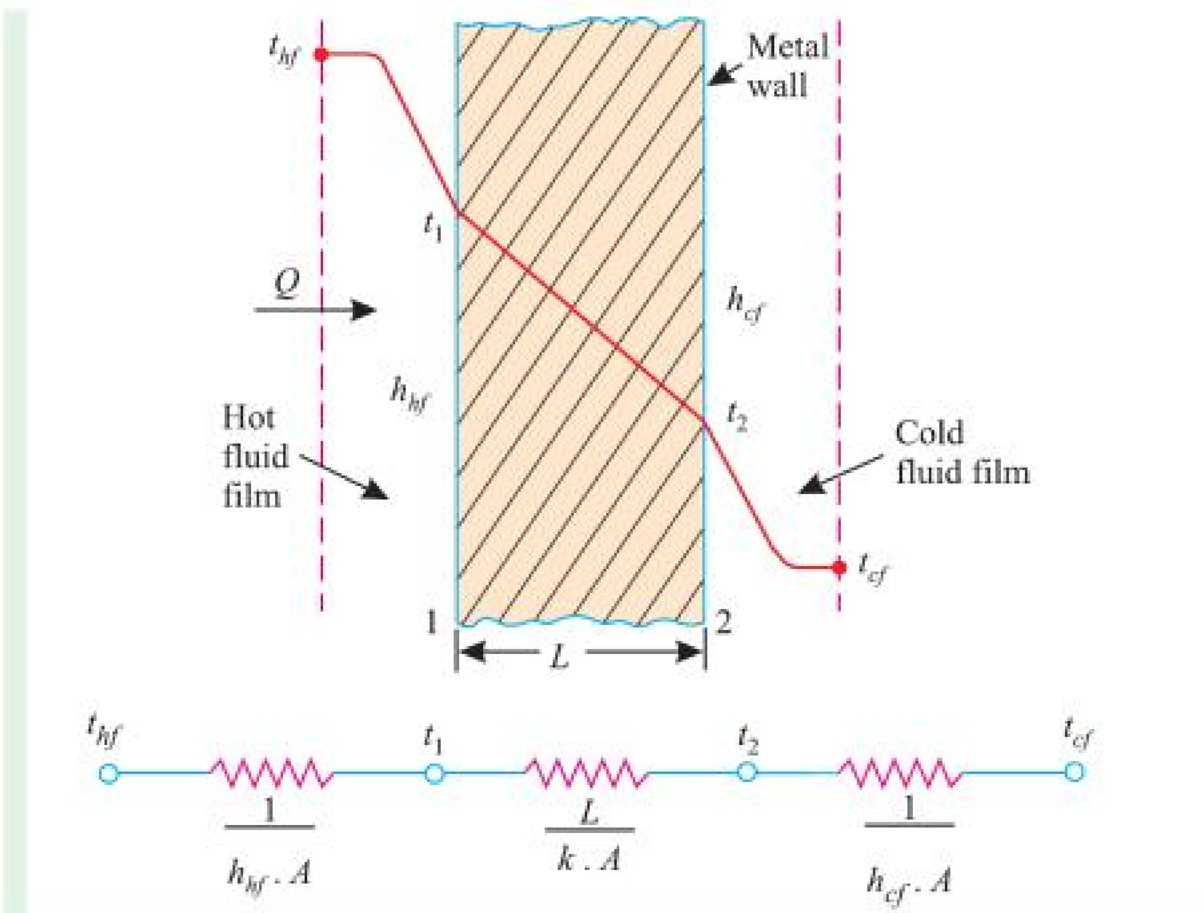


Fig. 2.9. The overall heat transfer through a plane wall.

The equations of heat flow through the fluid and the metal surface are given by

$$Q = h_{hf} \cdot A\,(t_{hf} - t_1) \quad \text{...(i)}$$

$$Q = \frac{k.A\,(t_1 - t_2)}{L} \quad \text{...(ii)}$$

$$Q = h_{cf} \cdot A\,(t_2 - t_{cf}) \quad \text{...(iii)}$$

By rearranging (*i*), (*ii*) and (*iii*), we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf}\,.A} \quad \text{...(iv)}$$

$$t_1 - t_2 = \frac{QL}{k\,.A} \quad \text{...(v)}$$

$$t_2 - t_{cf} = \frac{Q}{k_{cf} \cdot A} \quad ...(vi)$$

Adding (*iv*), (*v*) and (*vi*) we get

$$t_{hf} - t_{cf} = Q\left[\frac{1}{h_{hf} \cdot A} + \frac{L}{k \cdot A} + \frac{1}{h_{cf} \cdot A}\right]$$

or,
$$Q = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad ...(2.52)$$

If U is the overall coefficient of heat transfer, then

$$Q = U.A\,(t_{hf} - t_{cf}) = \frac{A\,(t_{hf} - t_{cf})}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}}$$

or,
$$U = \frac{1}{\frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}}} \quad ...(2.53)$$

It may be noticed from the above equation that if the individual coefficients differ greatly in magnitude only a change in the *least* will have any significant effect on the rate of heat transfer.

Example 2.1. *Discuss the effects of various parameters on the thermal conductivity of solids.*
(AMIE Summer, 2001)

Solution. The following are the effects of various parameters on the thermal conductivity of solids.

1. Chemical composition. *Pure metals have very high thermal conductivity. Impurities or alloying elements reduce the thermal conductivity considerably.* [Thermal conductivity of pure copper is 385 W/m° C, and that for pure nickel is 93 W/m° C. But monel metal (an alloy of 30% Ni and 70% Cu) has k of 24 W/m° C. Again for copper containing traces of Arsenic the value of k is reduced to 142 W/m° C.]

2. Mechanical forming. Forging, drawing and bending or *heat treatment of metals* cause considerable variation in thermal conductivity. For example, *the thermal conductivity of hardened steel is lower than that of annealed state.*

3. Temperature rise. The value of k for most metals *decreases with temperature rise* since at elevated temperatures the thermal vibrations of the lattice become higher that retard the motion of free electrons.

4. Non-metallic solids. Non-metallic solids have k *much lower* than that for metals. For many of the building materials (concrete, stone, brick, glass wool, cork etc.) the thermal conductivity may vary from sample to sample due to variations in structure, composition, density and porosity.


Fire brick

5. Presence of air. The thermal conductivity is *reduced* due to the presence of air filled pores or cavities.

6. Dampness. Thermal conductivity of a damp material is *considerably higher* than that of dry material.

7. Density. Thermal conductivity of insulating powder, asbestos etc. increases with density growth. Thermal conductivity of snow is also proportional to its density.

Example 2.2. *The inner surface of a plane brick wall is at 60°C and the outer surface is at 35°C. Calculate the rate of heat transfer per m² of surface area of the wall, which is 220 mm thick. The thermal conductivity of the brick is 0.51 W/m°C.*

(AMIE Winter, 2000)

Solution. Temperature of the inner surface of the wall, $t_1 = 60°C$

Temperature of the outer surface of the wall, $t_2 = 35°C$

The thickness of the wall, $L = 220$ mm = 0.22 m

Thermal conductivity of the brick,

$k = 0.51$ W/m°C

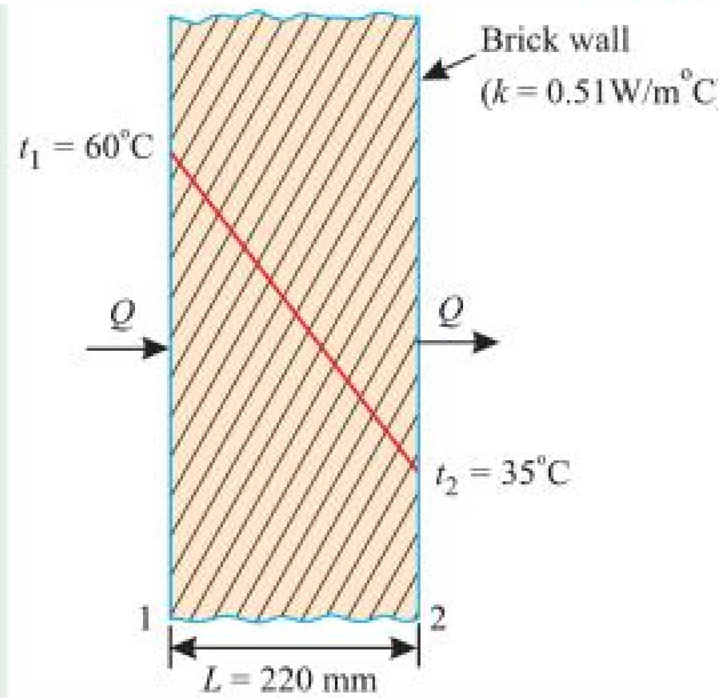


Fig. 2.10.

Rate of heat transfer per m², q :

Rate of heat transfer per unit area,

$$q = \frac{Q}{A} = \frac{k\,(t_1 - t_2)}{L}$$

or

$$q = \frac{0.51 \times (60 - 35)}{0.22} = \mathbf{57.95\ W/m^2\ (Ans.)}$$

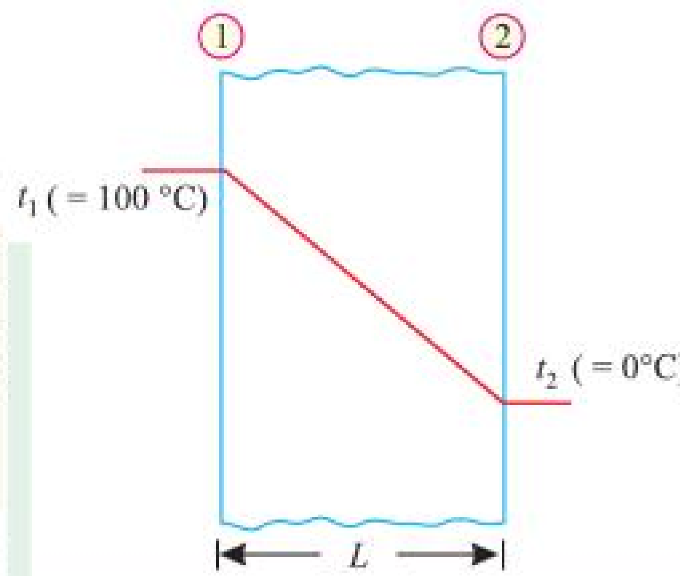


Fig. 2.11.

Example 2.3. *Consider a slab of thickness $L = 0.25$ m. One surface is kept at 100°C and the other surface at 0°C. Determine the net flux across the slab if the slab is made from pure copper. Thermal conductivity of copper may be taken as 387.6 W/m K.*

(AMIE Winter, 1998)

Solution. *Given* : $L = 0.25$ m; $t_1 = 100°C$; $t_2 = 0°C$; $k = 387.6$ W/m K.

From Fourier's law,

$$Q = -kA\,\frac{dt}{dx} \quad \text{...[Eqn. (1.1)]}$$

Net flux,
$$q = \frac{Q}{A} = -k \cdot \frac{(t_2 - t_1)}{L}$$

$$= -387.6 \times \frac{(0 - 100)}{0.25}$$

$$= \mathbf{1.55 \times 10^5\ W/m^2}\quad \textbf{(Ans.)}$$

Example 2.4. *A reactor's wall, 320 mm thick, is made up of an inner layer of fire brick (k = 0.84 W/m°C) covered with a layer of insulation (k = 0.16 W/m°C). The reactor operates at a temperature of 1325°C and the ambient temperature is 25°C.*

(i) Determine the thickness of fire brick and insulation which gives minimum heat loss;

(ii) Calculate the heat loss presuming that the insulating material has a maximum temperature of 1200°C.

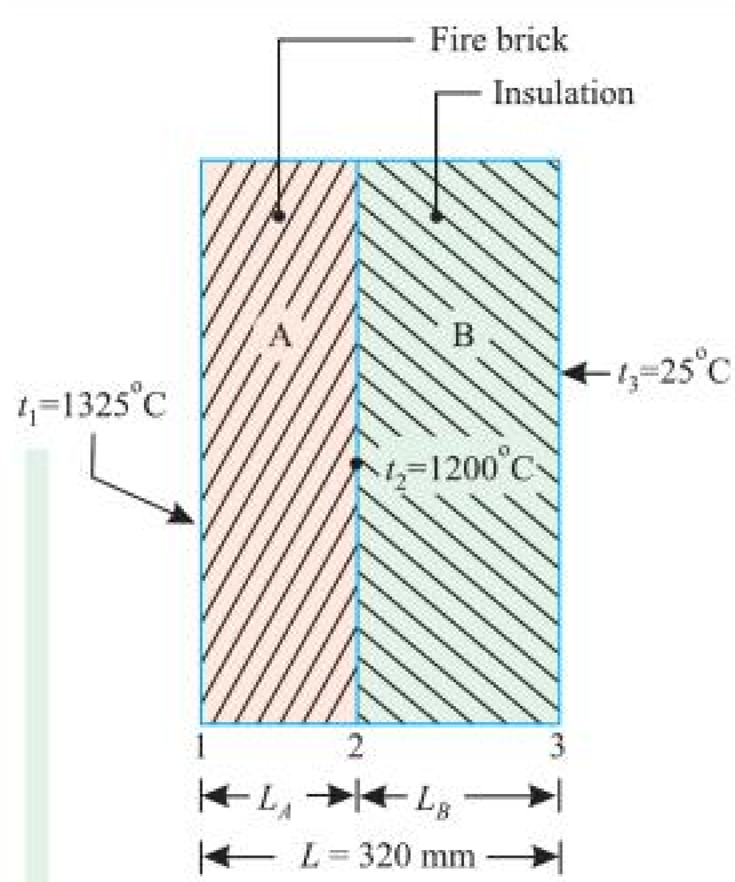


Fig. 2.12.

If the calculated heat loss is not acceptable, then state whether addition of another layer of insulation would provide a satisfactory solution.

Solution. Refer to Fig. 2.12.

Given : $t_1 = 1325°C; t_2 = 1200°C, t_3 = 25°C$

$$L_A + L_B = L = 320 \text{ mm or } 0.32 \text{ m}$$

$$\therefore \quad L_B = (0.32 - L_A); \qquad ...(i)$$

$$k_A = 0.84 \text{ W/m°C};$$

$$k_B = 0.16 \text{ W/m°C}.$$

(*i*) **$L_A : L_B$:**

The heat flux, under steady state conditions, is constant throughout the wall and is same for each layer. Then for *unit area* of wall,

$$q = \frac{t_1 - t_3}{L_A / k_A + L_B / k_B} = \frac{t_1 - t_2}{L_A / k_A} = \frac{t_2 - t_3}{L_B / k_B}$$

Considering first two quantities, we have

$$\frac{(1325 - 25)}{L_A / 0.84 + L_B / 0.16} = \frac{(1325 - 1200)}{L_A / 0.84}$$

or, $$\frac{1300}{1.190 L_A + 6.25 (0.32 - L_A)} = \frac{105}{L_A}$$

or, $$\frac{1300}{1.190 L_A + 2 - 6.25 L_A} = \frac{105}{L_A}$$

or, $$\frac{1300}{2 - 5.06 L_A} = \frac{105}{L_A}$$

or, $$1300 L_A = 105 (2 - 5.06 L_A)$$

or, $$1300 L_A = 210 - 531.3 L_A$$

or, $$L_A = \frac{210}{(1300 + 531.3)} = 0.1146 \text{ m or } \mathbf{114.6 \text{ mm}}$$

$\therefore$ Thickness of insulation $L_B = 320 - 114.6 = \mathbf{205.4 \text{ mm}}$ **(Ans.)**

(*ii*) **Heat loss per unit area, q :**

Heat loss per unit area, $$q = \frac{t_1 - t_2}{L_A / k_A} = \frac{1325 - 1200}{0.1146 / 0.84} = \mathbf{916.23 \text{ W/m}^2} \textbf{ (Ans.)}$$

If another layer of insulating material is added, the heat loss from the wall will reduce; consequently the temperature drop across the fire brick lining will drop and the interface temperature t_2 will rise. As the interface temperature is *already fixed*, therefore, a *satisfactory solution will not be available by adding another layer of insulation*.

Example 2.5. *A wall of a furnace is made up of inside layer of silica brick 120 mm thick covered with a layer of magnesite brick 240 mm thick. The temperatures at the inside surface of silica brick wall and outside surface of magnesite brick wall are 725°C and 110°C respectively. The contact thermal resistance between the two walls at the interface is 0.0035°C/W per unit wall area. If thermal conductivities of silica and magnesite bricks are 1.7 W/m°C and 5.8 W/m°C, calculate.*

(*i*) *The rate of heat loss per unit area of walls, and*

(*ii*) *The temperature drop at the interface.*

Solution. Refer Fig. 2.13.

Given : $L_A = 120 \text{ mm} = 0.12 \text{ m};$

$$L_B = 240 \text{ mm} = 0.24 \text{ m};$$

$k_A = 1.7$ W/m°C; $k_B = 5.8$ W/m°C

The contact thermal resistance $(R_{th})_{cont.}$ = 0.0035°C/W

The temperature at the inside surface of silica brick wall, $t_1 = 725$°C

The temperature at the outside surface of the magnesite brick wall, $t_4 = 110$°C

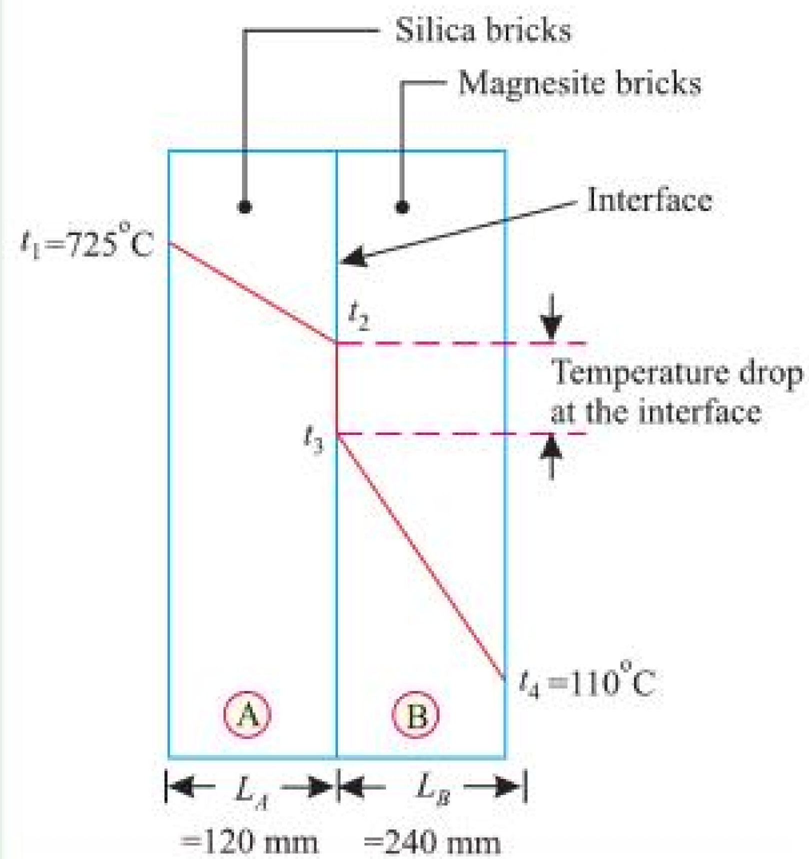


Fig. 2.13.

(i) The rate of heat loss per unit area of wall, q :

$$q = \frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t}{R_{th-A} + (R_{th})_{cont.} + R_{th-B}}$$

$$= \frac{(t_1 - t_4)}{L_A/k_A + 0.0035 + L_B/k_B}$$

$$= \frac{(725 - 110)}{0.12/1.7 + 0.0035 + 0.24/5.8}$$

$$= \frac{615}{0.0706 + 0.0035 + 0.0414}$$

$$= 5324.67 \text{ W/m}^2$$

∴ The rate of heat loss per unit area of wall, q = **5324.67 W/m²** **(Ans.)**

(ii) The temperature drop at the interface, $(t_2 - t_3)$:

As the same heat flows through each layer of composite wall, therefore,

$$q = \frac{t_1 - t_2}{L_A/k_A} = \frac{t_3 - t_4}{L_B/k_B}$$

or, $$5324.67 = \frac{(725 - t_2)}{0.12/1.7}$$

or, $$t_2 = 725 - 5324.67 \times \frac{0.12}{1.7} = 349.14°\text{C}$$

Similarly, $$5324.67 = \frac{(t_3 - 110)}{0.24/5.8}$$

or, $$t_3 = 110 + 5324.67 \times \frac{0.24}{5.8} = 330.33°\text{C}$$

Hence, the temperature drop at the interface = $t_2 - t_3$

= 349.14 – 330.33 = **18.81°C** **(Ans.)**

Example 2.6. *An exterior wall of a house may be approximated by a 0.1 m layer of common brick (k = 0.7 W/m°C) followed by a 0.04m layer of gypsum plaster (k = 0.48 W/m°C). What thickness of loosely packed rock wool insulation (k = 0.065 W/m°C) should be added to reduce the heat loss or (gain) through the wall by 80 per cent ?* **(AMIE Summer, 1999)**

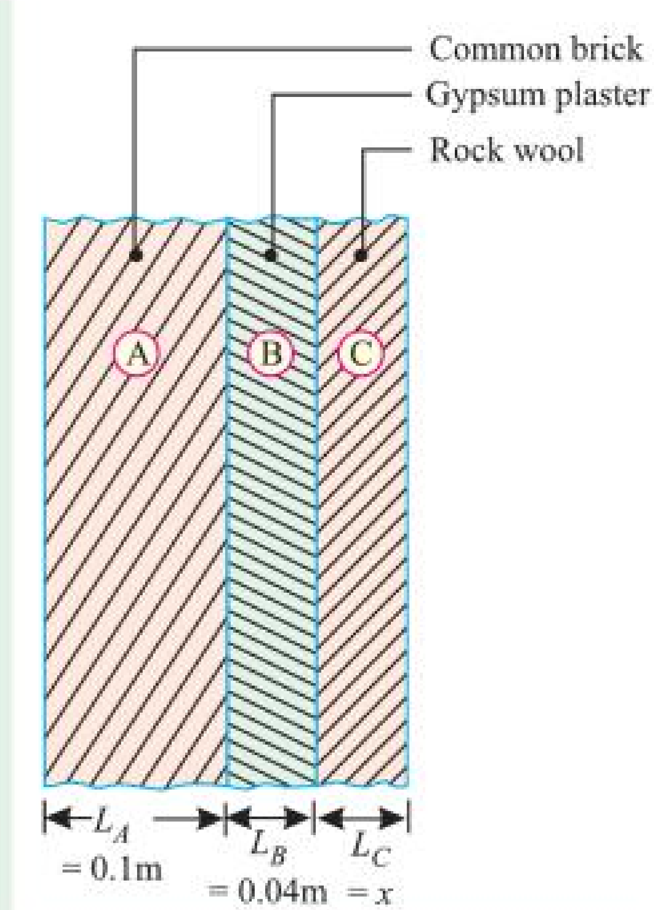


Fig. 2.14.

Solution. Refer to Fig. 2.14.

Thickness of common brick, $L_A = 0.1$ m

Thickness of gypsum plaster, $L_B = 0.04\,\text{m}$

Thickness of rock wool, $L_C = x\,(\text{in m}) = ?$

Thermal conductivities :

Common brick, $k_A = 0.7\,\text{W/m°C}$;

Gypsum plaster, $k_B = 0.48\,\text{W/m°C}$;

Rock wool, $k_C = 0.065\,\text{W/m°C}$.

Case I. *Rock wool insulation not used :*

$$Q_1 = \frac{A(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}} \quad ...(i)$$

Case II. *Rock wool insulation used :*

$$Q_2 = \frac{A(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} \quad ...(ii)$$

But, $Q_2 = (1 - 0.8)\,Q_1 = 0.2\,Q_1$ ...(*given*)

$$\therefore \quad \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} = 0.2 \times \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}}$$

or, $$\frac{0.1}{0.7} + \frac{0.04}{0.48} = 0.2\left[\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}\right]$$

or, $$0.1428 + 0.0833 = 0.2\,[0.1428 + 0.0833 + 15.385x]$$

or, $$0.2261 = 0.2\,(0.2261 + 15.385\,x)$$

or, $$x = 0.0588\,\text{m or } 58.8\,\text{mm}$$

Thus, the *thickness of rock wool insulation should be* **58.8 mm** **(Ans.)**

Induction furnace.

Example 2.7. *A furnace wall consists of 200 mm layer of refractory bricks, 6 mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400 W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find :*

(*i*) *To how many millimeters of insulation brick is the air layer equivalent?*

(*ii*) *What is the temperature of the outer surface of the steel plate?*

(AMIE Winter, 1996)

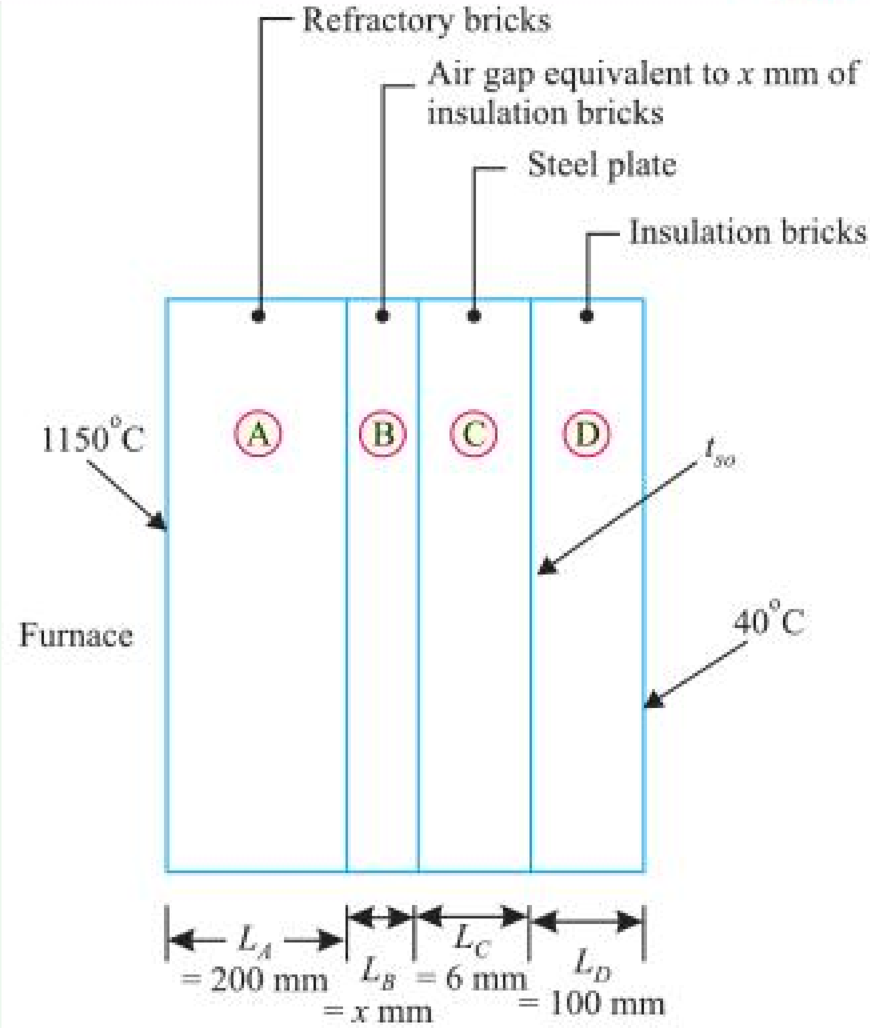


Fig. 2.15.

Solution. Refer Fig. 2.15.

Thickness of refractory bricks,

L_A = 200 mm = 0.2 m

Thickness of steel plate,

L_C = 6 mm = 0.006 m

Thickness of insulation bricks, L_D = 100 mm = 0.1 m

Difference of temperature beetween the innermost and outermost sides of the wall,

$$\Delta t = 1150 - 40 = 1110°C$$

Thermal conductivities :

k_A = 1.52 W/m°C; $k_B = k_D$ = 0.138 W/m°C; k_C = 45 W/m°C

Heat loss from the wall, q = 400 W/m²

(*i*) **The value of x ($= L_C$) :**

We know, $$Q = \frac{A \cdot \Delta t}{\Sigma \frac{L}{k}} \quad \text{or} \quad \frac{Q}{A} = q = \frac{\Delta t}{\Sigma \frac{L}{k}}$$

or, $$400 = \frac{1110}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{L_D}{k_D}}$$

or, $$400 = \frac{1110}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$

$$= \frac{1110}{0.1316 + 0.0072x + 0.00013 + 0.7246} = \frac{1110}{0.8563 + 0.0072x}$$

or, $$0.8563 + 0.0072\,x = \frac{1110}{400} = 2.775$$

or, $x = \dfrac{2.775 - 0.8563}{0.0072} = \mathbf{266.5\ mm}$ **(Ans.)**

(*ii*) Temperature of the outer surface of the steel plate t_{so}:

$$q = 400 = \frac{(t_{so} - 40)}{L_D / k_D}$$

or, $400 = \dfrac{(t_{so} - 40)}{(0.1/0.138)} = 1.38\ (t_{so} - 40)$

or, $t_{so} = \dfrac{400}{1.38} + 40 = \mathbf{329.8°C}$ **(Ans.)**

Example 2.8. *A furnace wall is composed of 220 mm of fire brick, 150 mm of common brick, 50 mm of 85% magnesia and 3 mm of steel plate on the outside. If the inside surface temperature is 1500°C and outside surface temperature is 90°C, estimate the temperatures between layers and calculate the heat loss in kJ/h- m². Assume, k (for fire brick) = 4kJ/m-h-°C, k(for common brick) = 2.8 kJ/m-h-°C, k (for 85% magnesia) = 0.24 kJ/m-h-°C, and k (steel) = 240 kJ/m-h-°C.*

(AMIE, Winter, 1997)

Solution. *Given* : L_A = 220 mm = 0.22 m; L_B = 150 mm = 0.15 m; L_C = 50 mm = 0.05m; L_D = 3 mm = 0.003 m

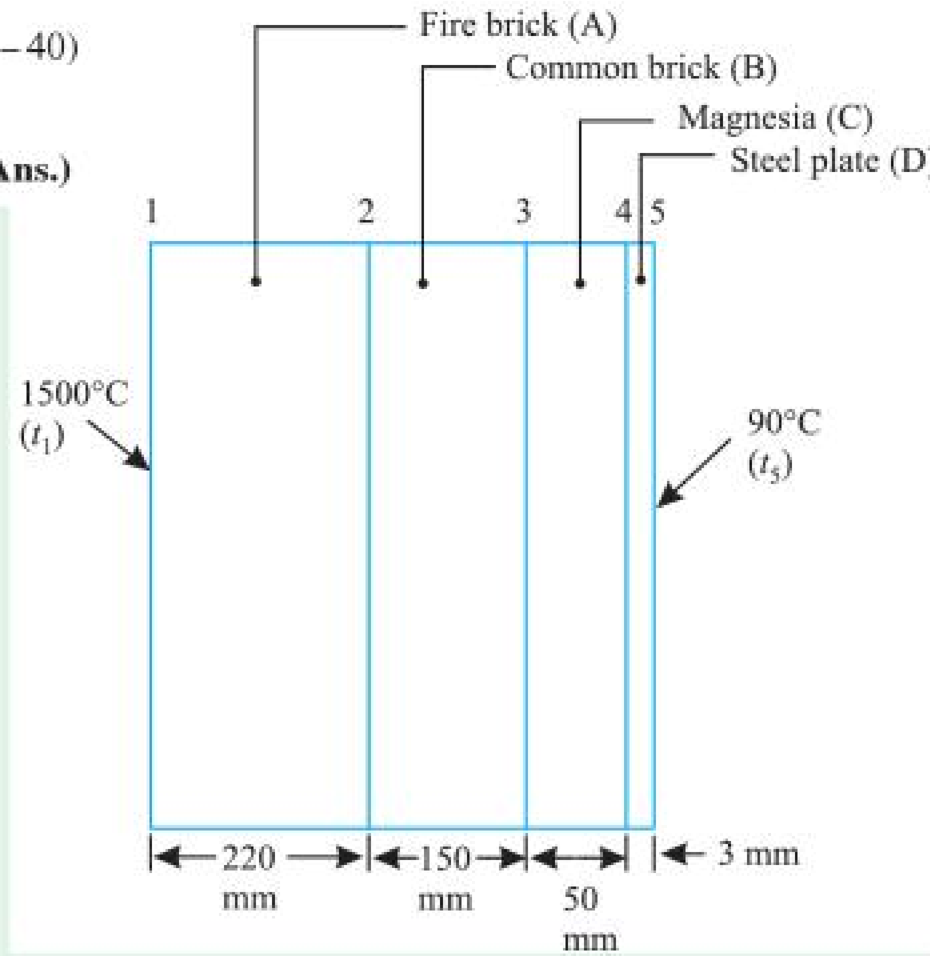


Fig. 2.16.

$$t_1 = 1500°C,\ t_5 = 90°C;$$
$$k_A = 4\ kJ/mh°C;\ k_B = 2.8\ kJ/mh°C$$
$$k_C = 0.24\ kJ/mh°C;\ k_D = 240\ kJ/mh°C.$$

Heat loss in kJ/hm² :

The equivalent thermal resistances of various layers are :

$$R_{th-A} = \frac{L_A}{k_A} = \frac{0.22}{4} = 0.055\ m^2h°C/kJ$$

$$R_{th-B} = \frac{L_B}{k_B} = \frac{0.15}{2.8} = 0.05357\ m^2h°C/kJ$$

$$R_{th-C} = \frac{L_C}{k_C} = \frac{0.05}{0.24} = 0.2083\ m^2h°C/kJ$$

$$R_{th-D} = \frac{L_D}{k_D} = \frac{0.003}{240} = 1.25 \times 10^{-5}\ m^2h°C/kJ$$

Total thermal resistance,

$$(R_{th})_{total} = 0.055 + 0.05357 + 0.2083 + 1.25 \times 10^{-5} = 0.3169\ m^2h°C/kJ$$

Heat loss, $q = \dfrac{(t_1 - t_5)}{(R_{th})_{total}} = \dfrac{(1500 - 90)}{0.3169} = \mathbf{4449.35\ kJ/hm^2}$ **(Ans.)**

Temperatures between layers :

Also, $q = \dfrac{t_4 - t_5}{R_{th-D}}$

or $t_4 = t_5 + q\,R_{th-D} = 90 + 4449.35 \times 1.25 \times 10^{-5} = \mathbf{90.056°C}$ **(Ans.)**

Similarly, $t_3 = t_4 + q\,R_{th-C} = 90.056 + 444.35 \times 0.2083 = \mathbf{1016.86°C}$

and $t_2 = t_3 + q\,R_{th-B} = 1016.86 + 4449.35 \times 0.05357 = \mathbf{1255.2°C}$

[Check $t_1 = t_2 + q\,R_{th-A} = 1255.2 + 4449.35 \times 0.55 \simeq 1500°C$]

Example 2.9. *A metal piece of length l has a cross-section of a sector of a circle of radius r and included angle of θ. Its two ends are maintained at temperatures t_1 and t_2 ($t_1 > t_2$). Find the expression for heat flow through the metal piece, assuming that the conductivity of metal varies with temperature according to relation,*

$$k = k_0 (1 - \beta t).$$

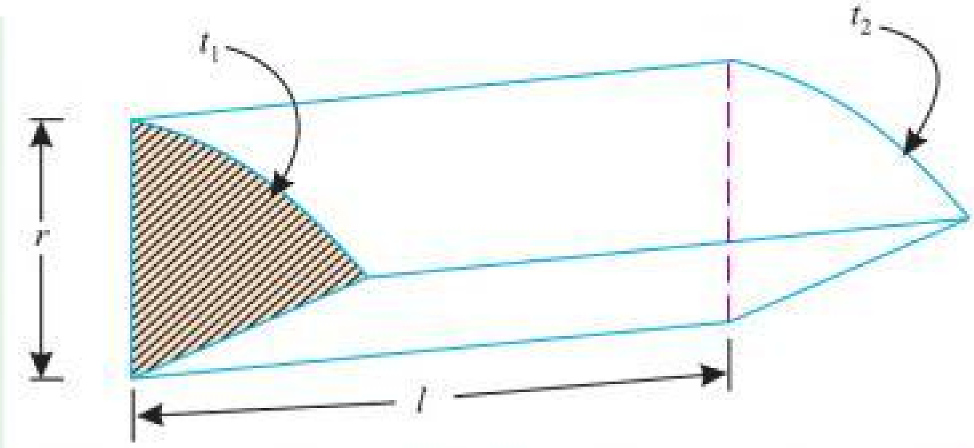


Fig. 2.17.

Also assume that $\dfrac{\partial t}{\partial \theta} = 0$ and $\dfrac{\partial t}{\partial r} = 0$ and outer surfaces of the slab except the end surfaces are completely insulated.

What will be the rate of heat transfer if l = 600 mm, r = 120 mm, θ = 60°, t_1 = 125°C, t_2 = 25°C and k_0 = 115 W/m°C and β = 10^{-4} ?

Solution. As per given conditions,

$$Q = \frac{kA(t_1 - t_2)}{l}$$

The area through which heat is flowing is given by,

$$A = \pi r^2 \times \frac{\theta}{2\pi} = \frac{r^2\theta}{2}$$

where θ is in radians.

$$k_m = k_0 (1 - \beta t_m) \text{ where } t_m = \frac{t_1 + t_2}{2}$$

as its variation is linear.

$$\therefore\ Q = \frac{k_m}{l}\left(\frac{r^2 \cdot \theta}{2}\right)(t_1 - t_2) \qquad ...(i)$$

Insulating fire bricks.

Rate of heat transfer, *Q* :

Given : l = 600 mm = 0.6 m; r = 120 mm = 0.12 m, θ = 60° = $\frac{\pi}{3}$ rad., t_1 = 125°C, t_2 = 25°C, k_0 = 115 W/m°C and β = 10^{-4}.

$$\therefore\ k_m = k_0 (1 - \beta t_m) = 115\left[1 - 10^{-4}\left(\frac{125 + 25}{2}\right)\right] = 114.14 \text{ W/m°C}$$

Subsituting the proper values in the expression (*i*), we have

$$Q = \frac{114.14}{0.6}\left(\frac{0.12^2 \times \frac{\pi}{3}}{2}\right)(125 - 25) = \mathbf{143.43\ W}$$ **(Ans.)**

Example 2.10. *(i) Derive an expression for the heat loss per* m^2 *of the surface area for a furnace wall (Fig. 2.18), when the thermal conductivity varies with temperature according to the relation :*

$$k = (a + bt^2)\ W/m°C, \qquad \text{where } t \text{ is in } °C$$

(ii) Find the rate of heat transfer through the wall, if $L = 0.2$ *m,* $t_1 = 300°C$, $t_2 = 30°C$ *and* $a = 0.3$ *and* $b = 5 \times 10^{-6}$. **(Maharashtra University)**

Solution. (*i*) The rate of heat transfer through the wall per m² is given by

$$q = \frac{k_m (t_1 - t_2)}{L}$$

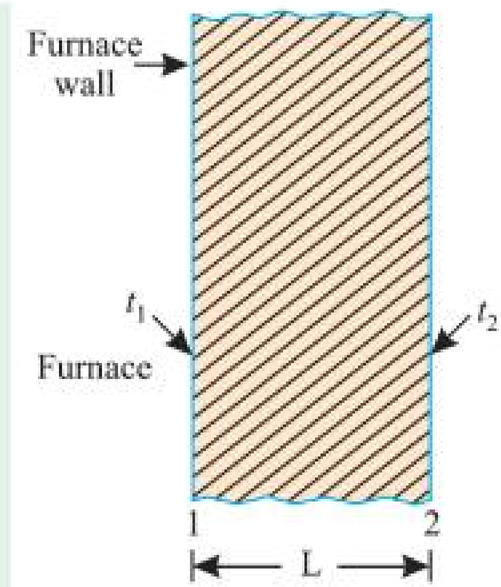


Fig. 2.18.

where $k_m = \frac{-1}{(t_1 - t_2)} \int_{t_1}^{t_2} k.dt$, where $k = f(t)$ ...[Eqn. (2.47)]

$$= \frac{-1}{(t_1 - t_2)} \int_{t_1}^{t_2} (a + bt^2)\, dt$$

$$= \frac{-1}{(t_1 - t_2)} \left[at + \frac{bt^3}{3} \right]_{t_1}^{t_2}$$

$$= \frac{-1}{(t_1 - t_2)} \left[a(t_2 - t_1) + \frac{b}{3}(t_2^3 - t_1^3) \right]$$

$$= \frac{-1}{(t_1 - t_2)} (t_2 - t_1) \left[a + \frac{b}{3}(t_2^2 + t_1 t_2 + t_1^2) \right]$$

$$= a + \frac{b}{3}\left[t_1^2 + t_1 t_2 + t_2^2 \right]$$

$$\therefore \qquad q = \left[a + \frac{b}{3}(t_1^2 + t_1 t_2 + t_2^2) \right]\left[\frac{t_1 - t_2}{L} \right]$$ ***...Required expression.*** **(Ans.)**

(*ii*) **Rate of heat transfer per** m^2**,** q **:**

Thickness of wall, $L = 0.2$m; $t_1 = 300°C$; $t_2 = 30°C$; $a = 0.3$ and $b = 5 \times 10^{-6}$.

Substituting these values in the said equation, we have

$$q = \left[0.3 + \frac{5 \times 10^{-6}}{3}(300^2 + 300 \times 30 + 30^2) \right]\left[\frac{(300 - 30)}{0.2} \right]$$

$$= \left(0.3 + \frac{5 \times 10^{-6}}{3} \times 99900 \right) \times 1350 = 629.77\ W/m^2$$

Hence, rate of heat transfer per m² through the wall = **629.77 W/m²** **(Ans.)**

Example 2.11. *The surfaces of a plane wall of thickness L are maintained at temperatures* t_1 *and* t_2. *The thermal conductivity of wall material varies according to the relation:* $k = k_0 t^2$.

(*i*) *Derive an expression to find the steady state conduction through the wall.*

(*ii*) *Find the temperature at which mean thermal conductivity be evaluated in order to get the same heat flow by its substitution in the simplified Fourier's equation.*

Solution. Thickness of wall $= L$

Temperatures of surfaces $= t_1, t_2$

Relation of variation of thermal conductivity $k = k_0 t^2$

(i) Expression for heat conduction through wall :

Heat conduction through a plane wall is given by (Fourier's law)

$$Q = -k\,A\frac{dt}{dx}$$

$$= -k_0 t^2 \cdot A\frac{dt}{dx}$$

By rearranging and integrating, we get

$$\int_0^L Q.dx = -k_0\,A\int_{t_1}^{t_2} t^2\,dt$$

$$Q\,|x|_0^L = -k_0\,A\left[\frac{t^3}{3}\right]_{t_1}^{t_2}$$

$$QL = \frac{-k_0 A}{3}\left(t_2^{\,3} - t_1^{\,3}\right)$$

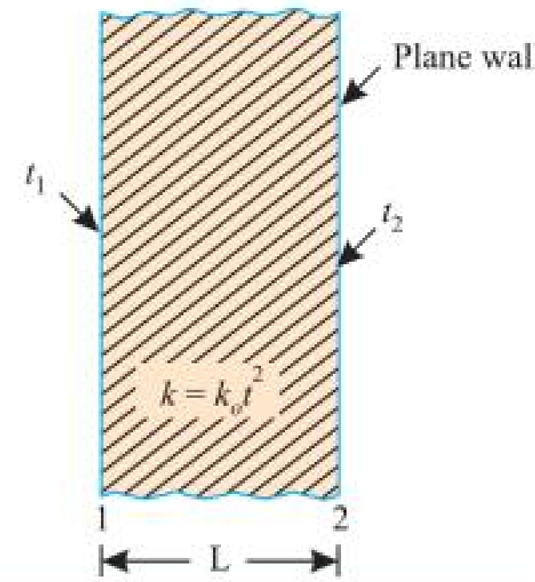


Fig. 2.19.

or, $$Q = \frac{k_0\,A}{3L}\left(t_1^{\,3} - t_2^{\,3}\right)$$ ***...Required expression.***

(ii) Temperature, t_m :

If the above heat flow is to be obtained by substituting mean value of thermal conductivity in the simplified Fourier's equation, we have

$$\frac{k_0\,A}{3L}\left(t_1^{\,3} - t_2^{\,3}\right) = \frac{k_m\,A\,(t_1 - t_2)}{L}$$

$$= \frac{k_0\,t_m^2\,A\,(t_1 - t_2)}{L}$$

or, $$t_m^{\,2} = \left[\frac{k_0\,A}{3L}(t_1^{\,3} - t_2^{\,3})\right] \times \left[\frac{L}{k_0\,A\,(t_1 - t_2)}\right]$$

$$= \frac{t_1^{\,3} - t_2^{\,3}}{3(t_1 - t_2)} = \frac{(t_1 - t_2)(t_1^{\,2} + t_1 t_2 + t_2^{\,2})}{3(t_1 - t_2)} = \frac{t_1^{\,2} + t_1\,t_2 + t_2^{\,2}}{3}$$

∴ $$t_m = \sqrt{\frac{t_1^{\,2} + t_2^{\,2} + t_1\,t_2}{3}}$$ ***...Required temperature.* (Ans.)**

Example 2.12. *The variation of thermal conductivity of a wall material is given by*

$$k = k_0\,(1 + \alpha t + \beta t^2)$$

If the thickness of the wall is L and its two surfaces are maintained at temperatures t_1 and t_2, find an expression for the steady state one-dimensional heat flow through the wall.

Solution. The rate of heat transfer through a wall per unit area is given by

$$q = -k \cdot \frac{dt}{dx} \quad \text{...Fourier's equation}$$

$$= -k_0\,(1 + \alpha t + \beta t^2) \cdot \frac{dt}{dx}$$

or, $$q.\,dx = -k_0\,(1 + \alpha t + \beta t^2)\,.\,dt$$

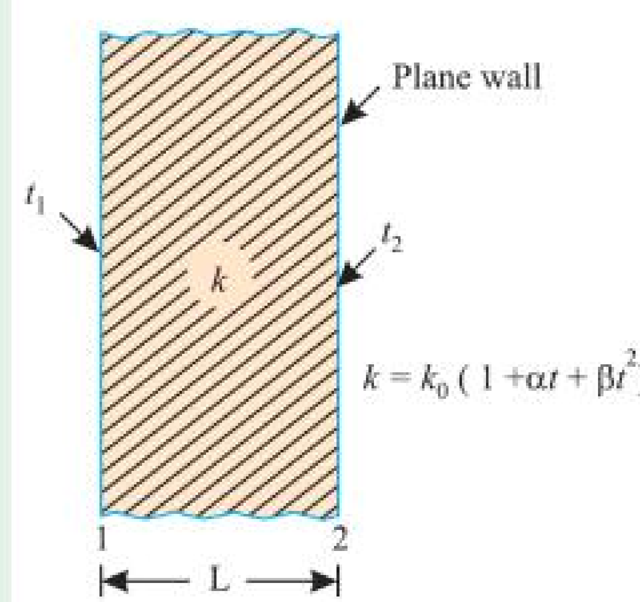


Fig. 2.20.

Integrating both sides we get

$$q \int_0^L dx = -k_0 \int_{t_1}^{t_2} (1 + \alpha t + \beta t^2)\, dt$$

$$q \times L = -k_0 \left[\left(t + \alpha.\frac{t^2}{2} + \beta.\frac{t^3}{3} \right) \right]_{t_1}^{t_2}$$

$$q \times L = -k_0 \left[(t_2 - t_1) + \frac{\alpha}{2}(t_2^2 - t_1^2) + \frac{\beta}{3}(t_2^3 - t_1^3) \right]$$

$$q = -\frac{k_0}{L} \left[(t_2 - t_1) + \frac{\alpha}{2}(t_2 - t_1)\,(t_2 + t_1) + \frac{\beta}{3}(t_2 - t_1)(t_1^2 + t_2^2 + t_1 t_2) \right]$$

$$q = -\frac{k_0\,(t_2 - t_1)}{L} \left[1 + \frac{\alpha}{2}(t_1 + t_2) + \frac{\beta}{3}(t_1^2 + t_2^2 + t_1 t_2) \right]$$ ***...Required expression.***

(Ans.)

Example 2.13. *It is proposed to carry pressurized water through a pipe imbeded in a 1.2 m thick wall whose surfaces are held at constant temperatures of 200°C and 60°C respectively. It is desired to locate the pipe in wall where the temperature is 120°C, find how far from the hot surface should the pipe be imbedded ? The thermal conductivity of the wall material varies with the temperature according to the relation, k = 0.28 (1 + 0.036t) where t is in degree celsius and k is in W/m°C.*

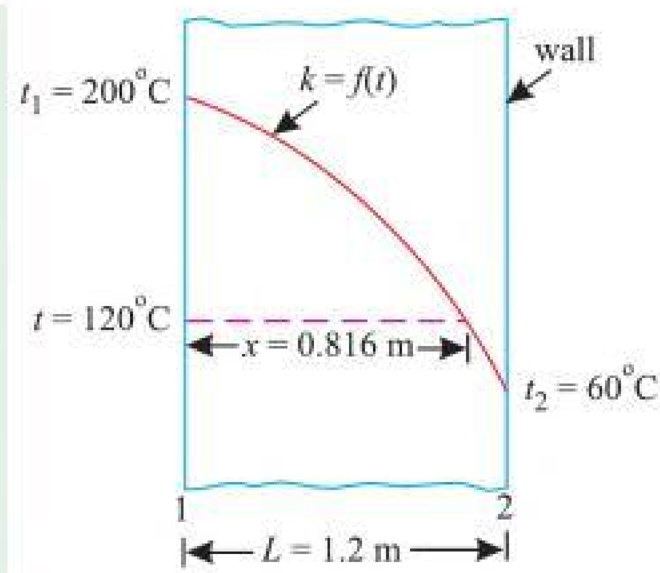


Fig. 2.21.

Solution. Thickness of wall, $L = 1.2$ m

Temperatures of wall surfaces $t_1 = 200°C$; $t_2 = 60°C$

Temperature, $t = 120°C$

Relation for conductivity $k = 0.28\,(1 + 0.036t)$

Pipework inside a factory.

The rate of heat transfer through a plane wall of variable thermal conductivity is given by

$$Q = k_m . \frac{A}{L}(t_1 - t_2)$$

$$= k_0\left[1 + \frac{\alpha}{2}(t_1 + t_2)\right]\frac{A}{L}(t_1 - t_2)$$

Now, when $t_2 = 60°C$, $L = 1.2$ m

and, when $t_2 = 120°C$, $L = x$ (unknown)

Substituting the values and equating the two expressions, we have

$$0.28\left[1 + \frac{0.036}{2}(200 + 60)\right]\frac{A}{1.2}(200 - 60) = 0.28\left[1 + \frac{0.036}{2}(200 + 120)\right]\frac{A}{x}(200 - 120)$$

$$185.54 = \frac{151.42}{x} \quad \text{or} \quad \frac{151.42}{185.54} = 0.816 \text{ m}$$

Hence the pipe should be imbedded 0.816 m *from the hot wall surface*. **(Ans.)**

Example 2.14. *Find the steady state heat flux through the composite slab as shown in the Fig. 2.22 and the interface temperature. The thermal conductivities of the two materials vary with temperature as given below :*

$k_A = 0.05\,(1 + 0.0065t)$ W/m°C; $k_B = 0.04\,(1 + 0.0076t)$ W/m°C, where temperatures are in °C. **[M.U.]**

Solution. $t_1 = 600°C$; $t_3 = 300°C$

$L_A = 50 \text{ mm} = 0.05\text{m}$

$L_B = 100 \text{ mm} = 0.1\text{m}$

$$k_{mA} = k_{OA}\left[1 + \alpha_A\left(\frac{t_1 + t_2}{2}\right)\right]$$

$$= 0.05\left[1 + 0.0065\left(\frac{t_1 + t_2}{2}\right)\right]$$

$$k_{mB} = k_{OB}\left[1 + \alpha_B\left(\frac{t_2 + t_3}{2}\right)\right]$$

$$= 0.04\left[1 + 0.0075\left(\frac{t_2 + t_3}{2}\right)\right]$$

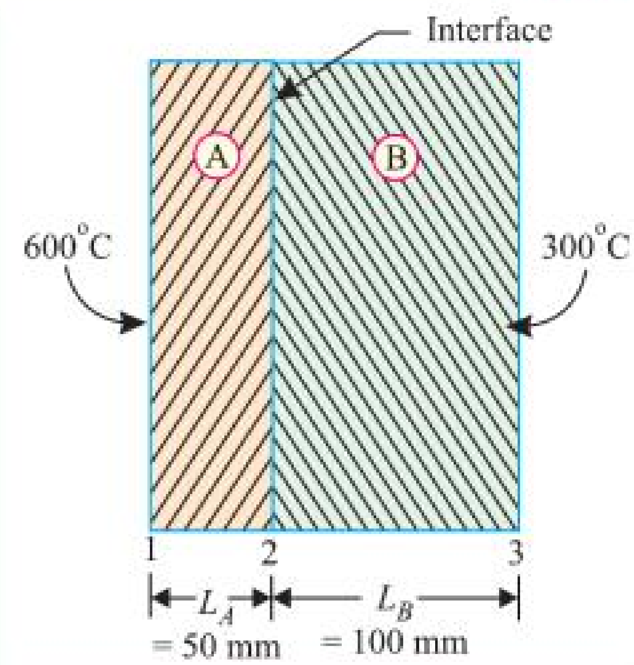


Fig. 2.22.

Interface temperature, t_2 :

Rate of heat transfer per m^2,

$$q = \frac{Q}{A} = \frac{(t_1 - t_2)}{(L_A / k_{mA})} = \frac{(t_2 - t_3)}{(L_B / k_{mB})} \qquad ...(1)$$

Now substituting the values of k_{mA} and k_{mB} in eqn (1), we get

$$\frac{(600 - t_2)}{\dfrac{0.05}{0.05\left[1 + 0.0065\left(\dfrac{600 + t_2}{2}\right)\right]}} = \frac{(t_2 - 300)}{\dfrac{0.1}{0.04\left[1 + 0.0075\left(\dfrac{t_2 + 300}{2}\right)\right]}}$$

or, $$(600 - t_2)\left[1 + 0.0065\left(\frac{600 + t_2}{2}\right)\right] = 0.4(t_2 - 300)\left[1 + 0.0075\left(\frac{t_2 + 300}{2}\right)\right]$$

or, $$(600 - t_2)\left[\frac{5.9 + 0.0065t_2}{2}\right] = (t_2 - 300)\left[\frac{4.25 + 0.0075t_2}{2}\right] \times 0.4$$

or, $(600 - t_2)(5.9 + 0.0065\, t_2) = (t_2 - 300)(1.7 + 0.003\, t_2)$

or, $3540 + 3.9 t_2 - 5.9 t_2 - 0.0065 t_2^2 = 1.7\, t_2 + 0.003\, t_2^2 - 510 - 0.9\, t_2$

or, $0.0095\, t_2^2 + 2.8\, t_2 - 4050 = 0$

or, $t_2^2 + 294.7\, t_2 - 426315 = 0$

or,
$$t_2 = \frac{-294.7 \pm \sqrt{294.7^2 + 4 \times 426316}}{2}$$
$$= \frac{-294.7 \pm 1338.7}{2} = 522^\circ\text{C}$$

$$\therefore \quad k_{mA} = 0.05\left[1 + 0.0065\left(\frac{600 + 522}{2}\right)\right] = 0.2323 \text{ W/m°C}$$

Rate of heat transfer per m^2, q :

The steady state heat flow through the composite slab,

$$q = \frac{(t_1 - t_2)}{(L_A / k_{mA})} = \frac{(600 - 522)}{(0.05/0.2323)} = \mathbf{362.39\ W/m^2\ (Ans.)}$$

Example 2.15. *The composite wall of a furnace is made up with 120 mm of fire clay [k = 0.25 (1 + 0.0009 t) W/m°C] and 600 mm of red brick (k = 0.8 W/m°C). The inside surface temperature is 1250°C and the outside air temperature is 40°C. Determine :*

(i) The temperature at the layer interface, and

(ii) The heat loss for 1m² of furnace wall.

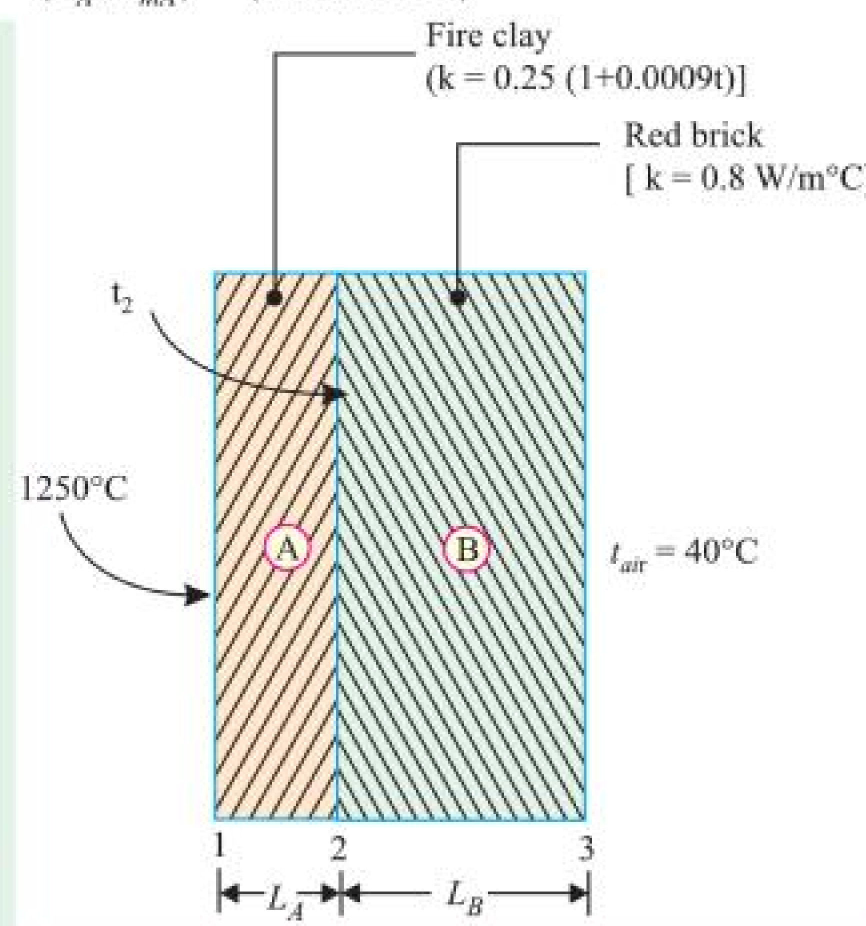


Fig. 2.23.

Solution. Refer Fig. 2.23.

L_A = 120 mm = 0.12 m; L_B = 600 mm = 0.6 m; $k_A = 0.25\,[1 + 0.0009\, t]$; k_B = 0.8 W/m°C; $\Delta t = (t_1 - t_{air}) = 1250 - 40 = 1210$°C.

(*i*) The temperature at layer interface, t_2 :

Average/mean thermal conductivity of fire clay,

$$(k_A)_m = 0.25\left[1 + 0.0009\left(\frac{1250 + t_2}{2}\right)\right]$$
$$= 0.25\,[1 + 0.00045\,(1250 + t_2)]$$

$\therefore$ Thermal resistance of fire clay,

$$R_{th-A} = \frac{L_A}{(k_A)_m A} = \frac{0.12}{0.25[1 + 0.00045(1250 + t_2)] \times 1} = \frac{1}{2.083 + 0.000937(1250 + t_2)}$$

Similarly, thermal resistance of red brick,

$$R_{th-B} = \frac{L_B}{k_B \cdot A} = \frac{0.6}{0.8 \times 1} = 0.75$$

Heat loss for $1 m^2$ of furnace wall,

$$Q = \frac{\Delta t}{\Sigma R_{th}} = \frac{\Delta t}{R_{th-A} + R_{th-B}}$$

$$= \frac{1210}{\dfrac{1}{2.083 + 0.000937(1250 + t_2)} + 0.75} \quad ...(i)$$

Under steady state conditions the same amount of heat flows through each layer. Then considering heat flow through the red brick, we have

$$Q = \frac{(t_2 - 40)}{k_B} = \frac{(t_2 - 40)}{0.8} \quad ...(ii)$$

From expression (*i*) and (*ii*), we obtain

$$\frac{1210}{\dfrac{1}{2.083 + 0.000937\,(1250 + t_2)} + 0.75} = \frac{(t_2 - 40)}{0.8}$$

or, $$\frac{1210[2.083 + 0.000937(1250 + t_2)]}{1 + 0.75[2.083 + 0.000937(1250 + t_2)]} = \frac{t_2 - 40}{0.8}$$

or, $$\frac{1210[3.254 + 0.000937 t_2]}{1 + 1.562 + 0.878 + 0.000703 t_2} = \frac{(t_2 - 40)}{0.8}$$

or, $$\frac{3937.34 + 1.134 t_2}{3.44 + 0.000703 t_2} = \frac{(t_2 - 40)}{0.8}$$

or, $$0.8\,(3937.34 + 1.134\, t_2) = (t_2 - 40)\,(3.44 + 0.000703\, t_2)$$

or, $$3149.87 + 0.907\, t_2 = 3.44\, t_2 + 0.000703\, t_2^2 - 137.6 - 0.0281\, t_2$$

or, $$0.000703\, t_2^2 + 2.505\, t_2 - 3287.47 = 0$$

or, $$t_2 = \frac{-2.505 + \sqrt{(2.505)^2 + 4 \times 0.000703 \times 3287.47}}{2 \times 0.000703} = \mathbf{1020.24°C} \text{ (Ans.)}$$

(*ii*) Heat loss, *Q* :

Heat loss for 1 m^2 of the furnace wall,

$$Q = \frac{(t_2 - 40)}{R_{th-B}} = \frac{(1020.24 - 40)}{0.75} = \mathbf{1306.98\ W} \text{ (Ans.)}$$

Example 2.16. *Find the heat flow rate through the composite wall as shown in Fig. 2.24 Assume one dimensional flow.*

k_A = 150 W/m°C,
k_B = 30 W/m°C,
k_C = 65 W/m°C, and
k_D = 50 W/m°C

(M.U. Winter, 2000)

Solution. The thermal circuit for heat flow in the given composite system (shown in Fig. 2.24) has been illustrated in Fig. 2.25.

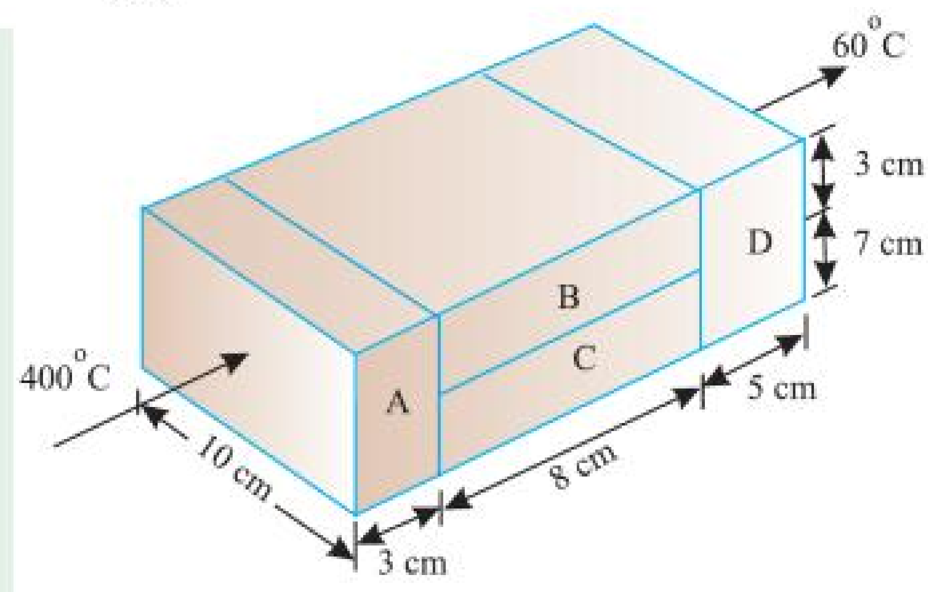


Fig. 2.24.

Furnance view from top.

Thickness :

$L_A = 3$ cm $= 0.03$ m; $L_B = L_C = 8$ cm $= 0.08$ m; $L_D = 5$ cm $= 0.05$ m

Areas :

$$A_A = 0.1 \times 0.1 = 0.01\text{m}^2; \qquad A_B = 0.1 \times 0.03 = 0.003\text{m}^2$$
$$A_C = 0.1 \times 0.07 = 0.007\text{m}^2; \quad A_D = 0.1 \times 0.1 = 0.01\text{m}^2$$

Heat flow rate, Q :

The thermal resistances are given by,

$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$

$$R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

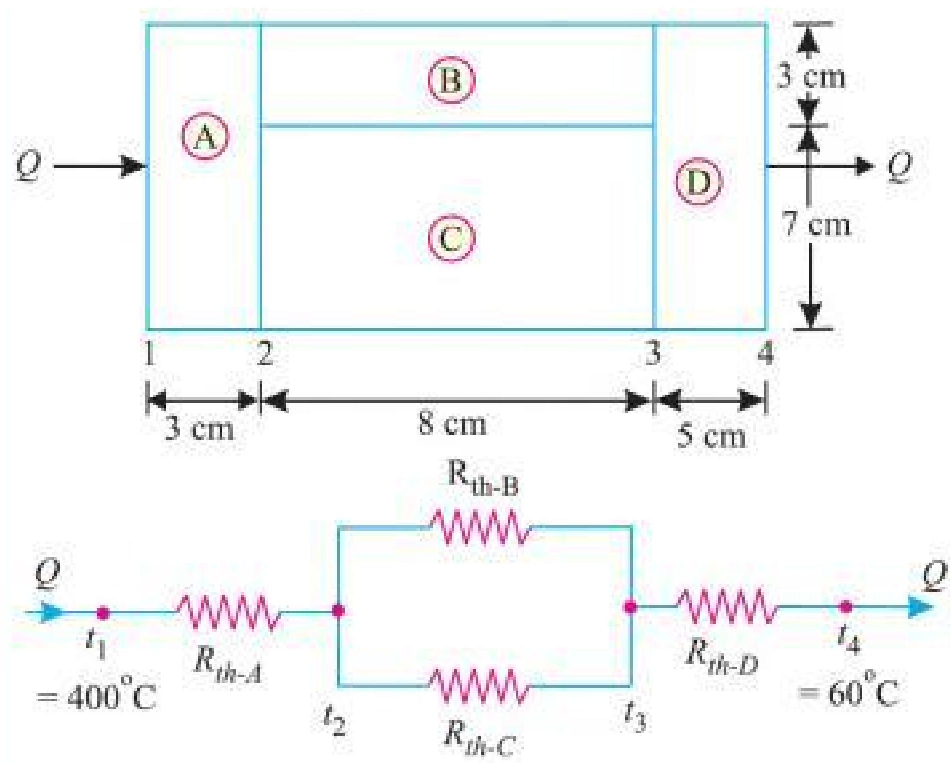


Fig. 2.25. Thermal circuit.

$$R_{th-C} = \frac{L_C}{k_C\, A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$

$$R_{th-D} = \frac{L_D}{k_D\, A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistance R_{th-B} and R_{th-C} is given by

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

$\therefore$ $$(R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

Now, the total thermal resistance is given by

$$(R_{th})_{\text{total}} = R_{th-A} + (R_{th})_{\text{eq.}} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

$\therefore$ $$Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400 - 60)}{0.267} = \mathbf{1273.4\ W}\ \textbf{(Ans.)}$$

Example 2.17. *The insulation boards for air-conditioning purposes are made of three layers, middle being of packed grass 10 cm thick (k = 0.02 W/m°C) and the sides are made of plywood each of 2 cm thickness (k = 0.12 W/m°C). They are glued with each other.*

(i) Determine the heat flow per m² area if one surface is at 35°C and other surface is at 20°C. Neglect the resistance of glue.

(ii) Instead of glue, if these three pieces are bolted by four steel bolts of 1 cm diameter at the corner (k = 40 W/m°C) per m² area of the board then find the heat flow per m² area of the combined board. **(M.U., 2001)**

Solution. (*i*) When the layers are glued : Refer Fig. 2.26.

Thickness of each of the plywood layer, $L_A = L_C = 2$ cm $= 0.02$ m

Thickness of grass layer,

$L_B = 10$ cm $= 0.1$ m

Thermal conductivities :

$k_A = k_C = 0.12$ W/m°C;

$k_B = 0.02$ W/m°C;

Temperatures : $t_1 = 35$°C; $t_4 = 20$°C

Heat flow per *m*² area, *q* :

$$q = \frac{(t_1 - t_4)}{R_{th-A} + R_{th-B} + R_{th-C}}$$

$$= \frac{(t_1 - t_4)}{\frac{L_A}{k_A \cdot A} + \frac{L_B}{k_B \cdot A} + \frac{L_C}{k_C \cdot A}}$$

$$= \frac{(35 - 20)}{\frac{0.02}{0.12 \times 1} + \frac{0.1}{0.02 \times 1} \times \frac{0.02}{0.12 \times 1}}$$

$$= \frac{15}{0.167 + 5.0 + 0.167} = \mathbf{2.81\ W/m^2}\ \textbf{(Ans.)}$$

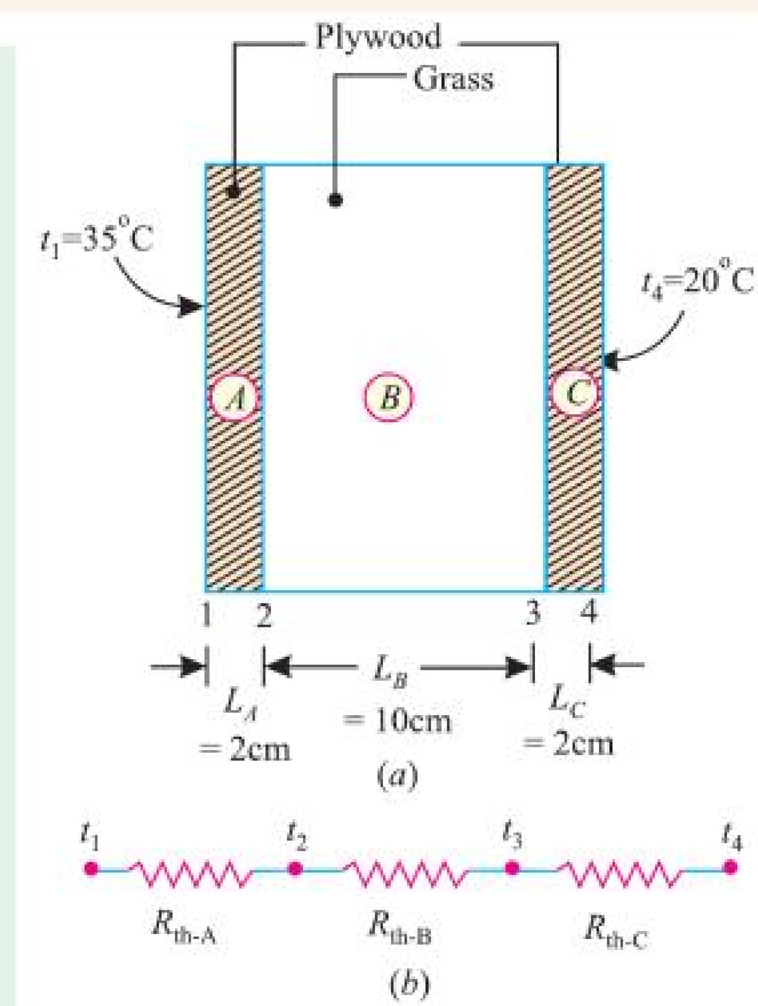


Fig. 2.26.

(*ii*) When the layers are joined by steel bolts : Refer to Fig. 2.27.

Number of steel bolts used = 4

Diameter of each bolt, d_b = 1cm = 0.01 m

$\therefore$ Area of each bolt, $A_b = \frac{\pi}{4} \times 0.01^2 = 7.854 \times 10^{-5}$ m^2

Thermal conductivity of bolt material, k_D = 40 W/m°C

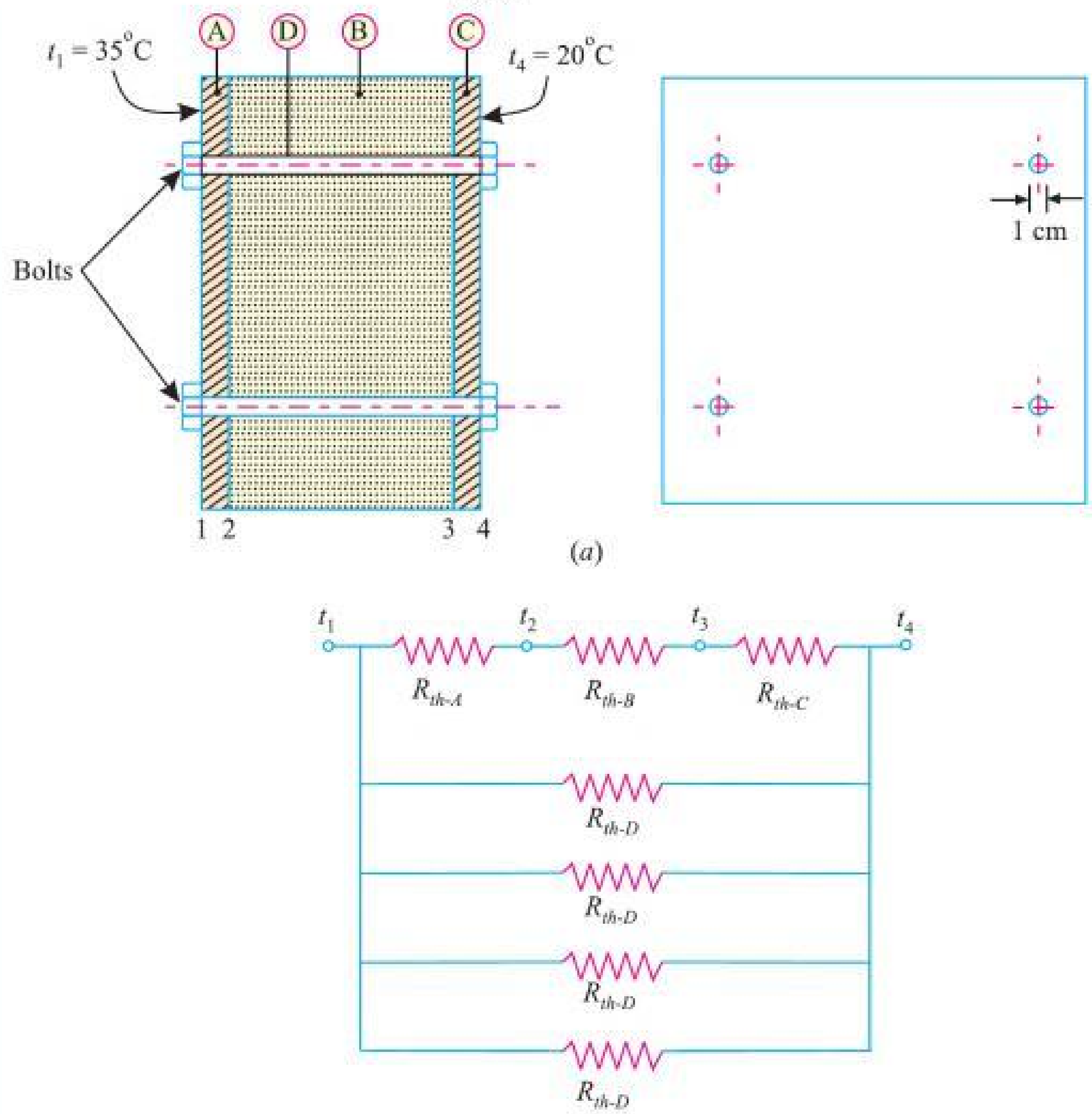
The equivalent thermal resistance $(R_{th})_{eq.}$ of the thermal circuit for the system is given by

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{(R_{th-A} + R_{th-B} + R_{th-C})} + \frac{4}{R_{th-D}}$$

where $$R_{th-D} = \frac{(L_A + L_B + L_C)}{k_D \cdot A_b} = \frac{0.02 + 0.1 + 0.02}{40 \times 7.854 \times 10^{-5}} = 44.56°\text{C/W}$$

$\therefore$ $$\frac{1}{(R_{th})_{eq}} = \frac{1}{(0.167 + 5.0 + 0.167)} + \frac{4}{44.56} = 0.187 + 0.089 = 0.276$$

or, $$(R_{th})_{eq.} \text{ or } (R_{th})_{total} = \frac{1}{0.276} = 3.623°\text{C/W}$$



(*b*) Thermal circuit for the system.

Fig. 2.27.

Heat flow per m² area, q :

$$q = \frac{(t_1 - t_4)}{(R_{th})_{total}} = \frac{(35 - 20)}{3.623} = \mathbf{4.14\ W/m^2\ (Ans.)}$$

Example 2.18. *Two slabs, each 120 mm thick, have thermal conductivities of 14.5 W/m°C and 210 W/m°C. These are placed in contact, but due to roughness, only 30 percent of area is in contact and the gap in the remaining area is 0.025 mm thick and is filled with air. If the temperature of the face of the hot surface is at 220°C and the outside side surface of other slab is at 30°C, determine :*

(i) Heat flow through the composite system.

(ii) The contact resistance and temperature drop in contact.

Assume that the conductivity of air is 0.032 W/m°C and that half of the contact (of the contact area) is due to either metal.

Solution. L_A = 120 mm = 0.12 m; L_{A_1} = 0.025 mm = 0.000025 m

L_B = 120 mm = 0.12 m; L_{B_1} = 0.025 mm = 0.000025 m

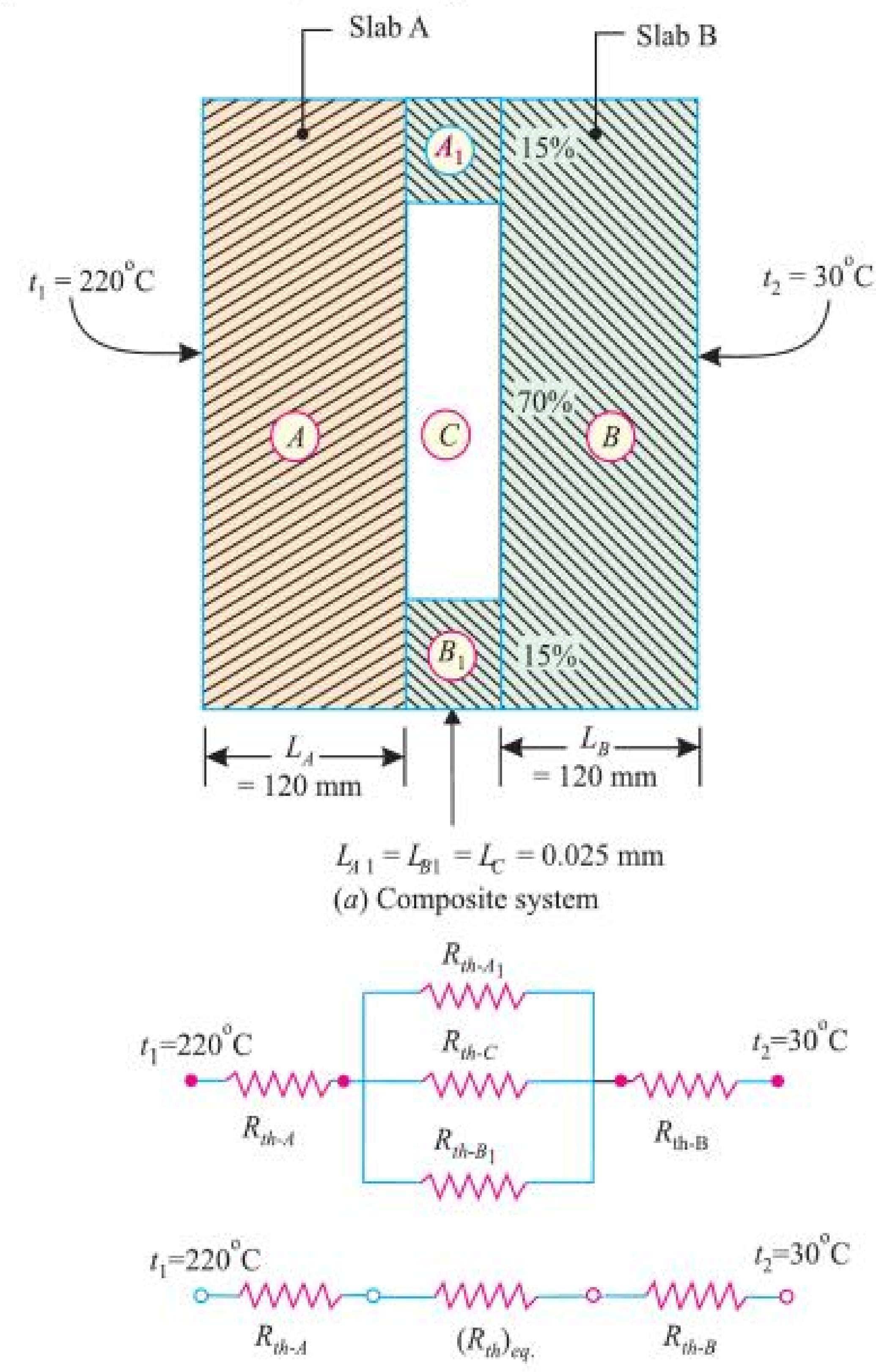


Fig. 2.28.

L_C = 0.025 mm = 0.000025 m;

$k_A = k_{A_1}$ = 14.5 W/m°C;

$k_B = k_{B_1}$ = 210 W/m°C;

k_C = 0.032 W/m°C; t_1 = 220°C; t_2 = 30°C

Rammed induction furnace.

(i) Heat flow through the system, Q :

$$R_{th-A} = \frac{0.12}{14.5 \times 1}$$

$$R_{th-A_1} = \frac{0.000025}{14.5 \times 0.15}, \; R_{th-C} = \frac{0.000025}{0.032 \times 0.7}$$

$$R_{th-B_1} = \frac{0.000025}{210 \times 0.15}; \; R_{th-B} = \frac{0.12}{210 \times 1}$$

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-A_1}} + \frac{1}{R_{th-C}} + \frac{1}{R_{th-B_1}}$$

$$\frac{1}{(R_{th})_{eq}} = \frac{14.5 \times 0.15}{0.000025} + \frac{0.032 \times 0.7}{0.000025} + \frac{210 \times 0.15}{0.000025}$$

or, $(R_{th})_{eq} = 7.419 \times 10^{-7}$

or, $(R_{th})_{total} = R_{th-A} + (R_{th})_{eq} + R_{th-B}$

$$= \frac{0.12}{14.5 \times 1} + 7.419 \times 10^{-7} + \frac{0.12}{210 \times 1} = 8.84 \times 10^{-3}$$

Hence, $Q = \dfrac{(\Delta t)_{overall}}{(R_{th})_{total}} = \dfrac{(220 - 30)}{8.84 \times 10^{-3}}$ = 21493 W or **21.493 kW (Ans.)**

(ii) The contact resistance and temperature drop in contact :

The contact resistance = **7.419×10^{-7} °C/W (Ans.)**

The temperature drop in contact = Q × contact resistance

= 21493 × 7.419 × 10^{-7} = **0.0159°C (Ans.)**

Example 2.19. *A mild steel tank of wall thickness 12 mm contains water at 95°C. The thermal conductivity of mild steel is 50 W/m°C, and the heat transfer coefficients for the inside and outside the tank are 2850 and 10 W/m²°C, respectively. If the atmospheric temperature is 15°C, calculate :*

(i) The rate of heat loss per m² of the tank surface area.

(ii) The temperature of the outside surface of the tank.

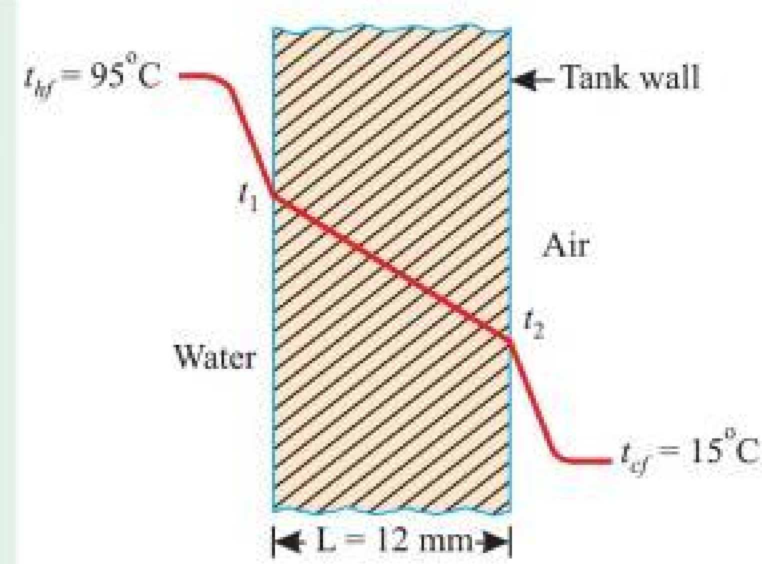


Fig. 2.29.

Solution. Refer to Fig. 2.29.

Thickness of mild steel tank wall

L = 12 mm = 0.012 m

Temperature of water, t_{hf} = 95°C

Temperature of air, t_{cf} = 15°C


Bricked induction furnace.

Thermal conductivity of mild steel,

$k = 50$ W/m°C

Heat transfer coefficients :

Hot fluid (water), $h_{hf} = 2850$ W/m^2°C

Cold fluid (air), $h_{cf} = 10$ W/m^2°C.

(*i*) Rate of heat loss per m² of the tank surface area, *q*:

Rate of heat loss per m^2 of tank surface,

$$q = UA\,(t_{hf} - t_{cf})$$

The overall heat transfer coefficient, U is found from the relation,

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L}{k} + \frac{1}{h_{cf}} = \frac{1}{2850} + \frac{0.012}{50} + \frac{1}{10}$$

$$= 0.0003508 + 0.00024 + 0.1 = 0.1006$$

$$\therefore \quad U = \frac{1}{0.1006} = 9.94 \text{ W/m}^2\,°\text{C}$$

$$\therefore \quad q = 9.94 \times 1 \times (95 - 15) = \mathbf{795.2\ W/m^2\ (Ans.)}$$

(*ii*) Temperature of the outside surface of the tank, t_2:

We know that, $q = h_{cf} \times 1 \times (t_2 - t_{cf})$

or, $795.2 = 10\,(t_2 - 15)$

or, $t_2 = \frac{795.2}{10} + 15 = \mathbf{94.52°C}$ **(Ans.)**

Example 2.20. *An electric hot plate is maintained at a temperature of 350°C, and is used to keep a solution boiling at 95°C. The solution is contained in a cast-iron vessel of wall thickness 25 mm, which is enamelled inside to a thickness of 0.8 mm. The heat transfer coefficient for the boiling solution is 5.5 kW/m²K, and the thermal conductivities of the cast iron and enamel are 50 and 1.05 W/mK, respectively. Calculate :*

(i) The overall heat transfer coefficient.

(ii) The rate of heat transfer per unit area. **(GATE, 1993)**

Solution. *Given :* $t_{heater} = 350°C$; $t_{solution} = 95°C$; $(\Delta x)_{C.I.} = 25$ mm $= 0.025$ m;

$(\Delta x)_{enamel} = 0.8$ mm $= 0.8 \times 10^{-3}$ m; $h_{solution} = 5.5$ kW/m^{2}K; $k_{C.I.} = 50$ W/mK;

$k_{enamel} = 1.05$ W/mK.

Refer Fig. 2.30.

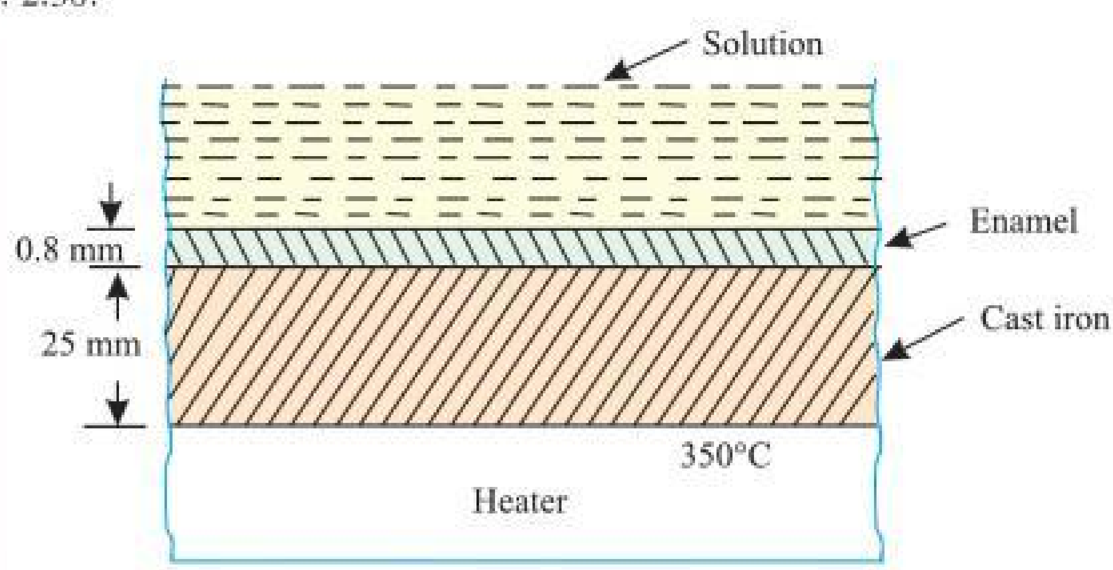


Fig. 2.30.

(*i*) The overall heat transfer coefficient, U :

$$\frac{1}{U} = \frac{(\Delta x)_{C.I.}}{k_{C.I.}} + \frac{(\Delta x)_{\text{enamel}}}{k_{\text{enamel}}} + \frac{1}{h_{\text{solution}}}$$

$$= \left(\frac{0.025}{50} + \frac{0.8 \times 10^{-3}}{1.05} + \frac{1}{5.5 \times 10^3}\right) = 1.444 \times 10^{-3} \text{ W}$$

$\therefore$ $U =$ **692.5 W/m²K** **(Ans.)**

(*ii*) The rate of heat transfer per unit area, Q :

$$Q = UA\,(t_{\text{heater}} - t_{\text{solution}})$$

$$= 692.5 \times 1 \times (350 - 9.5) = 176587.5 \text{ W/m}^2 \simeq \mathbf{176.6 \text{ kW/m}^2}$$ **(Ans.)**

Example 2.21. *The maximum operating temperature of a kitchen oven is set at 310°C. Due to seasonal variations, the kitchen temperature may vary from 12°C to 32°C. If the average heat transfer coefficient between the outside oven surface and kitchen air is 12 W/m²°C, determine the necessary thickness of fibre glass (k = 0.036 W/m°C) insulation to ensure that the outside surface temperature of oven does not exceed 45°C. Assume that the steady state conditions prevail and the thermal resistance of metal wall is negligible.*

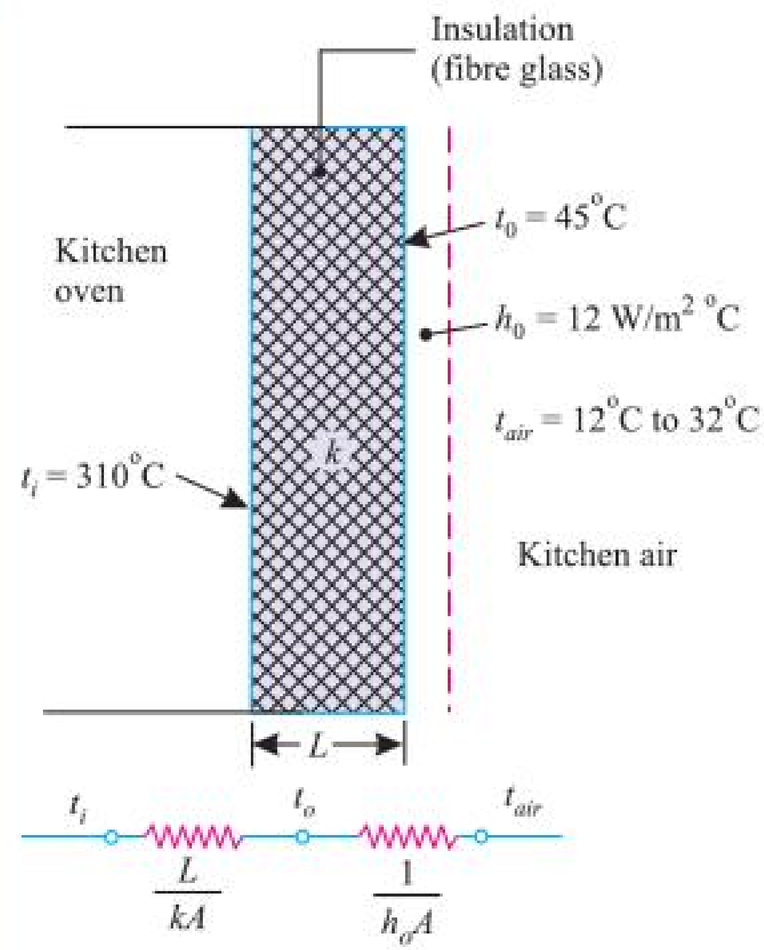


Fig. 2.31.

Solution. Refer Fig. 2.31.

Maximum temperature of kitchen oven,

$$t_i = 310^\circ\text{C}$$

Outside surface temperature of oven,

$$t_0 = 45^\circ\text{C}$$

Kitchen air temperature,

$$t_{air} = 12^\circ\text{C to } 32^\circ\text{C}$$

Thermal conductivity of insulating material (fibre glass)

$$k = 0.036 \text{W/m°C}$$

Heat transfer coefficient,

$$h_o = 12 \text{W/m}^2\text{°C}.$$

Thickness of insulation (fibre glass), *L* :

The rate of heat transfer per unit area of the wall is given as

$$q = \frac{Q}{A} = \frac{t_i - t_{air}}{L/k + 1/h_0}$$

Further, as the steady state conditions prevail, heat flow through each section is same.

$$\therefore \quad \frac{t_i - t_{air}}{L/k + 1/h_0} = \frac{t_0 - t_{air}}{1/h_0}$$

or, $$\frac{1}{h_0}(t_i - t_{air}) = (t_0 - t_{air})\ [L/k + 1/h_0]$$

$$= \frac{L}{k}(t_0 - t_{air}) + \frac{1}{h_0}(t_0 - t_{air})$$

or $\quad \frac{1}{h_0}[(t_i - t_{air}) - (t_0 - t_{air})] = \frac{L}{k}(t_0 - t_{air})$

or $$\frac{1}{h_0}(t_i - t_0) = \frac{L}{k}(t_0 - t_{air})$$

or $$L = \frac{k}{h_0}\left[\frac{t_i - t_0}{t_0 - t_{air}}\right]$$

The thickness of insulation (fibre glass) will be large for t_{air} = 32°C.

$\therefore$ $$L = \frac{0.036}{12}\left[\frac{310 - 45}{45 - 32}\right] = 0.06115 \text{ m or } \mathbf{61.15\text{ mm}} \quad \textbf{(Ans.)}$$

Example 2.22. *Hot gases at 1020°C flow past the upper surface of a gas turbine blade (the blade to be considered as a flat plate 1.2 mm thick) and the lower surface is cooled by air bled off the compressor. The thermal conductivity of blade material is 12 W/m°C and the heat transfer coefficients (convective) at the upper and lower surfaces are 2750 W/m²°C and 1400 W/m²°C respectively. Assuming steady state conditions have reached and the metallurgical considerations limit the blade temperature to 900°C, estimate the temperature of coolant-air.*

Solution. Temperature of hot gases (fluid) t_{hf} = 1020°C

Thickness of blade, L = 1.2 mm = 0.0012 m

Thermal conductivity of blade material, k = 12 W/m°C

Convective heat transfer coefficients :

Upper surface, h_{hf} = 2750 W/m²°C

Lower surface, h_{cf} = 1400 W/m²°C

Temperature at the upper surface of the blade,

t_1 = 900°C.

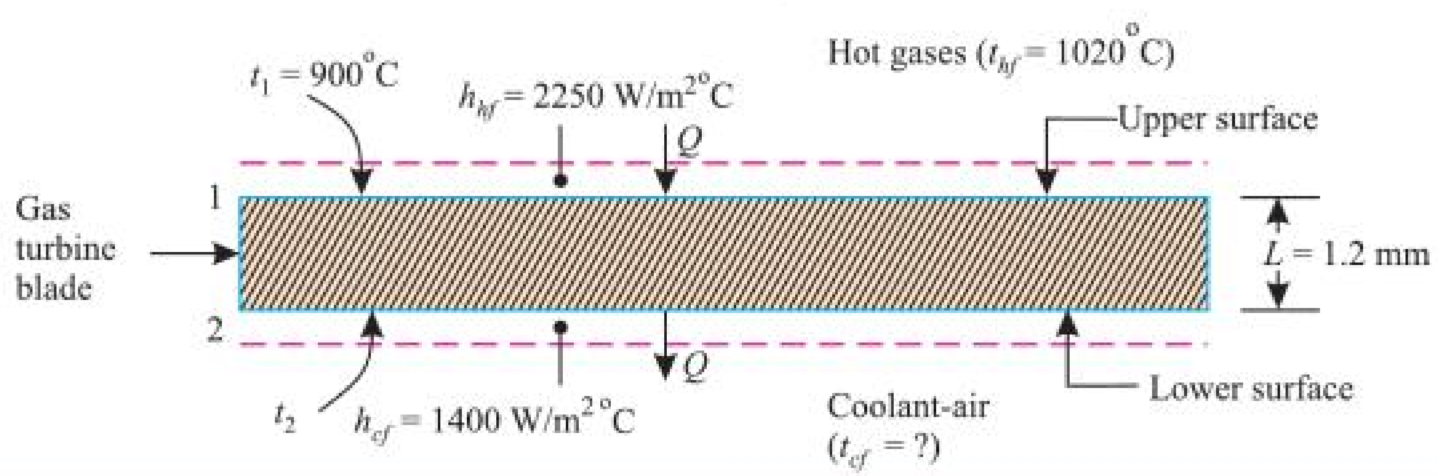


Fig. 2.32.

Temperature of the coolant air, t_{cf} :

The rate of heat transfer per unit area,

$$Q = h_{hf} \cdot A\,(t_{hf} - t_1)$$
$$= 2750 \times 1\,(1020 - 900) = 330000 \text{ W/m}^2$$

Since the heat transfer takes place under steady state conditions, therefore, this heat would be conducted across the gas turbine blade. Using Fourier's law of heat conduction, we have

$$Q = \frac{kA\,(t_1 - t_2)}{L}$$

where, t_2 = Temperature of the lower surface.

or, $$330000 = \frac{12 \times 1 \times (900 - t_2)}{0.0012}$$

or, $t_2 = 900 - \frac{330000 \times 0.0012}{12} = 867°C$

As the heat conducted across the blade would be transferred to the coolant-air, therefore,

$$Q = h_{cf} \cdot A\,(t_2 - t_{cf})$$

$$330000 = 1400 \times 1\,(867 - t_{cf})$$

$$\therefore \quad t_{cf} = 867 - \frac{330000}{1400} = \mathbf{631.28°C}$$

(Ans.)

Gas turbine blades.

Example 2.23. *A metal plate of 4mm thickness (k = 95.5 W/m°C) is exposed to vapour at 100°C on one side and cooling water at 25°C on the opposite side. The heat transfer coefficients on vapour side and water side are 14500 W/m²°C and 2250 W/m²°C respectively. Determine :*

(i) The rate of heat transfer,

(ii) The overall heat transfer coefficient, and

(iii) Temperature drop at each side of heat transfer.

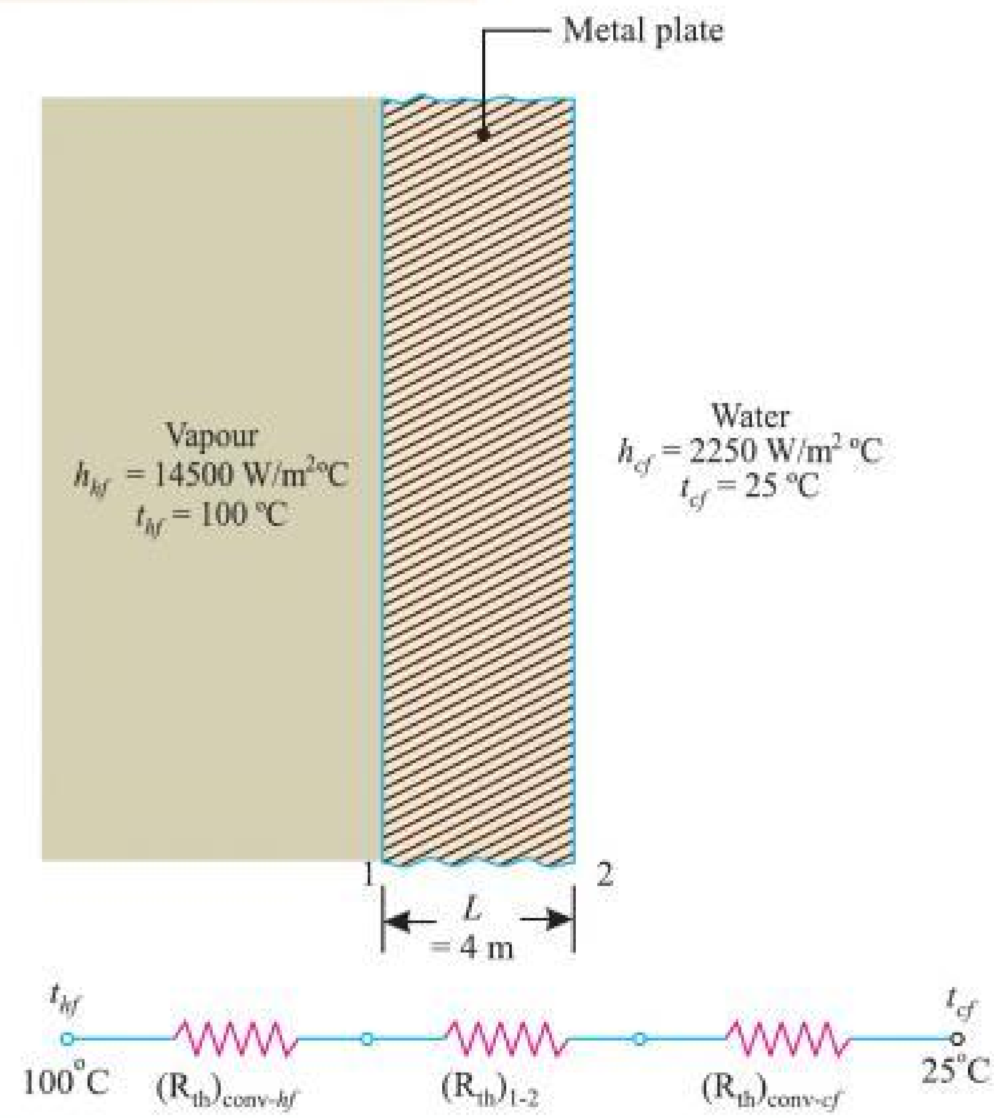


Fig. 2.33.

Solution. Thickness of metal plate, $L = 4\,\text{mm} = 0.004\,\text{m}$

Thermal conductivity of plate material, $k = 95.5\,\text{W/m°C}$

Temperature of vapour (hot fluid), $h_{hf} = 100°\text{C}$

Temperature of water (cold fluid), $t_{cf} = 25°\text{C}$

Heat transfer coefficients :

Vapour side, $h_{hf} = 14500\,\text{W/m}^2\text{°C}$

Water side, $h_{cf} = 2250\,\text{W/m}^2\text{°C}$

(*i*) The rate of heat transfer per m², *q* :

$$q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(t_{hf} - t_{cf})}{(R_{th})_{total}}$$

$$= \frac{(t_{hf} - t_{cf})}{(R_{th})_{conv.-hf} + (R_{th})_{1-2} + (R_{th})_{conv.-cf}}$$

$$= \frac{(100 - 25)}{\dfrac{1}{h_{hf}} + \dfrac{L}{k} + \dfrac{1}{h_{cf}}}$$

$$= \frac{75}{\dfrac{1}{14500} + \dfrac{0.004}{95.5} + \dfrac{1}{2250}}$$

$$= \frac{75}{6.896 \times 10^{-5} + 4.188 \times 10^{-5} + 44.444 \times 10^{-5}}$$

$$= 1.35 \times 10^5\,\text{W/m}^2$$

Hence, rate of heat transfer, $\boldsymbol{q = 1.35 \times 10^5\,\text{W/m}^2}$ **(Ans.)**

(*ii*) The overall heat transfer coefficient, U:

The rate of heat transfer through a composite system is given by

$$Q = U.A.\,(\Delta t)_{overall}$$

or, $$U = \frac{Q}{A.(\Delta t)} = \frac{q}{\Delta t} = \frac{1.35 \times 10^5}{(100 - 25)} = \mathbf{1800\ W/m^2\,°C}\ \textbf{(Ans.)}$$

(*iii*) Temperature drop at each side of heat transfer :

We know that $q = q_{hf} = q_{1-2} = q_{cf} = 1.35 \times 10^5\,\text{W/m}^2$

Now, $$q_{hf} = \frac{(\Delta t)_{hf}}{(R_{th})_{conv.-cf}}$$

or, $$(\Delta t)_{hf} = 1.35 \times 10^5 \times \frac{1}{14500} = 9.31°\text{C}$$

i.e., Temperature drop in vapour film = **9.31°C (Ans.)**

Similarly, $$q_{1-2} = \frac{(\Delta t)_{1-2}}{(R_{th})_{1-2}} \text{ or } (\Delta t)_{1-2} = 1.35 \times 10^5 \times \frac{0.004}{95.5} = 5.65°\text{C}$$

i.e., Temperature drop in the *metal* = **5.65°C (Ans.)**

and, $$q_{cf} = \frac{(\Delta t)_{cf}}{(R_{th})_{conv-cf}}$$

or, $$(\Delta t)_{cf} = 1.35 \times 10^5 \times \frac{1}{2250} = 60°\text{C}$$

i.e., Temperature drop in the *water film* = **60°C (Ans.)**

Example 2.24. *The interior of a refrigerator having inside dimensions of 0.5 m × 0.5 m base area and 1m height, is to be maintained at 6°C. The walls of the refrigerator are constructed of two mild steel sheets 3mm thick (k = 46.5 W/m°C) with 50 mm of glass wool insulation (k = 0.046 W/m°C) between them. If the average heat transfer coefficients at the outer and inner surfaces are 11.6 W/m²°C and 14.5 W/m²°C respectively, calculate :*

(i) The rate at which heat must be removed from the interior to maintain the specified temperature in the kitchen at 25°C, and

(ii) The temperature on the outer surface of the metal sheet.

Solution. Refer to Fig. 2.34.

$$L_A = L_C = 3\text{mm} = 0.003\,\text{m}$$
$$L_B = 50\,\text{mm} = 0.05\,\text{m}$$
$$k_A = k_C = 46.5\ \text{W/m°C};$$
$$k_B = 0.046\ \text{W/m° C}$$
$$h_0 = 11.6\ \text{W/m}^2\text{°C};\ h_i = 14.5\ \text{W/m}^2\text{°C}$$
$$t_0 = 25\text{°C};\ t_i = 6\text{°C}.$$

The total area through which heat is coming into the refrigerator,

$$A = 0.5 \times 0.5 \times 2 + 0.5 \times 1 \times 4 = 2.5\ \text{m}^2$$

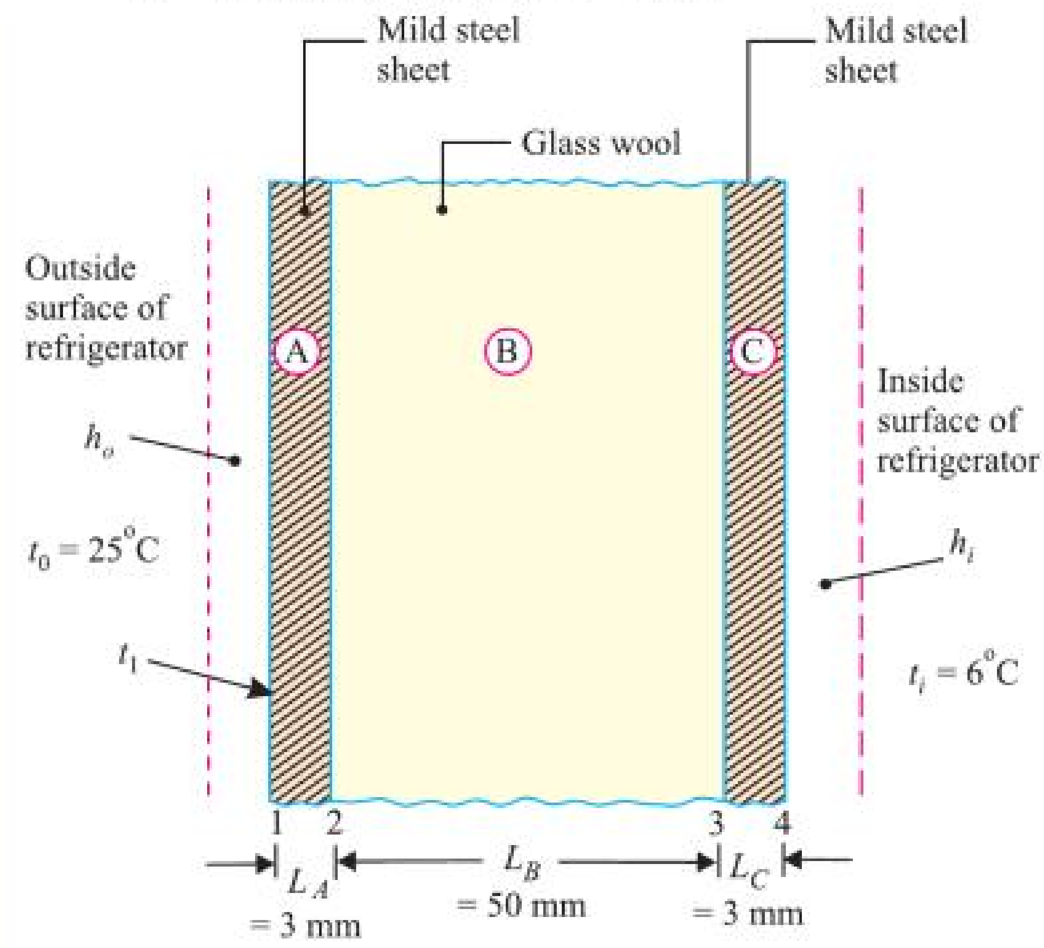


Fig. 2.34.

(i) The rate of removal of heat, Q :

$$Q = \frac{A(t_0 - t_i)}{\frac{1}{h_0} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_i}}$$

$$= \frac{2.5(25-6)}{\frac{1}{11.6} + \frac{0.003}{46.5} + \frac{0.05}{0.046} + \frac{0.003}{46.5} + \frac{1}{14.5}} = \mathbf{38.2\ W} \quad \textbf{(Ans.)}$$

(*ii*) The temperature at the outer surface of the metal sheet, *t* :

$$Q = h_0 A\,(25 - t_1)$$

or, $$38.2 = 11.6 \times 2.5\,(25 - t_1)$$

or, $$t_1 = 25 - \frac{38.2}{11.6 \times 2.5} = \mathbf{23.68°\,C} \quad \textbf{(Ans.)}$$

Example 2.25. *Calculate the rate of heat flow per m² through a furnace wall consisting of 200 mm thick inner layer of chrome brick, a centre layer of kaolin brick 100 mm thick and an outer layer of masonry brick 100 mm thick. The unit surface conductance at the inner surface is 74 W/m²°C and the outer surface temperature is 70°C. The temperature of the gases inside the furnace is 1670°C. What temperatures prevail at the inner and outer surfaces of the centre layer ?*

Take : $k_{chrome\ brick} = 1.25$ *W/m°C;* $k_{kaolin\ brick} = 0.074$ *W/m°C;* $k_{masonry\ brick} = 0.555$ *W/m°C*

Assume steady heat flow. **(M.U.)**

Solution. Thickness of chrome bricks, L_A = 200 mm = 0.2 m

Thickness of kaolin bricks, L_B = 100 mm = 0.1 m

Thickness of masonry bricks, L_C = 100 mm = 0.1m

Thermal conductivities : k_A = 1.25 W/m°C;

k_B = 0.074 W/m°C; k_C = 0.555 W/m°C;

The unit surface conductance, h_{hf} = 74 W/m²°C

Temperature of hot fluid, $t_{hf}\ (= t_g)$ = 1670°C

Temperature of the outer surface, t_4 = 70°C

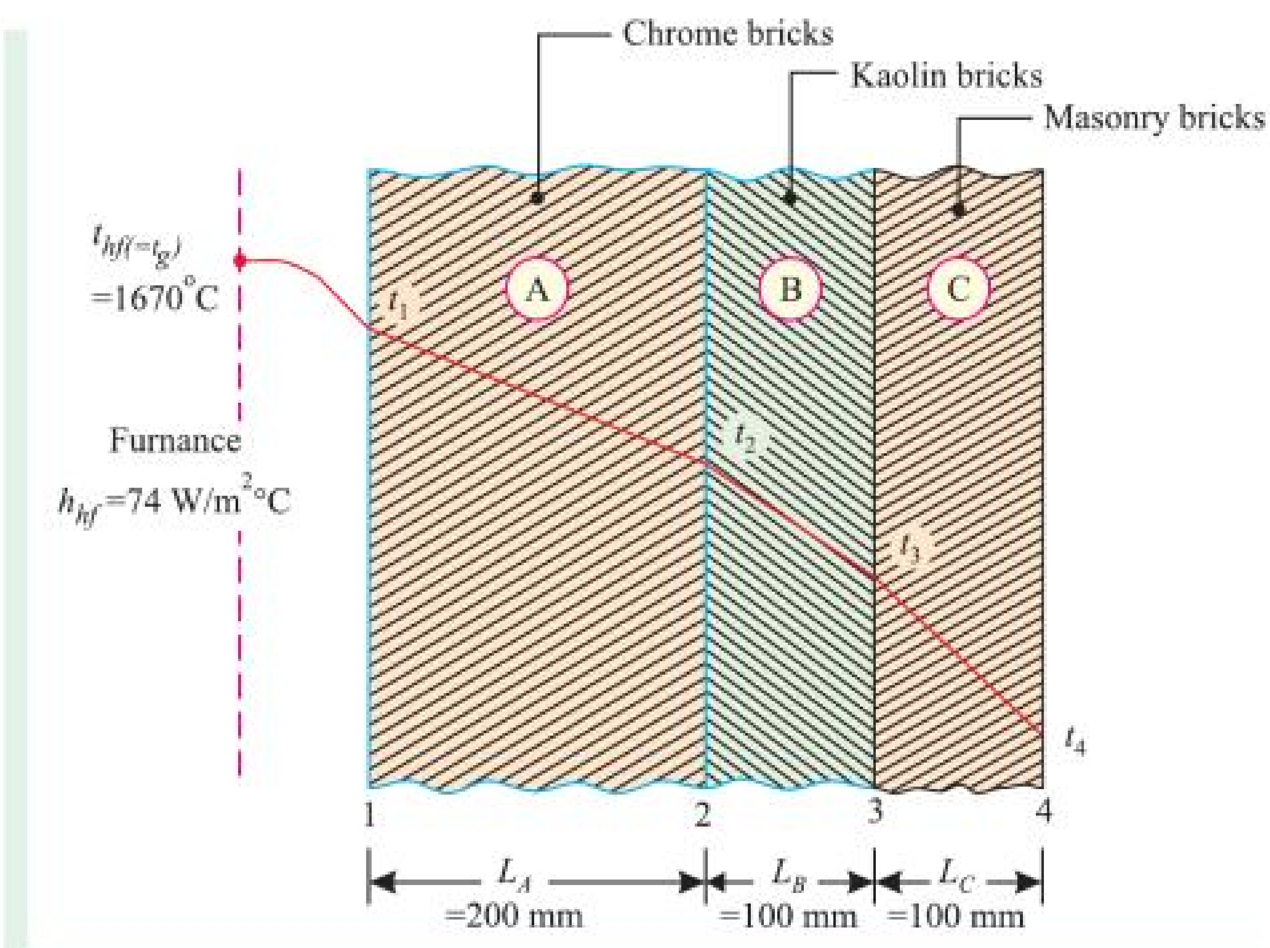


Fig. 2.35

(*i*) Rate of heat flow per m², *q* :

$$q = \frac{(t_{hf} - t_4)}{\dfrac{1}{h_{hf}} + \dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}}$$

$$= \frac{(1670 - 70)}{\frac{1}{74} + \frac{0.2}{1.25} + \frac{0.1}{0.074} + \frac{0.1}{0.555}}$$

$$= \frac{1600}{0.0135 + 0.16 + 1.351 + 0.1802}$$

$$= \mathbf{938.58\ W/m^2} \qquad \textbf{(Ans.)}$$

Close-up view of turbine blades.

(ii) Temperatures; t_2, t_3:

The heat flow is given by

$$q = \frac{(t_{hf} - t_1)}{1/h_{hf}} = \frac{(t_1 - t_2)}{L_A/k_A} = \frac{(t_2 - t_3)}{(L_B/k_B)}$$

$$\therefore \qquad 938.58 = \frac{1670 - t_1}{1/74}$$

or $\qquad t_1 = 1670 - 938.58 \times \frac{1}{74} = 1657.3°C$

Similarly, $\quad 938.58 = \frac{(1657.3 - t_2)}{0.2/1.25}$ or $t_2 = 1657.3 - 938.58 \times \frac{0.2}{1.25} = \mathbf{1507.1°C}$ **(Ans)**

$938.58 = \frac{(1507.1 - t_3)}{(0.1/0.074)}$ or $t_3 = 1507.1 - 938.58 \times \frac{0.1}{0.074} = \mathbf{238.7°C}$ **(Ans.)**

Example 2.26. *A cold storage room has walls made of 220 mm of brick on the outside, 90 mm of plastic foam, and finally 16 mm of wood on the inside. The outside and inside air temperatures are 25°C and –3°C respectively. If the inside and outside heat transfer coefficients are respectively 30 and 11 W/m²°C, and the thermal conductivities of brick, foam and wood are 0.99, 0.022 and 0.17 W/m°C respectively, determine :*

(i) The rate of heat removal by refrigeration if the total wall area is 85 m²;

(ii) The temperature of the inside surface of the brick.

Solution. Refer Fig. 2.36.

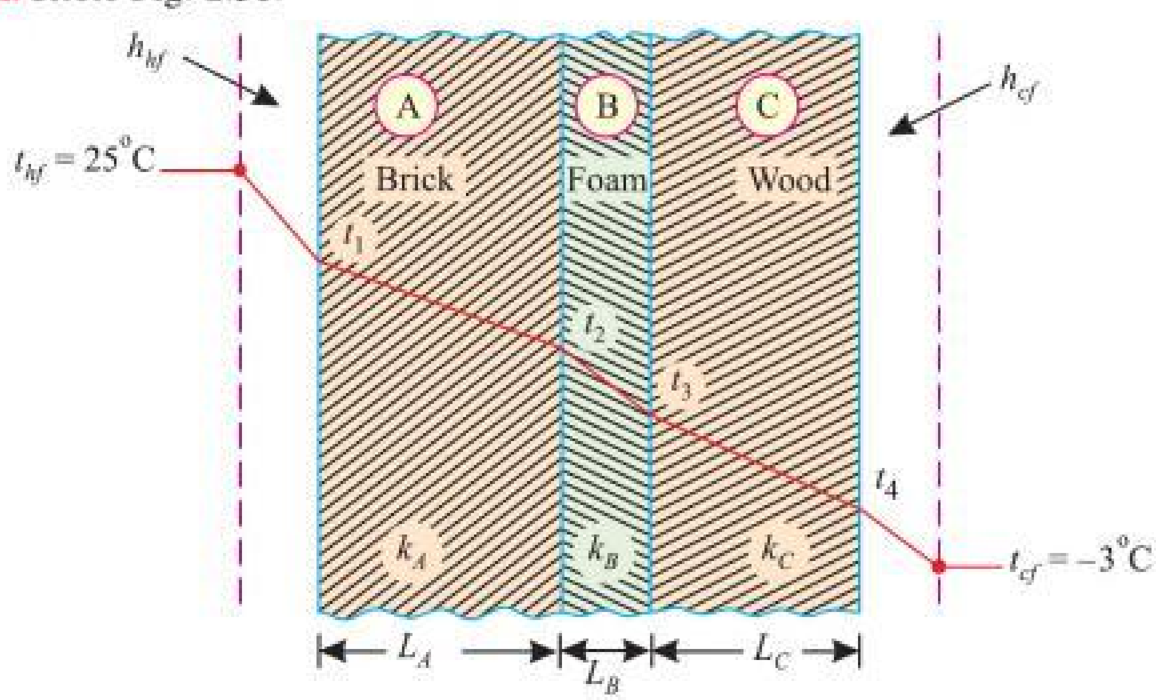


Fig. 2.36.

Thickness of brick wall, $\qquad L_A = 220$ mm $= 0.22$ m

Thickness of plastic foam, $\qquad L_B = 90$ mm $= 0.09$ m

Thickness of wood, $L_C = 16 \text{ mm} = 0.016 \text{ m}$

Temperature of hot fluid (air), $t_{hf} = 25°C$

Temperature cold fluid (air), $t_{cf} = -3°C$

Heat transfer coefficients :

Hot fluid (air), $h_{hf} = 11 \text{ W/m}^2°C$

Cold fluid (air), $h_{cf} = 30 \text{ W/m}^2°C$

Thermal conductivities :

Brick, $k_A = 0.99 \text{ W/m°C}$

Foam, $k_B = 0.022 \text{ W/m°C}$

Wood, $k_C = 0.17 \text{ W/m°C}$

Total wall area, $A = 85 \text{ m}^2$

(*i*) Rate of heat transfer, *Q* :

$$Q = UA\,(t_{hf} - t_{cf})$$

The overall heat transfer co-efficient (U) may be found from the following relation :

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}$$

$$= \frac{1}{11} + \frac{0.22}{0.99} + \frac{0.09}{0.022} + \frac{0.06}{0.17} + \frac{1}{30}$$

$$= 0.091 + 0.222 + 4.091 + 0.094 + 0.033 = 4.531$$

$$\therefore \quad U = \frac{1}{4.531} = 0.2207 \text{ W/m}^2\,°C$$

$$\therefore \quad Q = 0.2207 \times 85\,[25 - (-3)] = \mathbf{525.26\ W} \quad \textbf{(Ans.)}$$

(*ii*) Temperature of inside surface of the brick, t_2 :

$$Q = U.A\,(t_{hf} - t_2)$$

or,

$$525.26 = \left[\frac{1}{\frac{1}{h_{hf}} + \frac{L_A}{k_A}}\right] A\,(t_{hf} - t_2)$$

$$= \left[\frac{1}{\frac{1}{11} + \frac{0.22}{0.99}}\right] \times 85\,(25 - t_2) = 271.45(25 - t_2)$$

$$\therefore \quad t_2 = 25 - \frac{525.26}{271.45} = \mathbf{23.06°C} \quad \textbf{(Ans.)}$$

Example 2.27. *A furnace wall is made of composite wall of total thickness 550 mm. The inside layer is made of refractory material (K = 2.3 W/mK) and outside layer is made of an insulating material (K = 0.2 W/mK). The mean temperature of the glass inside the furnace is 900°C and inter-face temperature is 520°C. The heat transfer coefficient between the gases and inner surface can be taken as 230 W/m² °C and between the outside surface and atmosphere as 46 W/m²°C. Taking air temperature = 30°C, calculate :*

(i) Required thickness of each layer,

(ii) The rate of heat loss per m² area, and

(iii) The temperatures of surface exposed to gases and of surface exposed to atmosphere.

(B.U., Dec., 2002)

Solution. *Given* : Total thickness of wall, $L_A + L_B = 550$ mm $= 0.55$ m; $k_A = 2.3$ W/mK; $k_B = 0.2$ W/mK; $t_{hf} = 900°C$; $t_2 = 520°C$; $h_{hf} = 230$ W/m^2°C; $h_{cf} = 46$ W/m^2°C; $t_{cf} = 30°C$.

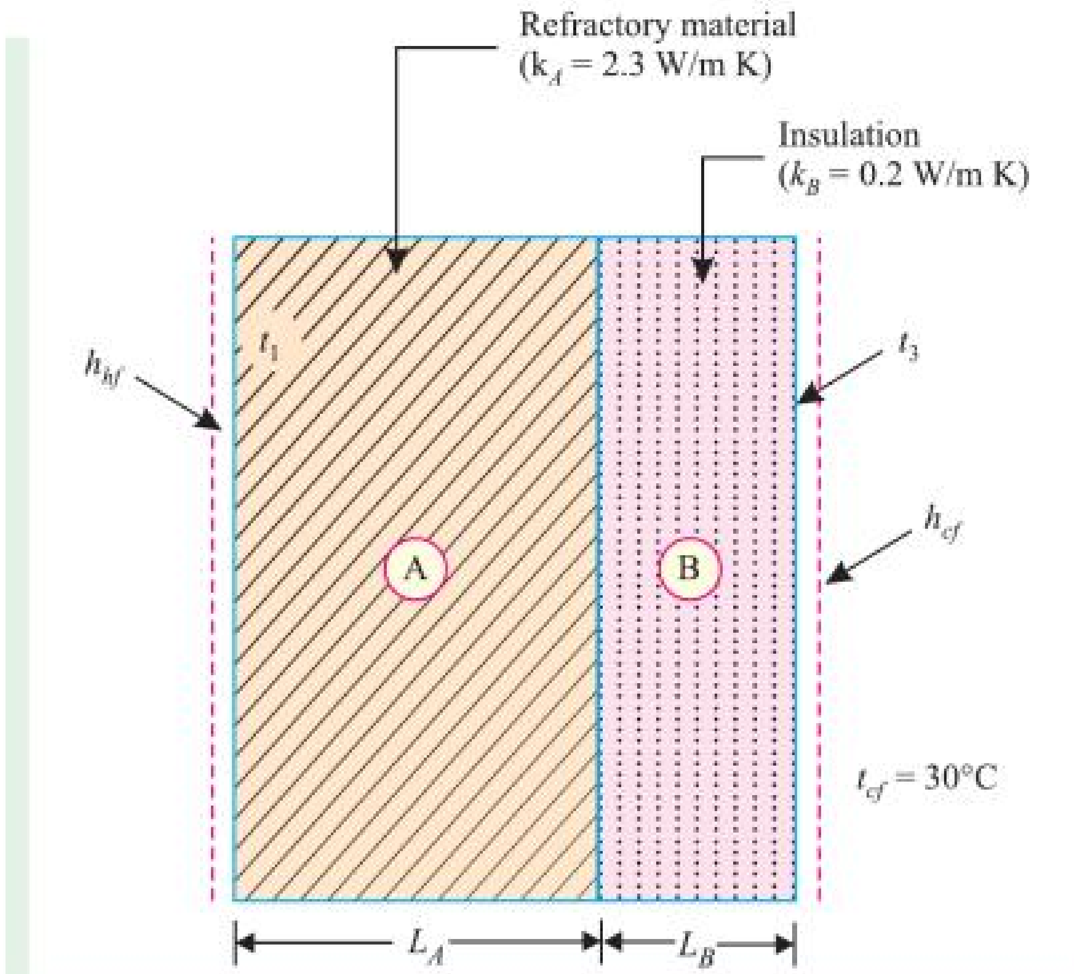


Fig. 2.37.

Thickness of each layer, L_A, L_B :

The heat flow rate,

$$q = \frac{Q}{A} = \frac{t_{hf} - t_2}{\dfrac{1}{h_{hf}} + \dfrac{L_A}{k_A}}$$

$$= \frac{t_2 - t_{cf}}{\dfrac{0.55 - L_A}{k_B} + \dfrac{1}{h_{cf}}}$$

Equating (*i*) and (*ii*) we get,

$$\frac{t_{hf} - t_2}{\dfrac{1}{h_{hf}} + \dfrac{L_A}{k_A}} = \frac{t_2 - t_{cf}}{\dfrac{0.55 - L_A}{k_B} + \dfrac{1}{h_{cf}}}$$

$$\frac{900 - 520}{\dfrac{1}{230} + \dfrac{L_A}{2.3}} = \frac{520 - 30}{\dfrac{0.55 - L_A}{0.22} + \dfrac{1}{46}}$$

$$380\left[\frac{0.55 - L_A}{0.2} + \frac{1}{46}\right] = 490\left[\frac{1}{230} + \frac{L_A}{2.3}\right]$$

$$1045 - 1900\, L_A + 8.26 = 2.13 + 213\, L_A$$

$$2113\, L_A = 1051.13$$

or, $L_A = 0.497$ m or **497 mm** **(Ans.)**

and, $L_B = 550 - 497$ m $=$ **53 mm** **(Ans.)**

(i) The rate of heat loss per m² area, q :

$$q = \frac{t_{hf} - t_2}{\frac{1}{h_{hf}} + \frac{L_A}{k_A}} = \frac{900 - 520}{\frac{1}{230} + \frac{0.497}{2.3}}$$

$$= \frac{380}{0.004348 + 0.216} = \mathbf{1724.5\ W/m^2} \quad \textbf{(Ans.)}$$

Example 2.28. *The inside temperature of furnace wall, 200 mm thick, is 1350°C. The mean thermal conductivity of wall material is 1.35 W/m°C. The heat transfer coefficient of the outside surface is a function of temperature difference and is given by*

$$h = 7.85 + 0.08\ \Delta t$$

where Δt is the temperature difference between outside wall surface and surroundings. Determine the rate of heat transfer per unit area if the surrounding temperature is 40°C.

Solution. Thickness of wall,

$$L = 200\ \text{mm} = 0.2\text{m}$$

Temperature of inner surface of wall,

$$t_1 = 1400°\text{C}$$

Temperature of air (cold fluid),

$$t_{cf} = 40°\text{C}$$

Mean thermal conductivity of wall material,

$$k = 1.35\ \text{W/m°C}$$

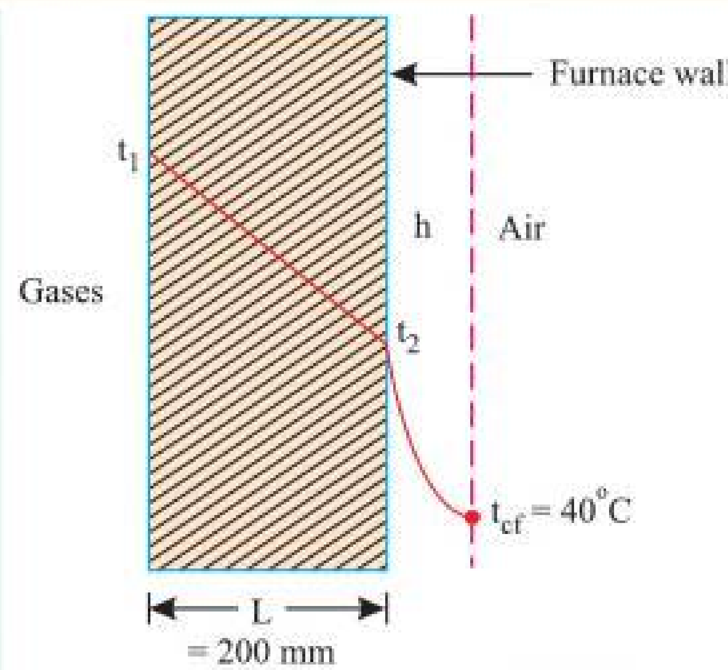


Fig. 2.38.

Rate of heat transfer per unit area, q :

$$q = \frac{(t_1 - t_2)}{L/k} = \frac{(t_2 - t_{cf})}{1/h}$$

$$\frac{(1350 - t_2)}{0.2/1.35} = h(t_2 - 40)$$

Furnace walls and electric wils.

or, $$6.75\,(1350 - t_2) = [7.85 + 0.08\,(t_2 - 40)]\,(t_2 - 40)$$

or, $$9112.5 - 6.75t_2 = 7.85\,(t_2 - 40) + 0.08\,(t_2 - 40)^2$$

or, $$9112.5 - 6.75t_2 = 7.85t_2 - 314 + 0.08\,(t_2^2 - 80t_2 + 1600)$$

or, $$t_2^2 - 80\,t_2 + 1600 = \frac{9112.5 - 6.75\,t_2 - 7.85\,t_2 + 314}{0.08}$$

$$= 117831 - 182.5\,t_2$$

or, $$t_2^2 + 102.5t_2 - 116231 = 0$$

or, $$t_2 = \frac{-102.5 \pm \sqrt{(102.5)^2 + 4 \times 116231}}{2}$$

$$= \frac{-102.5 \pm 689.5}{2} = 293.5°\text{C}$$

∴ $$q = \frac{(1350 - 293.5)}{0.2/1.35} = \mathbf{7131.37\ W/m^2}\ \textbf{(Ans.)}$$

Exampel 2.29. *The furnace wall consists of 120 mm wide refractory brick and 120 mm wide insulating fire brick separated by an air gap. The outside wall is covered with a 12 mm thickness of plaster. The inner surface of the wall is at 1090°C and the room temperature is 20°C. The heat transfer coefficient from the outside wall surface to the air in the room is 18 W/m²°C, and the resistance to heat flow of the air gap is 0.16 K/W. If the thermal conductivities of the refractory brick, insulating fire brick, and plaster are 1.6, 0.3 and 0.14 W/mK, respectively calculate :*

(i) Rate at which heat is lost per m² of the wall surface;

(ii) Each interface temperature; and

(iii) Temperature of the outside surface of the wall.

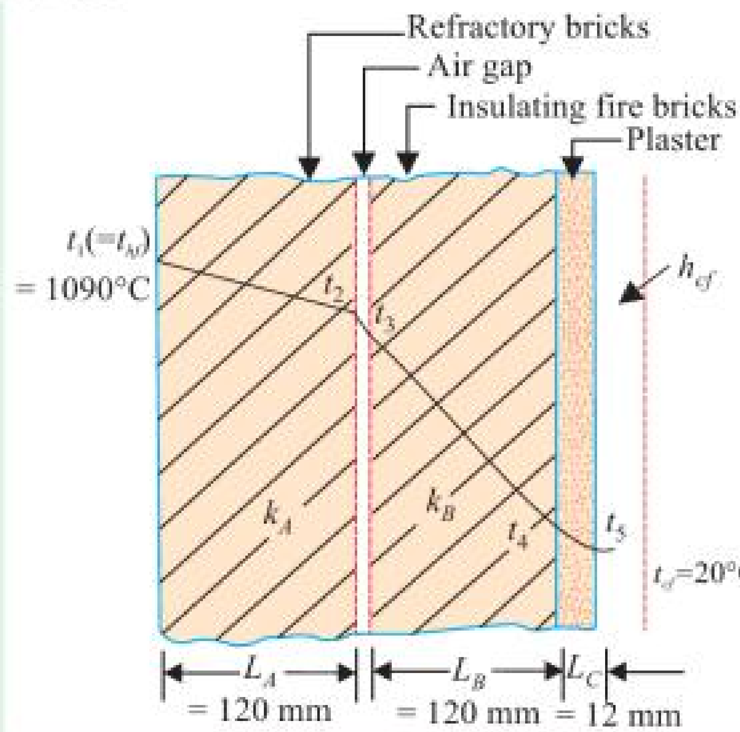


Fig. 2.39.

Solution. Refer Fig. 2.39.

Thickness of refractory brick, $L_A = 120\text{ mm} = 0.12\text{ m}$

Thickness of insulating fire brick, $L_B = 120\text{ mm} = 0.12\text{ m}$

Thickness of plaster, $L_C = 12\text{ mm} = 0.012\text{ m}$

Heat transfer coefficient from the outside wall surface to the air in the room,

$$h_{cf} = 18\text{ W/m}^2\text{°C}$$

Resistance of air gap to heat flow = 0.16 K/W

Thermal conductivities :

Refractory brick, $k_A = 1.6\text{ W/m°C}$

Insulating fire brick, $k_B = 0.3\text{ W/m°C}$

Plaster, $k_C = 0.14\text{ W/m°C}$.

Temperatures : $t_{hf} = 1090°\text{C}$; $t_{cf} = 20°\text{C}$

Consider 1m² of surface area.

(i) Rate of heat loss per m² of surface area, q :

$$q = \frac{(t_{hf} - t_{cf})}{\frac{L_A}{k_A} + \text{air gap resistance} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

$$= \frac{(1090 - 20)}{\frac{0.12}{1.6} + 0.16 + \frac{0.12}{0.3} + \frac{0.012}{0.14} + \frac{1}{18}}$$

$$= \frac{1070}{0.075 + 0.16 + 0.4 + 0.0857 + 0.0555} = 1378.5 \text{ W or } 1.3785 \text{ kW}$$

i.e., Rate of heat loss per m² of surface area = **1.3785 kW** **(Ans.)**

(ii) Temperatures at interfaces, t_2, t_3, t_4:

$$Q = 1378.5 = \frac{1090 - t_2}{L_A / k_A} = \frac{1090 - t_2}{0.12/1.6} = \frac{1090 - t_2}{0.075}$$

∴ $t_2 = 1090 - 1378.5 \times 0.075 =$ **986.6°C** **(Ans.)**

Also, $$Q = 1378.5 = \frac{t_2 - t_3}{\text{air gap resistance}} = \frac{986.6 - t_3}{0.16}$$

∴ $t_3 = 986.6 - 1378.5 \times 0.16 =$ **766.04°C** **(Ans.)**

Again, $$Q = 1378.5 = \frac{t_3 - t_4}{L_B / k_B} = \frac{766.04 - t_4}{0.12/0.3} = \frac{766.04 - t_4}{0.4}$$

∴ $t_4 = 766.04 - 1378.5 \times 0.4 =$ **214.64°C** **(Ans.)**

(iii) Temperature of the outside surface of the wall, t_5 :

$$Q = 1378.5 = \frac{t_4 - t_5}{L_C / k_C} = \frac{214.64 - t_5}{0.012/0.14} = \frac{214.64 - t_5}{0.0857}$$

∴ $t_5 = 214.64 - 1378.5 \times 0.0857 =$ **96.5°C** **(Ans.)**

Example 2.30. *A furnace wall is made up of three layers of thicknesses 250 mm, 100 mm and 150 mm with thermal conductivities of 1.65, k and 9.2 W/m°C respectively. The inside is exposed to gases at 1250°C with a convection coefficient of 25 W/m²°C and the inside surface is at 1100°C, the outside surface is exposed to air at 25°C with convection coefficient of 12 W/m²°C. Determine :*

(i) The unknown thermal conductivity 'k';

(ii) The overall heat transfer coefficient;

(iii) All surface temperatures.

Solution.
L_A = 250 mm = 0.25 m;
L_B = 100 mm = 0.1 m;
L_C = 150 mm = 0.15 m;
k_A = 1.65W/m°C;
k_C = 9.2 W/m°C;
t_{hf} = 1250°C, t_1 = 1100°C;
h_{hf} = 25 W/m²°C; h_{cf} = 12 W/m²°C.

(i) Thermal conductivity, k (= k_B) :

The rate of heat transfer per unit area of the furnace wall,

$$q = h_{hf}(t_{hf} - t_1)$$
$$= 25(1250 - 1100) = 3750 \text{ W/m}^2$$

Also,
$$q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}}$$

or
$$q = \frac{(t_{hf} - t_{cf})}{(R_{th})_{conv-hf} - R_{th-A} + R_{th-B} + R_{th-C} + (R_{th})_{conv-cf}}$$

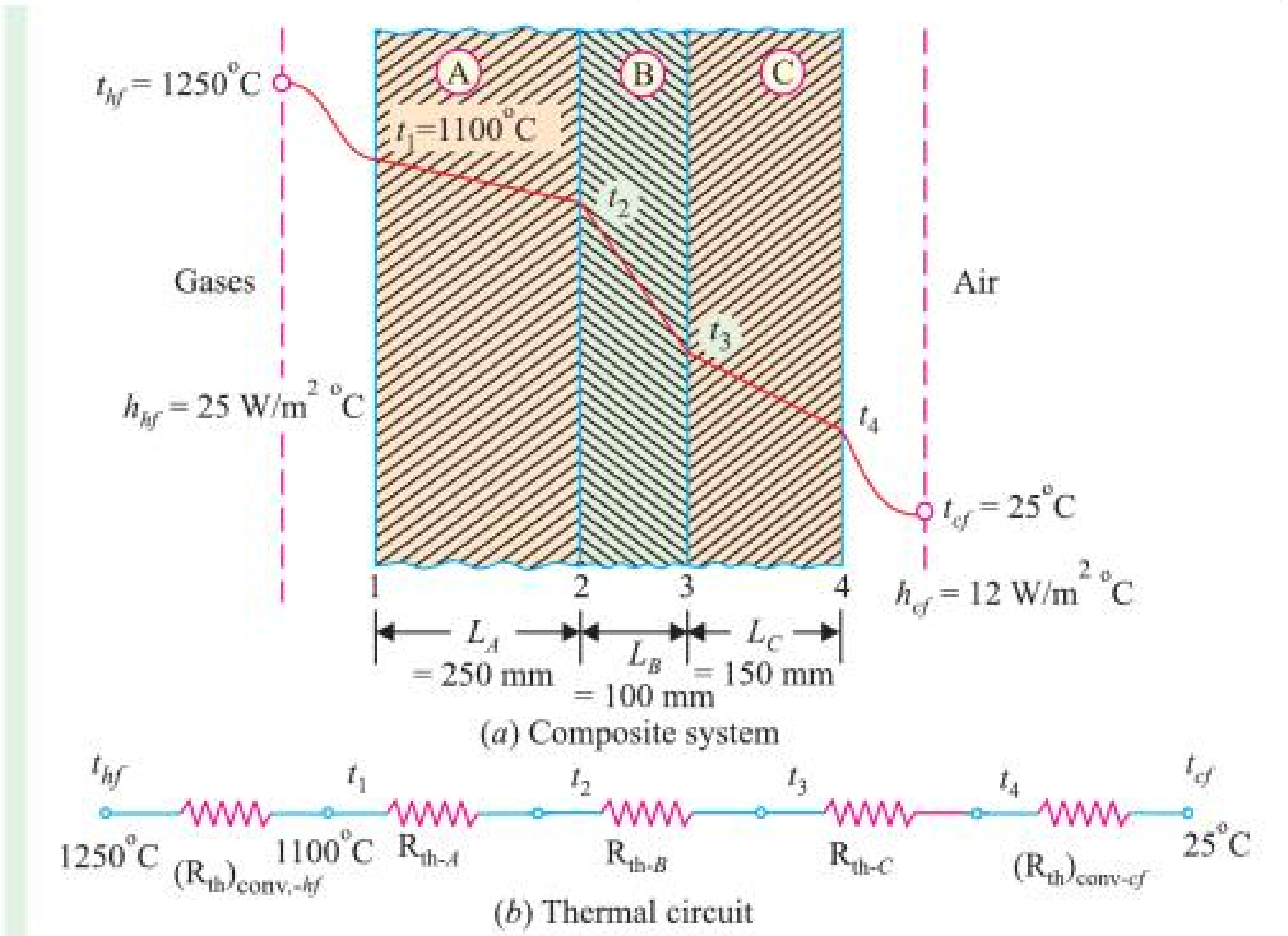


Fig. 2.40.

or,
$$3750 = \frac{(1250 - 25)}{\frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}}$$

or,
$$3750 = \frac{1225}{\frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{k_B} + \frac{0.15}{9.2} + \frac{1}{12}}$$

$$= \frac{1225}{0.04 + 0.1515 + \frac{0.1}{k_B} + 0.0163 + 0.0833} = \frac{1225}{0.2911 + \frac{0.1}{k_B}}$$

or,
$$3750\left(0.289 + \frac{0.1}{k_B}\right) = 1225$$

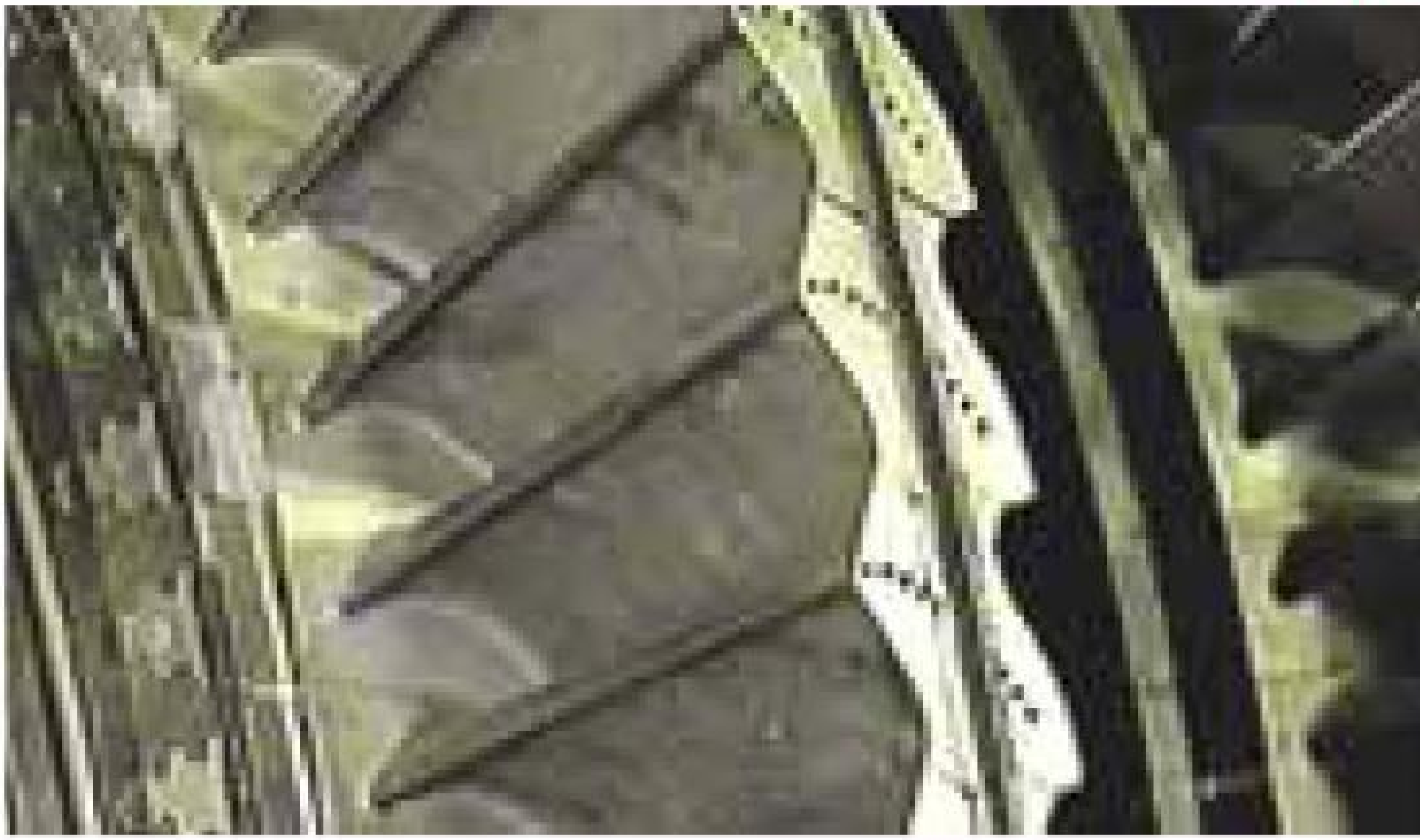
or,
$$\frac{0.1}{k_B} = \frac{1225}{3750} - 0.2911 = 0.0355$$

∴
$$k_B = k = \frac{0.1}{0.0355}$$

= 2.817 W/m°C (Ans.)

(*ii*) The overall heat transfer coefficient, *U* :

The overall heat transfer coefficient, $U = \frac{1}{(R_{th})_{total}}$



Effects of heat on turbine blades is an important consideration while designing.

Now, $$(R_{th})_{\text{total}} = \frac{1}{25} + \frac{0.25}{1.65} + \frac{0.1}{2.817} + \frac{0.15}{9.2} + \frac{1}{12}$$

$$= 0.04 + 0.1515 + 0.0355 + 0.0163 + 0.0833 = 0.3266 \text{ °C m}^2\text{/W}$$

∴ $$U = \frac{1}{(R_{th})_{\text{total}}} = \frac{1}{0.3266} = \mathbf{3.06\ W/m^2 °C\ (Ans.)}$$

(*iii*) All surface temperatures; t_1, t_2, t_3, t_4:

$$q = q_A = q_B = q_C$$

or, $$3750 = \frac{(t_1 - t_2)}{L_A / k_A} = \frac{(t_2 - t_3)}{L_B / k_B} = \frac{(t_3 - t_4)}{L_C / k_C}$$

or, $$3750 = \frac{(1100 - t_2)}{0.25 / 1.65}$$

or, $$t_2 = 1100 - 3750 \times \frac{0.25}{1.65} = \mathbf{531.8° C\ (Ans.)}$$

Similarly, $$3750 = \frac{(531.8 - t_3)}{0.1 / 2.817}$$

or, $$t_3 = 531.8 - 3750 \times \frac{0.1}{2.817} = \mathbf{398.6° C\ (Ans.)}$$

and $$3750 = \frac{(398.6 - t_4)}{(0.15 / 9.2)}$$

or, $$t_4 = 398.6 - 3750 \times \frac{0.5}{9.2} = \mathbf{337.5° C\ (Ans.)}$$

[Check using outside convection,

$$q = \frac{(337.5 - 25)}{1 / h_{cf}} = \frac{(337.5 - 25)}{1/12} = 3750 \text{ W/m}^2 \text{]}$$

Substituting the values in the above equation, we get,

$$1000 = 0.0225\,(t_{max} - 25)\left[\frac{1}{\frac{0.02}{50} + \frac{1}{200}} + \frac{1}{\frac{0.01}{0.2} + \frac{1}{50}}\right]$$

$$= 0.0225\,(t_{max} - 25)\left[\frac{1}{(0.0004 + 0.005)} + \frac{1}{(0.05 + 0.02)}\right]$$

$$= 0.0225\,(t_{max} - 25) \times 199.47$$

$\therefore$ $$t_{max} = 25 + \frac{100}{0.0225 \times 199.47} = \mathbf{247.8°\,C} \text{ (Ans.)}$$

(*ii*) Outer surface temperature of two slabs; t_1, t_2:

$$Q_A = \frac{k_A \,.\, A\,(t_{max} - t_1)}{L_A} = h_1 \,.\, A\,(t_1 - t_a)$$

or, $$\frac{50\,(247.8 - t_1)}{0.02} = 200\,(t_1 - 25)$$

or, $$2500\,(247.8 - t_1) = 200\,(t_1 - 25)$$

or, $$247.8 - t_1 = \frac{200}{2500}(t_1 - 25) = 0.08\,t_1 - 2$$

or, $$1.08\,t_1 = 249.8$$

$\therefore$ $$t_1 = \frac{249.8}{1.08} = \mathbf{231.3°\,C} \text{ (Ans.)}$$

Similarly, $$Q_B = \frac{k_B \,.\, A\,(t_{max} - t_2)}{L_B} = h_2 \,.\, A\,(t_2 - t_a)$$

or, $$\frac{0.2\,(247.8 - t_2)}{0.01} = 50\,(t_2 - 25)$$

or, $$20\,(247.8 - t_2) = 50\,(t_2 - 25)$$

or, $$(247.8 - t_2) = \frac{50}{20}(t_2 - 25) = 2.5\,t_2 - 62.5$$

or, $$3.5\,t_2 = 310.3$$

$\therefore$ $$t_2 = \frac{310.3}{3.5} = \mathbf{88.6°\,C} \text{ (Ans.)}$$

Equivalent electrical/thermal circuit is shown in Fig. 2.41 (*b*).

Example 2.32. *The following data relate to furnace of a steam boiler :*

Temperature of gases in the furnace .... *1300°C*

Temperature of air in the boiler room .... *30°C*

Thickness of refractory material .... *250 mm*

The heat transfer coefficient from gases to refractory wall .... *30 W/m²°C*

The heat transfer coefficient from outside surface to surrounding air .... *10W/m²°C*

Thermal conductivity of refractory material : k = 0.28 (1 + 0.000833 t) W/m°C

Thermal conductivity of diatomite layer : k = 0.113 (1 + 0.000206 t) W/m°C

Estimate the thickness of the diatomite layer of setting so that the loss of heat to the surroundings should not exceed 750 W/m².

(iv) Heat exchange by radiation, $q_{rad.}$:

Assuming the emissivity of hot gases as unity, the net radiation heat gain of wall per unit area is given by

$$q_{rad.} = \varepsilon\sigma\left((T_{hf})^4 - (T_1)^4\right)$$

$$= 0.82 \times 5.67 \times 10^{-8}\,[(810 + 273)^4 - (808 + 273)^4]$$

$$= 0.82 \times 5.67\left[\left(\frac{810 + 273}{100}\right)^4 - \left(\frac{808 + 273}{100}\right)^4\right] = \mathbf{471.2\ W/m^2}\ \textbf{(Ans.)}$$

(v) Convective heat transfer coefficient for the inside surface of the furnace wall ($h_{conv.}$) :

As convective and radiative heat transfers between gases and wall are in parallel, therefore

$$q = (q_{rad}) + (q_{conv.})_i$$

or $$(q_{conv.})_i = q - q_{rad.} = 538.9 - 471.2 = 67.7\ \text{W/m}^2{}^\circ\text{C}$$

∴ Convective heat transfer coefficient on the inside surface,

$$(h_{conv.})_i = \frac{(q_{conv.})_i}{t_{hf} - t_1} = \frac{67.7}{810 - 808} = \mathbf{33.85\ W/m^2\,{}^\circ C} \quad \textbf{(Ans.)}$$

Example 2.34. *The following data relate to a large rectangular combustion chamber for a furnace made of 220 mm common brick, lined on the inside with 220 mm thick layer of magnesite brick :*

Temperature of gases = 1300°C; Temperature of surrounding air = 40°C; Radiation coefficient, inside surface = 17.5 W/m²°C; convection coefficient, inside surface = 16.4 W/m²°C; Radiation coefficient, outside surface = 7.2 W/m² °C; convection co-efficient, outside surface = 11.5 W/m²°C; Thermal conductivity of common brick = 0.65 W/m°C; Thermal conductivity of magnesite brick = 3.5 W/m°C.

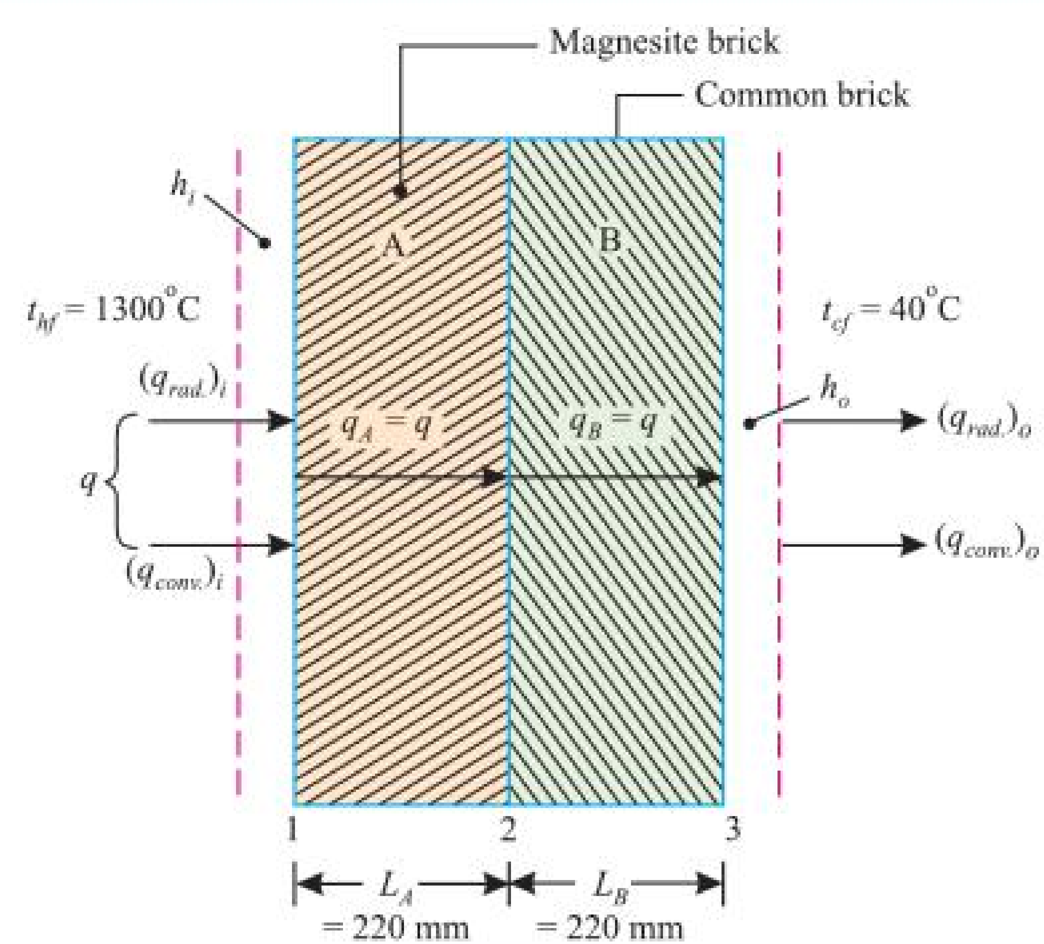


Fig. 2.44.

Determination of conduction heat transfer rate (Q) :

The conduction heat transfer rate is determined by utilizing the temperature distribution [Eqn. (2.57)] in conjunction with Fourier's equation as follows :

$$Q = -kA\frac{dt}{dr}$$

$$= -kA\frac{d}{dr}\left[t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r_1) - \frac{(t_1 - t_2)}{\ln(r_2/r_1)}\ln(r)\right]$$

[Substituting the value of t from Eqn. (2.57)]

$$= -k(2\pi r.L)\left[\frac{-(t_1 - t_2)}{r\ln(r_2/r_1)}\right]$$

$$= 2\pi k L\frac{(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi k L}}\left[=\frac{\Delta t}{R_{th}}\right] \quad \left(\text{where, } R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L}\right)$$

Hence,

$$Q = \frac{(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{2\pi k L}} \qquad ...(2.59)$$

Alternative method :

Refer to Fig. 2.45 Consider an element at radius 'r' and thickness '*dr*' for a length of the hollow cylinder through which heat is transmitted. Let *dt* be the temperature drop over the element.

Area through which heat is transmitted, $A = 2\pi\, r.\, L$.

Path length = *dr* (over which the temperature falls is *dt*)

$$\therefore \qquad Q = -kA\cdot\left(\frac{dt}{dr}\right)$$

$$= -k.2\pi r.L\frac{dt}{dr} \text{ per unit time}$$

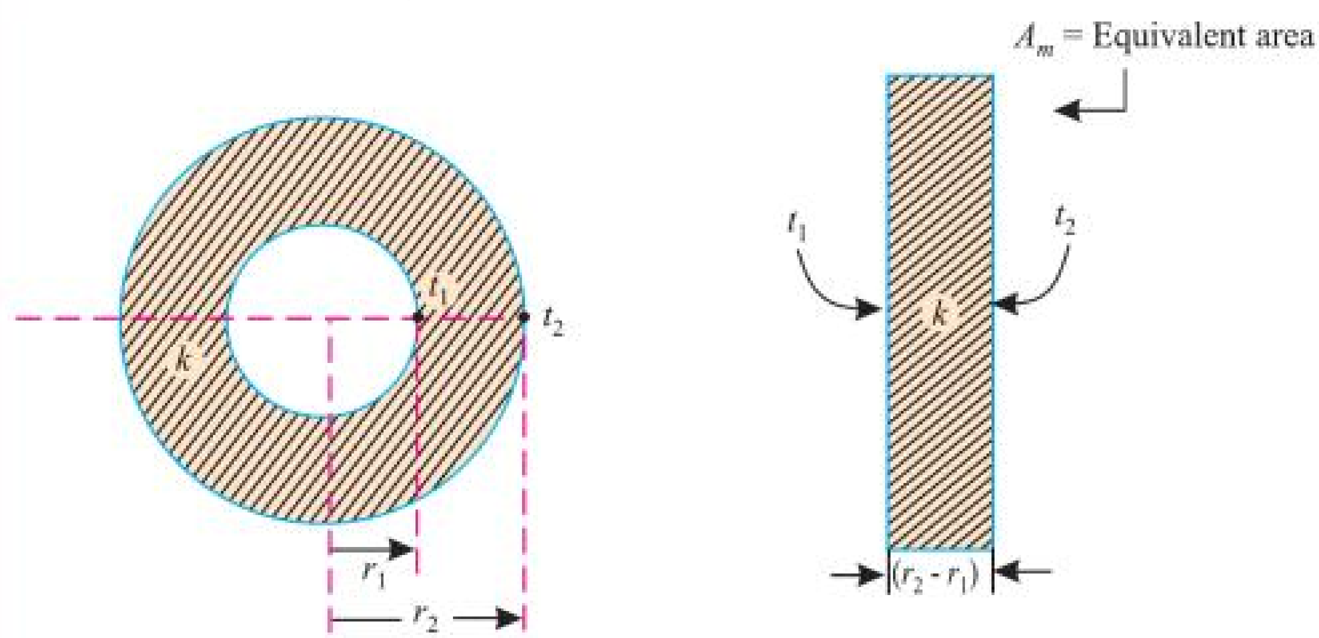
Induction eddy current heating.

$$= -\frac{1}{\beta} + \sqrt{\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{Q}{\beta k_0} \cdot \frac{\ln (r/r_1)}{\pi L}}$$

i.e.,
$$t = -\frac{1}{\beta} + \left[\left(t_1 + \frac{1}{\beta}\right)^2 - \frac{Q}{\beta k_0} \cdot \frac{\ln (r/r_1)}{\pi L}\right]^{\frac{1}{2}} \quad ...(2.66)$$

2.6.1.1. LOGARITHMIC MEAN AREA FOR THE HOLLOW CYLINDER

Invariably it is considered convenient to have an expression for the heat flow through a hollow cylinder of the same form as that for a plane wall. Then thickness will be equal to ($r_2 - r_1$) and the area A will be an equivalent area A_m as shown in the Fig. 2.46. Now, expressions for heat flow through the hollow cylinder and plane wall will be as follows :



(*a*) Hollow cylinder (*b*) Plane wall

Fig. 2.46.

$$Q = \frac{(t_1 - t_2)}{\dfrac{\ln (r_2/r_1)}{2\pi k L}} \quad \text{... Heat flow through cylinder.}$$

$$Q = \frac{(t_1 - t_2)}{\dfrac{(r_2 - r_1)}{k A_m}} \quad \text{... Heat flow through plane wall.}$$

A_m is so chosen that heat flow through cylinder and plane wall will be equal for the same thermal potential.

$\therefore$
$$\frac{(t_1 - t_2)}{\dfrac{\ln (r_2/r_1)}{2\pi k L}} = \frac{(t_1 - t_2)}{\dfrac{(r_2 - r_1)}{k A_m}}$$

or,
$$\frac{\ln (r_2/r_1)}{2\pi k L} = \frac{(r_2 - r_1)}{k A_m}$$

or,
$$A_m = \frac{2\pi L (r_2 - r_1)}{\ln (r_2/r_1)} = \frac{2\pi L r_2 - 2\pi L r_1}{\ln (2\pi L r_2 / 2\pi L r_1)}$$

or,
$$A_m = \frac{A_0 - A_i}{\ln (A_0 - A_i)} \quad ...(2.67)$$

where A_i and A_o are inside and outside surface areas of the cylinder.

Given :
$$r_1 = \frac{240}{2} = 120\,\text{mm} = 0.12\,\text{m}$$
$$r_2 = 120 + 50 = 170\,\text{mm} = 0.17\,\text{m}$$
$$r_3 = 120 + 50 + 40 = 210\,\text{mm} = 0.21\,\text{m}$$
$$k_A = 0.092\,\text{W/m°C};\ k_B = 0.062\,\text{W/m°C}$$
$$t_1 = 390°\text{C};\ t_3 = 40°\text{C}$$

Length of steam main, $L = 210$ m.

(*i*) Total heat loss per hour :

$$Q = \frac{2\pi L(t_1 - t_3)}{\left[\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}\right]} \qquad \text{[Eqn. (2.71)]}$$

$$= \frac{2\pi \times 210(390 - 40)}{\left[\frac{\ln(0.17/0.12)}{0.092} + \frac{\ln(0.21/0.17)}{0.062}\right]}$$

$$= \frac{461814}{(3.786 + 3.408)} = 64194.3\ \text{W}$$

$$= \frac{64194.3 \times 3600}{1000} = 231099.5\ \text{kJ/h}$$

i.e., The total heat loss per hour = **231099.5 kJ/h** **(Ans.)**

(*ii*) Total heat loss per m^2 of the pipe surface :

Total heat loss per m^2 of the surface

$$= \frac{231099.5}{2\pi r_1 . L} = \frac{231099.5}{2\pi \times 0.12 \times 210} = \mathbf{1459.55\ kJ/h} \quad \textbf{(Ans.)}$$

(*iii*) Total heat loss per m^2 of the outer surface :

Total heat loss per m^2 of the outer surface

$$= \frac{231099.5}{2\pi r_3 . L} = \frac{231099.5}{2\pi \times 0.21 \times 210} = \mathbf{834.03\ kJ/h} \quad \textbf{(Ans.)}$$

(*iv*) The temperature between two layers, t_2 :

$$Q = \frac{2\pi L(t_1 - t_2)}{\left[\frac{\ln(r_2/r_1)}{k_A}\right]}$$

$$64194.3 = \frac{2\pi \times 210(390 - t_2)}{\left[\frac{\ln(0.17/0.12)}{0.092}\right]} = 348.5(390 - t_2)$$

$$\therefore \qquad t_2 = 390 - \frac{64194.3}{348.5} = \mathbf{205.8°\,C} \quad \textbf{(Ans.)}$$

Example 2.41. *A steam pipe of outer diameter 120 mm is covered with two layers of lagging, inside layer 45 mm thick (k = 0.08W/m°C) and outside layer 30 mm thick (k = 0.12 W/m°C). The pipe conveys steam at a pressure of 20 bar with 50°C superheat. The outside temperature of lagging is 25°C. If the steam pipe is 30m long, determine :*

(*i*) *Heat lost per hour, and*

(*ii*) *Interface temperature of lagging.*

The thermal resistance of steam pipe may be neglected.

$t_1 = 320°$ C, $t_4 = 40°C$

$k_A = 58$ W/m°C

$k_B = 0.18$ W/m°C, $k_C = 0.09$ W/m°C

(*i*) Quantity of heat lost per meter (Q) and layer contact temperatures (t_2, t_3) :

Quantity of heat lost is given by

$$Q = \frac{2\pi L(t_1 - t_4)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{\ln(r_4/r_3)}{k_C}}$$

$$= \frac{2\pi \times 1 \times (320 - 40)}{\frac{\ln(0.08/0.075)}{58} + \frac{\ln(0.11/0.08)}{0.18} + \frac{\ln(0.16/0.11)}{0.09}} = \mathbf{296.5\ W/m} \quad \textbf{(Ans).}$$

Also, $$Q = \frac{2\pi \times 1(t_1 - t_2)}{\frac{\ln(r_2/r_1)}{k_A}} = \frac{2\pi \times 1(t_2 - t_3)}{\frac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi \times 1(t_3 - t_4)}{\frac{\ln(r_4/r_3)}{k_C}}$$

$\therefore$ $$296.5 = \frac{2\pi(320 - t_2)}{\frac{\ln(r_2/r_1)}{k_A}} \quad \text{or} \quad t_2 = 320 - \frac{296.5}{2\pi} \times \frac{\ln(r_2/r_1)}{k_A}$$

$$= 320 - \frac{296.5}{2\pi} \times \frac{\ln(0.08/0.75)}{58} = \mathbf{319.95°\ C} \quad \textbf{(Ans.)}$$

Similarly, $$296.5 = \frac{2\pi(319.95 - t_3)}{\frac{\ln(r_3/r_2)}{k_B}}$$

or, $$t_3 = 319.95 - \frac{296.5}{2\pi} \times \frac{\ln(r_3/r_2)}{k_B} = 319.95 - \frac{296.5}{2\pi} \times \frac{\ln(0.11/0.08)}{0.18} = \mathbf{236.5°\ C} \quad \textbf{(Ans.)}$$

(*ii*) Quality of steam coming out of one metre pipe, x :

Total heat of steam when it is saturated at 320°C = 2703 kJ/kg ...from steam tables

Heat carried by steam per minute after losing heat in the pipe

$$= 0.32\,(\text{kg/min}) \times 2703\,(\text{kJ/kg}) - \frac{296.5 \times 60}{1000}\,(\text{kJ/min}) = 847.17\ \text{kJ/min}$$

Now $847.17 = 0.32\,(h_f + xh_{fg})$

Corresponding to 320°C saturation temperature, from steam tables, we have

$h_f = 1463$ kJ/kg, $h_{fg} = 1240$ kJ/kg

$\therefore$ $847.17 = 0.32\,(1463 + x \times 1240) = 468.16 + 396.8x$

or, $$x = \frac{(847.17 - 468.16)}{396.8} = \mathbf{0.955} \quad \textbf{(Ans.)}$$

Example 2.44. *Thermal conductivity, k, of a certain material is given by $k = a + bT + cT^2$ where a, b, c are constants and T is the absolute temperature. Derive an expression for heat flow per unit length of a hollow cylinder made of this material. Assume that inner and outer radii of the cylinder are r_1 and r_2 respectively and the cylinder ends are perfectly insulated.* **(AMIE Summer, 2000)**

Solution. Refer to Fig. 2.56. Consider a hollow ring at radius r, and thickness dr of the hollow cylinder. The radius heat flow across the ring, per unit length, is given by

$$Q = -kA_r \frac{dT}{dr} = -k \times 2\pi r \times \frac{dT}{dr}$$

(ii) The temperature at the mid thickness, t_{mt} :

$$Q = \frac{(280 - t_{mt})}{\dfrac{\ln(r_{mt}/r_1)}{2\pi k_m \times 1}} \qquad \left[\begin{array}{l}\text{where } r_{mt} = \text{radius at mid thickness/plane} \\ = \left(\dfrac{r_1 + r_2}{2}\right) = \left(\dfrac{0.11 + 0.16}{2}\right) = 0.135\,\text{m}\end{array}\right]$$

$$300.06 = \frac{2\pi k_m (280 - t_{mt})}{\ln(0.135/0.11)} = \frac{2\pi \times 0.06\left[1 + 0.0018\left(\dfrac{280 + t_{mt}}{2}\right)(280 - t_{mt})\right]}{0.2048}$$

or, $$[1 + 0.0009(280 + t_{mt})](280 - t_{mt}) = \frac{300.06 \times 0.2048}{2\pi \times 0.06} = 163$$

or, $$(1 + 0.252 + 0.0009\, t_{mt})(280 - t_{mt}) = 163$$

or, $$(1.252 + 0.0009\, t_{mt})(280 - t_{mt}) = 163$$

or, $$350.56 - 1.252\, t_{mt} + 0.252\, t_{mt} - 0.0009\, t_{mt}^2 = 163$$

or, $$0.0009\, t_{mt}^2 + t_{mt} - 187.56 = 0$$

or, $$t_{mt} = \frac{-1 + \sqrt{1 + 4 \times 0.0009 \times 187.56}}{2 \times 0.0009} = 163.5°\text{C}$$

∴ Temperature at the mid thickness, t_{mt} = **163.5°C (Ans.)**

$$\left[\text{Check } Q = \frac{(280 - 163.5)}{ln(0.135/0.11)} \times 2\pi \times 0.06\left\{1 + 0.0018\left(\frac{280 + 163.5}{2}\right)\right\} = 300.06\,\text{W}\right]$$

Example 2.47. *An insulated steam pipe having outside diameter of 30 mm is to be covered with two layers of insulation, each having thickness of 20 mm. The thermal conductivity of one material is 5 times that of the other.*

Assuming that the inner and outer surface temperatures of composite insulation are fixed, how much will heat transfer be increased when better insulation material is next to the pipe than it is outer layer? **(M.U.)**

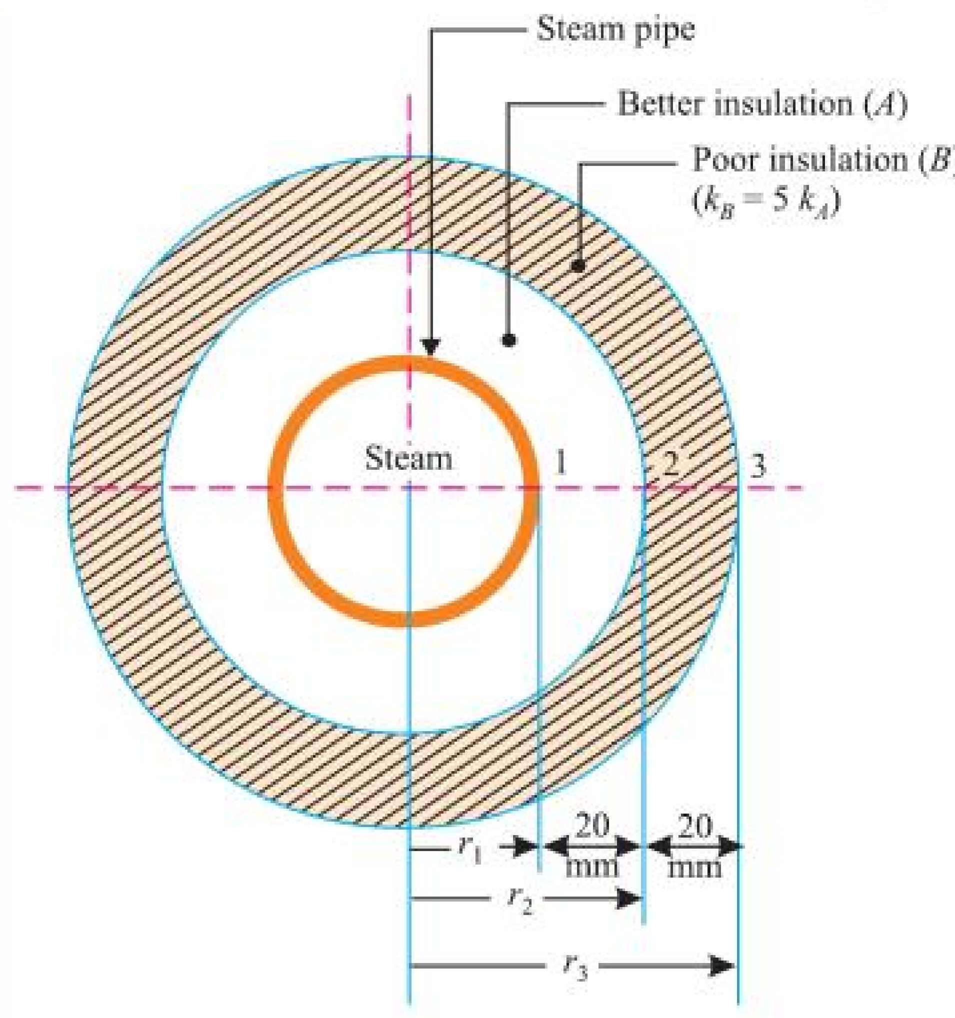


Fig. 2.59.

Solution. Case I. When better insulation is inside :

Refer to Fig. 2.59.

$$r_1 = \frac{30}{2} = 15\,\text{mm} = 0.015\,\text{m};$$

$$r_2 = 15 + 20 = 35\text{ mm} = 0.035\text{ m};$$

$$r_3 = 35 + 20 = 55\text{ mm} = 0.055\text{m}$$

$$k_B = 5k_A$$

Heat lost through the pipe is given by

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Heat loss/m length, Q/L :

$$\frac{Q}{L} = 2\pi r_1 \times U_i \times (t_{hf} - t_{cf})$$

$$= 2\pi \times 0.01 \times 2.788\,(200 - 30) = \mathbf{29.78\ W/m\ (Ans.)}$$

Example 2.50. *An aluminium pipe carries steam at 110°C. The pipe (k = 185 W/m°C) has an inner diameter of 100 mm and outer diameter of 120 mm. The pipe is located in a room where the ambient air temperature is 30°C and the convective heat transfer coefficient between the pipe and air is 15 W/m²°C. Determine the heat transfer rate per unit length of pipe.*

To reduce the heat loss from the pipe, it is covered with a 50 mm thick layer of insulation (k = 0.20 W/m°C). Determine the heat transfer rate per unit length from the insulated pipe. Assume that the convective resistance of the steam is negligible. **(AMIE Summer, 1999)**

Solution. Case I. Refer to Fig. 2.63.

Given : $r_1 = \frac{100}{2} = 50$ mm $= 0.05$ m

$r_2 = \frac{120}{2} = 60$ mm $= 0.06$ m

Temperature of steam (hot fluid), $t_{hf} = 110$°C

Temperature of ambient air (cold fluid), $t_{cf} = 30$°C

Thermal conductivity of pipe material, $k = 185$ W/m°C

Heat transfer coefficient between the pipe and air, $h_{cf} = 15$ W/m²°C

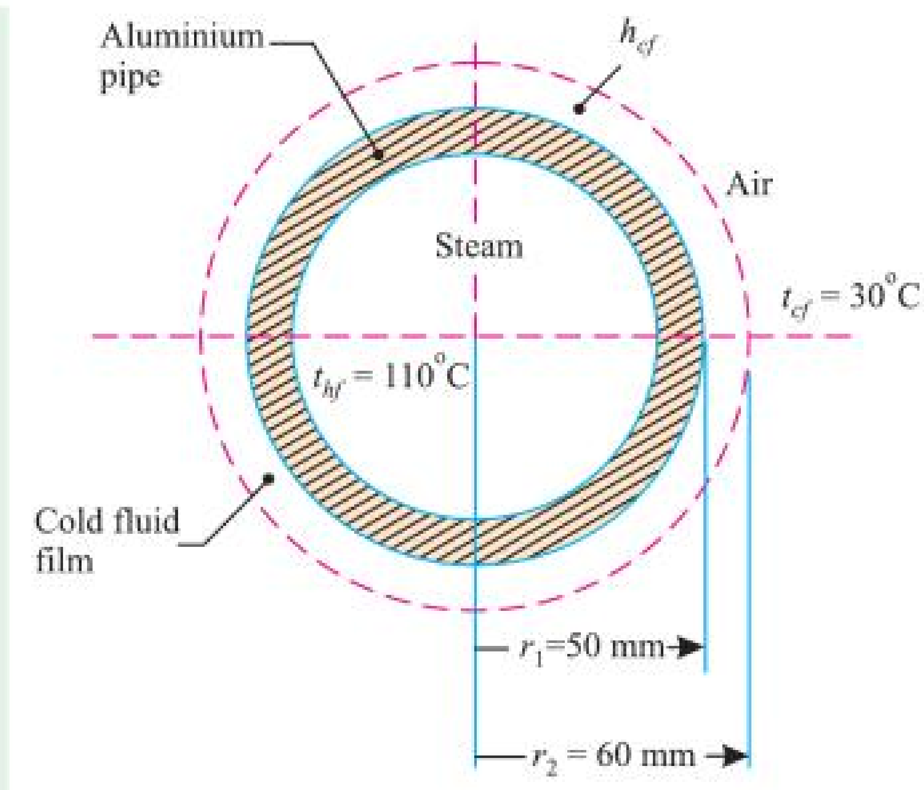


Fig. 2.63.

Heat transfer rate per unit length of pipe, Q/L :

Heat transfer rate is given by,

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\frac{\ln(r_2/r_1)}{k_A} + \frac{1}{h_{cf}.r_2}\right]} \quad \text{[Eqn. (2.69)]}$$

or,

$$\frac{Q}{L} = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\frac{\ln(r_2/r_1)}{k_A} + \frac{1}{h_{cf}.r_2}\right]} = \frac{2\pi(110 - 30)}{\left[\frac{\ln(0.06/0.05)}{185} + \frac{1}{15 \times 0.06}\right]} = 451.99 \text{ W/m}$$

i.e., Heat transfer rate per unit length of pipe = **451.99 W/m (Ans.)**

Case II : Refer to Fig. 2.64.

$r_1 = 50$ mm $= 0.05$ m; $r_2 = 60$ mm $= 0.06$ m

$r_3 = 60 + 50 = 110$ mm $= 0.11$ m; $k_A = 185$ W/m°C

$k_B = 0.20$ W/m°C; $h_{cf} = 15$ W/m²°C

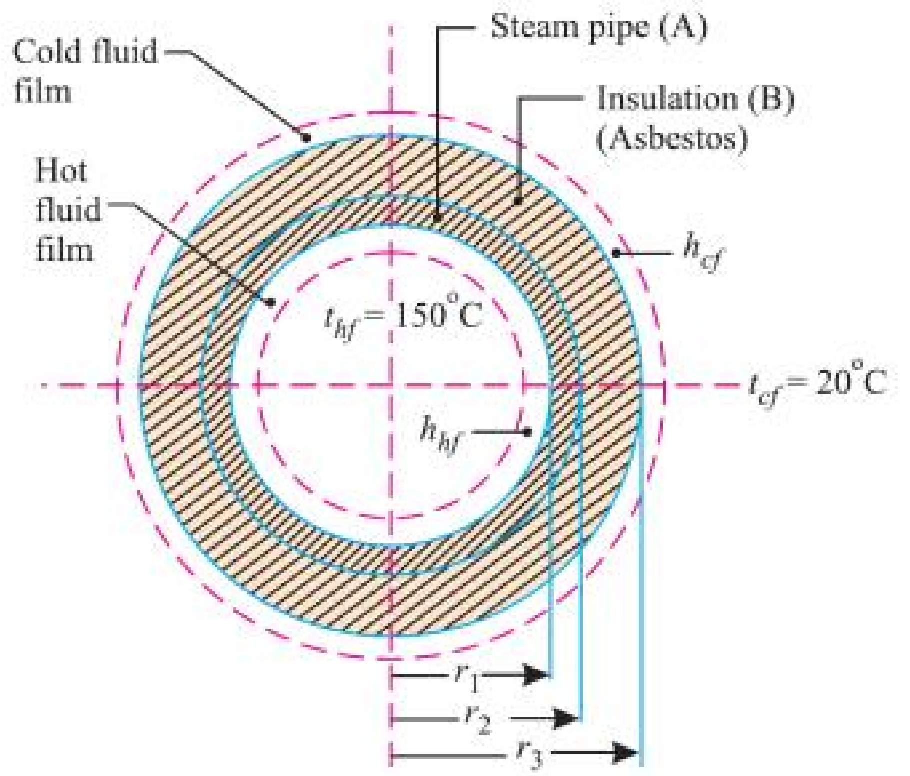


Fig. 2.65.

k_A = 42 W/m°C; k_B = 0.8 W/m°C

t_{hf} = 150°C; t_{cf} = 20°C

h_{hf} = 100 W/m²°C; h_{cf} = 30 W/m²°C

Heat loss = 2.1 kW/m²

Thickness of insulation (asbestos), $(r_3 - r_2)$:

Area for heat transfer = $2\pi\, r\, L$ (where L = length of the pipe)

$\therefore$ Heat loss $= 2.1 \times 2\pi\, rL$ kW

$= 2.1 \times 2\pi \times 0.075 \times L = 0.989\, L$ kW

$= 0.989 \text{ L} \times 10^3$ watts

(where r, mean radius $= \frac{150}{2} = 75$ mm or 0.075 m ... given]

Heat transfer rate in such a case is given by

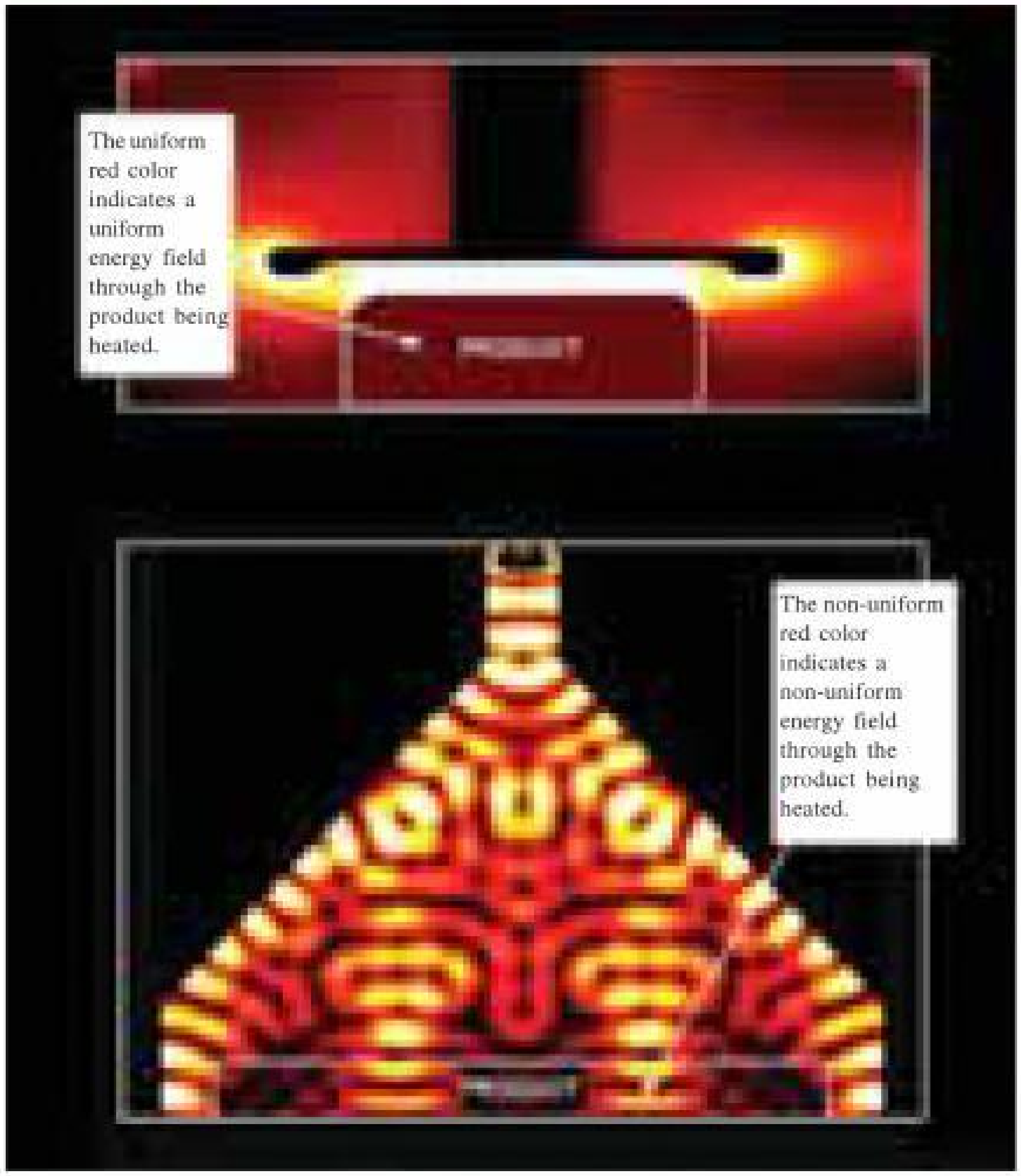
$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf}\,.\,r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf}\,.\,r_3}\right]} \quad \text{[Eqn. 2.69]}$$

$$0.989\; L \times 10^3 = \frac{2\pi L(150 - 20)}{\left[\frac{1}{100 \times 0.06} + \frac{\ln(0.08/0.06)}{42} + \frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30 \times r_3}\right]}$$

$$0.989 \times 10^3 = \frac{816.81}{\left[0.16666 + 0.00685 + \frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30\, r_3}\right]}$$

or, $$\frac{\ln(r_3/0.08)}{0.8} + \frac{1}{30\, r_3} = \frac{816.81}{0.989 \times 10^3} - (0.16666 + 0.00685) = 0.6524$$

or, $$1.25 \ln\;(r_3/0.08) + \frac{1}{30\, r_3} - 0.6524 = 0$$



Dielectric heating.

$$= \frac{2\pi(t_s - t_a)}{\frac{\ln(0.12/0.08)}{0.8} + \frac{\ln(0.13/0.12)}{1.2} + \frac{1}{10 \times 0.13}}$$

$$= \frac{2\pi(t_s - t_a)}{1.343} \quad \text{...(ii)}$$

The percentage decrease in heat flow due to extra addition of insulation can be calculated using eqns. (*i*) and (*ii*) as follows :

$$\frac{Q_1 - Q_2}{Q_1} = \left[\frac{(1/1.34) - (1/1.343)}{(1/1.34)}\right] = 0.00223 \text{ or } \mathbf{0.223\%} \quad \textbf{(Ans.)}$$

Example 2.53. *A steam pipe (k = 45 W/m°C) having 70 mm inside diameter and 85 mm outside diameter is lagged with two insulation layers; the layer in contact with the pipe is 35 mm asbestos (k = 0.15 W/m°C) and it is covered with 25 mm thick magnesia insulation (k = 0.075 W/m°C). The heat transfer coefficients for the inside and outside surfaces are 220 W/m²°C and 6.5 W/m²°C respectively. If the temperature of steam is 350°C and the ambient temperature is 30°C, calculate :*

(*i*) *The steady loss of heat for* 50 *m length of the pipe;*

(*ii*) *The overall heat transfer coefficients based on inside and outside surfaces of the lagged steam main.*

or, $$L = \frac{16410.24 \times 10^3}{5425.8} = 3024\,\text{m}$$

$\therefore$ Outer surface area of the superheater $= 2\pi r_2 L$

$$= 2\pi \times 0.0225 \times 3024 = \mathbf{427.5\ m^2}\ \ \textbf{(Ans.)}$$

Example 2.56. *A pipe having outer diameter 250 mm is insulated by a material of thermal conductivity of 0.48 W/m°C. The insulation of outside diameter 500 mm, due to restriction of space, is placed with an eccentricity of 60 mm. Determing the heat loss for a length of 10 m if inner and outer surfaces are at temperatures of 280°C and 50°C respectively.*

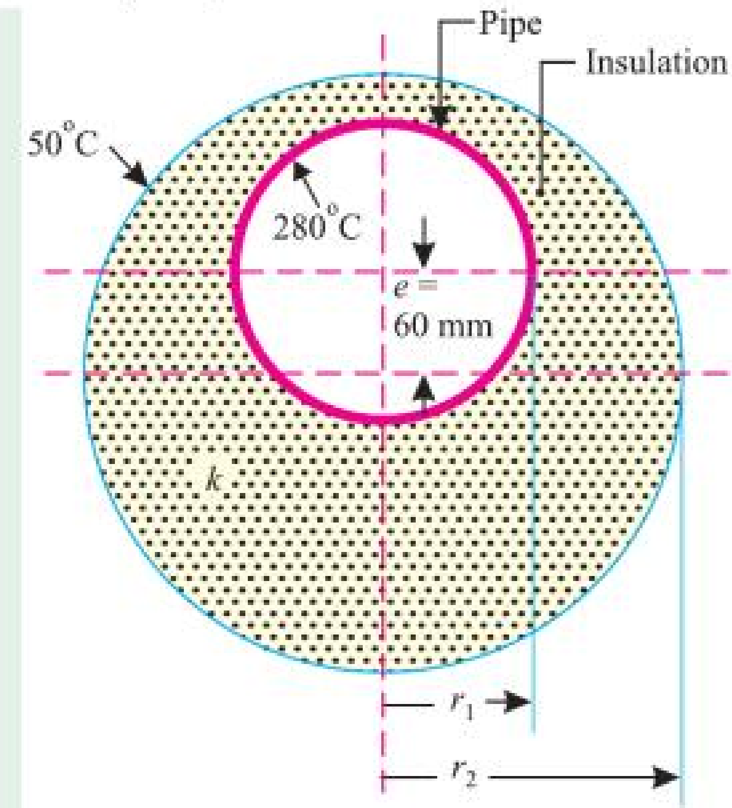


Fig. 2.71.

Solution. Refer to Fig. 2.71.

$$r_1 = \frac{250}{2} = 125\,\text{mm} = 0.125\,\text{m};$$

$$r_2 = \frac{500}{2} = 250\,\text{mm} = 0.25\,\text{m}$$

Eccentricity, $e = 60\,\text{mm} = 0.06\,\text{m}$

Length of pipe, $L = 10$ m

Thermal conductivity of insulation,

$k = 0.48$ W/m°C

Heat loss, Q :

$$Q = \frac{\Delta t}{R_{th}} \qquad \text{...(}i\text{)}$$

The thermal resistance (R_{th}) in this case is given by (from hand book)

$$R_{th} = \frac{1}{2\pi k L} \ln\left[\frac{\{(r_2 + r_1)^2 - e^2\}^{1/2} + \{(r_2 - r_1)^2 - e^2\}^{1/2}}{\{(r_2 + r_1)^2 - e^2\}^{1/2} - \{(r_2 - r_1)^2 - e^2\}^{1/2}}\right]$$

$$= \frac{1}{2\pi \times 0.48 \times 10} \ln\left[\frac{\{(0.250 + 0.125)^2 - (0.06)^2\}^{1/2} + \{(0.250 - 0.125)^2 - (0.06)^2\}^{1/2}}{\{(0.250 + 0.125)^2\,(0.06)^2\}^{1/2} - \{(0.250 - 0.125)^2 - (0.06)^2\}^{1/2}}\right]$$

$$= \frac{1}{2\pi \times 0.48 \times 10} \ln\left[\frac{0.3702 + 0.1096}{0.3702 - 0.1096}\right] = 0.0202\text{°C/W}$$

Substituting the proper values in expression (i), we have

$$Q = \frac{(280 - 50)}{0.0202} = \mathbf{11386\ W} \qquad \textbf{(Ans.)}$$

Example 2.57. *A current of 950 amperes is flowing through a long copper rod of 25 mm diameter, having an electrical resistance of 22 × 10⁻⁶ ohm per metre length. The rod is insulated to a radius of 17 mm with fibrous cotton (k = 0.058 W/m°C) which is further covered by a layer of plastic (k = 0.42 W/m°C). The heat transfer coefficient between the plastic and the surroundings is 20.5 W/m²°C and the temperature of surroundings is 15°C. Determine :*

(i) The thickness of the plastic layer which gives minimum temperature in a cotton insulation.

(ii) The temperature of copper rod and the maximum temperature in the plastic layer, for the condition as at (i).

Example 2.59. *Heat is conducted through a tapered circular rod of 200 mm length. The ends A and B having diameters 50 mm and 25 mm are maintained at 27°C and 227°C respectively. k (rod material) = 40 W/m°C. Find :*

(i) Heat conducted through the rod.

(ii) The temperature at the mid-point of the end.

Assume there is no temperature gradient at a particular cross-section and there is no heat transfer through the peripheral surface. **(M.U.)**

Solution. *Given* : $L = 200$ mm $= 0.2$ m; $D_1 = 50$ mm $= 0.05$ m; $D_2 = 25$ mm $= 0.025$ m; $t_1 = 227°C$; $t_2 = 27°C$; $k = 40$ W/m°C.

(*i*) Heat conducted through the rod, *Q* :

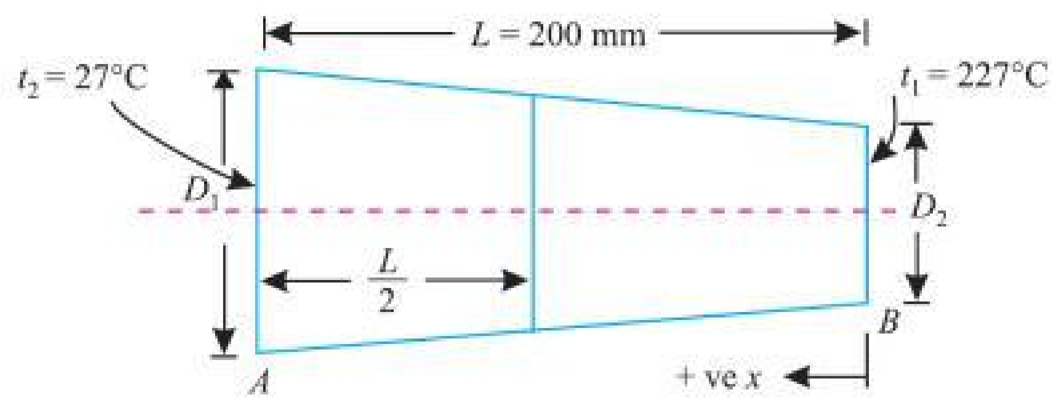


Fig. 2.74.

The heat flow through the rod is given by

$$Q = \frac{k \pi R_1 R_2 (t_1 - t_2)}{L} \quad ...(i)$$

$$= \frac{40 \times \pi \times 0.025 \times 0.0125 \times (227 - 27)}{0.2} = \mathbf{39.27\ W}\ \textbf{(Ans.)}$$

(*ii*) The temperature at the mid-point of the rod, *t* :

Now, $$R_x = R_2 + (R_1 - R_2)\frac{x}{L}$$

In power plants turbines and pipes function under high temperatures and pressures.

For steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, unidirectional heat flow in the radial direction $\{t \neq f(\theta, \phi)\}$ and with no heat generation $(q_g = 0)$, the above equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \cdot \frac{dt}{dr}\right) = 0$$

or, $$\frac{d}{dr}\left(r^2 \cdot \frac{dt}{dr}\right) = 0 \quad \text{as} \quad \frac{1}{r^2} \neq 0$$

or, $$r^2 \cdot \frac{dt}{dr} = C \text{ (a constant)} \qquad ...(2.72)$$

Integrating the above equation, we obtain

$$t = -\frac{C}{r} + C_1 \qquad ...(2.73)$$

(where C_1 = a constant of integration)

Using the following boundary conditions, we have

At $r = r_1, t = t_1$; At $r = r_2, t = t_2$

∴ $$t_1 = -\frac{C}{r_1} + C_1 \qquad ...(i)$$

$$t_2 = -\frac{C}{r_2} + C_1 \qquad ...(ii)$$

From (*i*) and (*ii*), we have

$$C = \frac{(t_1 - t_2)\, r_1 r_2}{r_1 - r_2}$$

and, $$C_1 = t_1 + \frac{(t_1 - t_2)\, r_1 r_2}{r_1 (r_1 - r_2)}$$

Substituting the values of these constants in eqn. (2.73), we get

$$t = -\frac{(t_1 - t_2)\, r_1 r_2}{r (r_1 - r_2)} + t_1 + \frac{(t_1 - t_2)\, r_1 r_2}{r_1 (r_1 - r_2)}$$

or, $$t = -\frac{(t_1 - t_2)}{r (1/r_2 - 1/r_1)} + t_1 + \frac{(t_1 - t_2)}{r_1 (1/r_2 - 1/r_1)}$$

or, $$t = t_1 + \frac{(t_1 - t_2)}{(1/r_2 - 1/r_1)}\left[\frac{1}{r_1} - \frac{1}{r}\right] \qquad ...(2.74)$$

or, $$\frac{t - t_1}{t_2 - t_1} = \frac{1/r - 1/r_1}{1/r_2 - 1/r_1}$$

or, $$\frac{t - t_1}{t_2 - t_1} = \frac{r_2}{r}\left[\frac{r - r_1}{r_2 - r_1}\right] \quad \text{[Dimensionless form]} \qquad ...(2.75)$$

From the eqn. (2.75) it is evident that the temperature distribution associated with radial conduction through a sphere is represented by a *hyperbola*.

Determination of conduction heat transfer rate, Q :

The conduction heat transfer rate is determined by using the temperature distribution expression [Eqn. (2.75)] in conjuction with Fourier's equation as follows :

$$Q = -kA\frac{dt}{dr}$$

$$= -k \,.\, 4\pi r^2 \,.\, \frac{d}{dr}\left[t_1 + \frac{(t_1 - t_2)}{(1/r_2 - 1/r_1)}\left(\frac{1}{r_1} - \frac{1}{r}\right)\right]$$

$$\therefore \quad Q = \frac{4\pi(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1^2} + \frac{(r_2 - r_1)}{k_A \cdot r_1 r_2} + \frac{(r_3 - r_2)}{k_B \cdot r_2 r_3} + \frac{1}{h_{cf} \cdot r_3^2}\right]} \quad ...(2.82)$$

If there are n concentric spheres then the above equation can be written as follows

$$Q = \frac{4\pi(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1^2} + \sum_{n=1}^{n=n}\left\{\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right\} + \frac{1}{h_{cf} \cdot r_{(n+1)}^2}\right]} \quad ...(2.83)$$

If inside and outside heat transfer coefficients are not considered, then the above equation can be written as follows :

$$Q = \frac{4\pi(t_1 - t_{n+1})}{\sum_{n=1}^{n=n}\left[\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right]} \quad ...(2.84)$$

Example 2.61. *A spherical shaped vessel of 1.4 m diameter is 90 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220°C. Thermal conductivity of the material of the sphere is 0.083 W/m°C.*

Solution. Refer to Fig. 2.78.

$$r_2 = \frac{1.4}{2} = 0.7 \text{ m.}$$

$$r_1 = 0.7 - \frac{90}{1000} = 0.61 \text{m}$$

$$t_1 - t_2 = 220°\text{C};$$

$$k = 0.083 \text{ W/m°C}$$

The rate of heat transfer/leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\frac{(r_2 - r_1)}{4\pi k r_1 r_2}\right]} \quad ...\text{Fig. (2.76)}$$

$$= \frac{220}{\left[\frac{(0.7 - 0.61)}{4\pi \times 0.083 \times 0.61 \times 0.7}\right]}$$

$$= 1088.67 \text{ W}$$

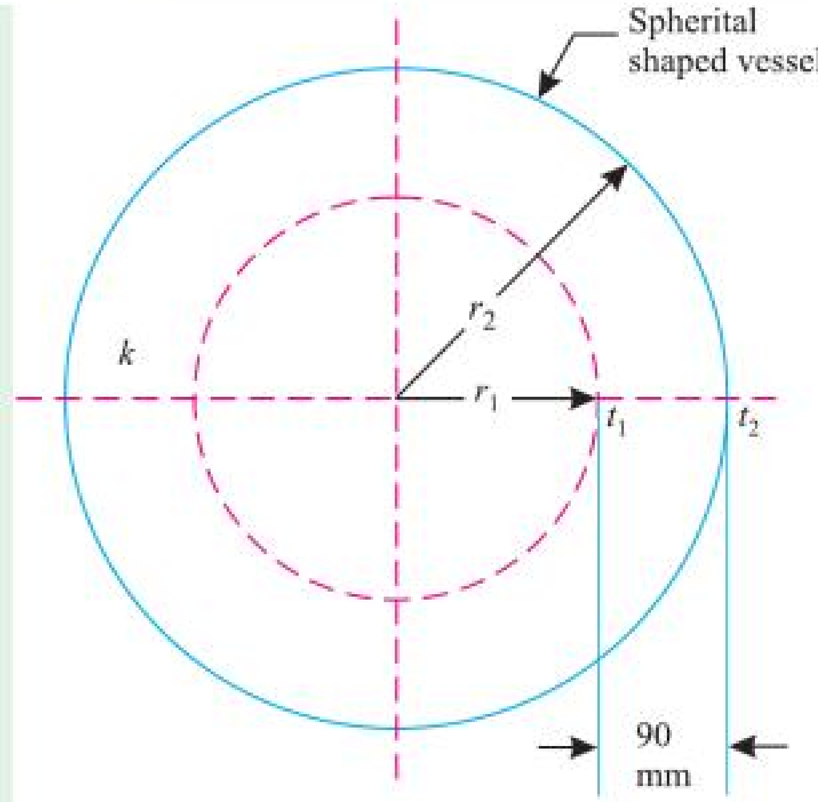


Fig. 2.78.

i.e., Rate of heat leakage = **1088.67 W** **(Ans.)**

Example 2.62. *A spherical thin walled metallic container is used to store liquid N_2 at – 196° C. The container has a diameter of 0.5 m and is covered with an evacuated reflective insulation composed of silica powder. The insulation is 25 mm thick and its outer layer is exposed to air at 27°C. The convective heat transfer coefficient on outer surface = 20 W/m²° C. Latent heat of evaporation of $N_2 = 2 \times 10^5$ J/kg. Density of N_2 = 804 kg/m³.*

k (silica powder) = 0.0017 W/m°C.

Find out the rate of heat transfer and rate of N_2 boil-off. **(N.U., 1998)**

Solution. *Given :* $t_1 = -196°\text{C}$; $t_2 = 27°\text{C}$; $r_1 = \frac{0.5}{2} = 0.25 \text{ m}$;

$$r_2 = r_1 + 0.025 = 0.25 + 0.025 = 0.275 \text{ m};$$

$$\frac{Q}{4\pi}\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = k_0\left[(t_2-t_1)+\alpha\left(\frac{t_2^2-t_1^2}{2}\right)+\frac{\beta}{3}(t_2^3-t_1^3)\right]$$

or, $$\frac{Q}{4\pi}\left(\frac{r_1-r_2}{r_1 r_2}\right) = -k_0\left[(t_1-t_2)+\frac{\alpha}{2}(t_1^2-t_2^2)+\frac{\beta}{3}(t_1^3-t_2^3)\right]$$

or, $$\frac{Q}{4\pi}\left(\frac{r_2-r_1}{r_1 r_2}\right) = -k_0(t_1-t_2)\left[1+\frac{\alpha}{2}(t_1+t_2)+\frac{\beta}{3}(t_1^2+t_1 t_2+t_2^2)\right]$$

or, $$Q = \frac{4\pi r_1 r_2}{r_2-r_1}\times k_0(t_1-t_2)\left[1+\frac{\alpha}{2}(t_1+t_2)+\frac{\beta}{3}(t_1^2+t_1 t_2+t_2^2)\right]$$

...Required expression (Ans.)

Example 2.66. *The inside and outside surfaces of a hollow sphere, having inner and outer radii r_1 and r_2 respectively, are maintained at uniform temperatures t_1 and t_2. Find the rate of heat transfer through the sphere if the conductivity of the material of which the sphere is made varies according to relation :*

$$k = k_1 + (k_2 - k_1)\,[(t-t_1)/(t_2-t_1)]$$

Solution. Fourier's equation for unidirectional steady state heat conduction is given as :

$$Q = -kA\frac{dt}{dr} \qquad ...(i)$$

where $A = 4\pi r^2$ (area normal to radial direction).

Now, by substituting the values of A and k in eqn. (*i*), we get

$$Q = -[k_1 + (k_2-k_1)\{(t-t_1)/(t_2-t_1)\}]\times 4\pi r^2 \times \frac{dt}{dr}$$

or, $$Q.\frac{dr}{r^2} = -4\pi[k_1+(k_2-k_1)\{(t-t_1)/(t_2-t_1)\}]\,dt$$

Integrating both sides, we get

$$Q\int_{r_1}^{r_2}\frac{dr}{r^2} = -4\pi\int_{t_1}^{t_2}\left[k_1+(k_2-k_1)\left\{\frac{(t-t_1)}{(t_2-t_1)}\right\}\right]dt$$

$$-Q\left[\frac{1}{r}\right]_{r_1}^{r_2} = -4\pi\left[k_1 t+\frac{k_2-k_1}{t_2-t_1}\left\{\frac{t^2}{2}-t\cdot t_1\right\}\right]_{t_1}^{t_2}$$

Furnace.

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Also, $L + 2r_1 = 5$ or $L + 2 \times 0.5 = 5 \quad \therefore L = 4.0$ m

$$\therefore \quad Q_{cyl.} = \frac{2940}{\left[1 + \frac{2 \times 0.5 \times 0.57 \times \ln(0.57/0.5)}{(0.57 - 0.5) \times 4}\right]}$$

$$= 2320.9 \text{ kJ/h} = \frac{2320.9 \times 1000}{3600} \text{ W} = 644.69 \text{ W}$$

From expression (*i*), we have

$$k = \frac{\ln(r_2/r_1)}{2\pi L(t_o - t_i)} \times Q_{cyl.}$$

$$= \frac{\ln(0.57/0.5)}{2\pi \times 4[25 - (-180)]} \times 644.69 = \mathbf{0.0164\ W/m°C} \textbf{ (Ans.)}$$

Example 2.69. *A cylindrical tank with hemispherical ends is used to store liquid oxygen at –183° C. The diameter of the tank is 1.5 m and the total length is 8 m. The tank is covered with a 10 cm thick layer of insulation. Determine the thermal conductivity of the insulation, so that the boil-off rate does not exceed 10.8 kg/h. The latent heat of vapourisation of liquid oxygen is 214 kJ/kg. Assume that the outer surface temperature of the insulation is 27°C and that the thermal resistance of the wall of the tank is negligible.* **(U.P.S.C., 1994)**

Solution. Heat generated during boiling of oxygen,

$$Q_{boil} = 10.8 \times 214 = 2311.2 \text{ kJ/h}$$

Let t_0 be the room temperature outside of insulation and t_i be the temperature of liquid oxygen inside the tank.

For the cylindrical section of the tank,

$$Q_{cy1} = \frac{2\pi k L_{cy1}(t_0 - t_i)}{\ln(r_2/r_1)}$$

or, $$(t_0 - t_i) = \frac{\ln(r_2/r_1)}{2\pi k L_{cy1}} \times Q_{cy1} \quad ...(i)$$

For the two spherical ends, $$Q_{ends} = \frac{t_0 - t_i}{(r_2 - r_1)/(4\pi k r_1 r_2)}$$

$$= \frac{4\pi k r_1 r_2 (t_0 - t_i)}{(r_2 - r_1)}$$

or, $$(t_0 - t_i) = \frac{r_2 - r_1}{4\pi k r_1 r_2} \times Q_{ends} \quad ...(ii)$$

From eqns. (*i*) and (*ii*), we get

$$\frac{\ln(r_2/r_1)}{2\pi k L_{cy1}} \times Q_{cy1} = \frac{r_2 - r_1}{4\pi k r_1 r_2} \times Q_{ends}$$

$$\therefore \quad Q_{ends} = \frac{2 r_1 r_2 \times \ln(r_2/r_1)}{(r_2 - r_1) L_{cy1}} \times Q_{cy1}$$

Now $$Q_{cy1} + Q_{ends} = Q_{boil} = 2311.2$$

or, $$Q_{cy1} + \frac{2 r_1 r_2 \times \ln(r_2/r_1)}{(r_2 - r_1) L_{cy1}} \times Q_{cy1} = 2311.2$$

$$\therefore \quad Q_{cy1} = \frac{2311.2}{1 + \left[\frac{2 r_1 r_2 \times \ln(r_2/r_1)}{(r_2 - r_1) L_{cy1}}\right]} \quad ...(iii)$$

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$$Q \text{ (with insulation)} = \frac{2\pi(T_1 - T_2)}{\frac{\ln(r_c/r_1)}{k} + \frac{1}{h_0 r_c}}$$

$$= \frac{2\pi(475 - 300)}{\frac{\ln(0.06143/0.03)}{0.172} + \frac{1}{2.8 \times 0.06143}} = \frac{1099.56}{4.167 + 5.814} = \mathbf{110.16\ W/m} \quad \textbf{(Ans.)}$$

$$Q \text{ (without insulation)} = h_0 \times 2\pi\, r_1 (T_1 - T_2)$$

$$= 2.8 \times 2\pi \times 0.03\,(475 - 300) = \mathbf{92.36\ W/m} \quad \textbf{(Ans.)}$$

Example 2.71. *A 10 mm cable is to be laid in atmosphere of 20°C with outside heat transfer coefficient 8.5 W/m²°C. The surface temperature of cable is likely to be 65°C due to heat generation within. Will the rubber insulation, k = 0.155 W/m °C, be effective ? If yes how much ?*

(AMIE Winter, 1999)

Solution. Refer to Fig. 2.85.

Given : $r_1 = \frac{10}{2} = 5\text{ mm};\ t_1 = 65°\text{ C};\ t_{air} = 20°\text{ C};\ k = 0.155\text{ W/m°C};\ h_0 = 8.5\text{ W/m}^2\text{°C}$

For a cable (cylinder),

$$r_c = \frac{k}{h_0} = \frac{0.155}{8.5} = 0.018235\text{ m} = 18.235\text{ mm},$$

r_c is greater than the radius of the cable.

Hence, the rubber insulation upto a thickness of 13.235 mm (18.235 – 5) *will be effective in heat dissipation*. **(Ans.)**

The maximum heat dissipation per m length of the cable will be

$$\frac{Q_{max}}{L} = \frac{2\pi(t_1 - t_{air})}{\frac{\ln(r_c/k_1)}{k} + \frac{1}{h_0 r_c}}$$

$$= \frac{2\pi(65 - 20)}{\frac{\ln(18.235/5)}{0.155} + \frac{1}{8.5 \times 0.018235}} = \frac{282.74}{8.348 + 6.452} = \mathbf{19.1\ W/m} \quad \textbf{(Ans.)}$$

Example 2.72. *A small electric heating application uses wire of 2 mm diameter with 0.8 mm thick insulation (k = 0.12 W/m°C). The heat transfer coefficient (h_o) on the insulated surface is 35 W/m²°C. Determine the critical thickness of insulation in this case and the percentage change in the heat transfer rate if the critical thickness is used, assuming the temperature difference between the surface of the wire and surrounding air remains unchanged.*

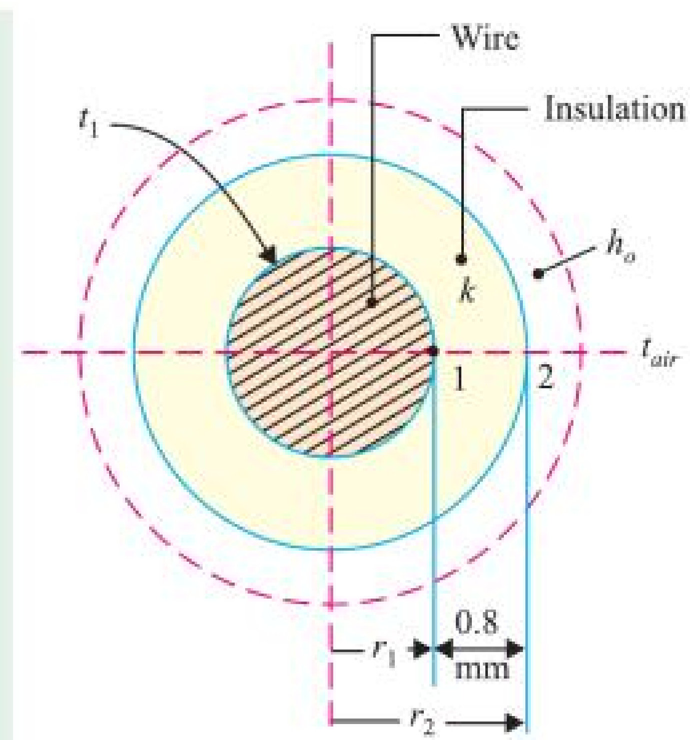

Fig. 2.88.

Solution. Refer to Fig. 2.88.

$$r_1 = \frac{2}{2} = 1\text{ mm} = 0.001\text{ m}$$

$$r_2 = 1 + 0.8 = 1.8\text{ mm} = 0.0018\text{ m}$$

$$k = 0.12\text{ W/m°C},\ h_o = 35\text{ W/m}^2\text{ °C}.$$

Critical thickness of insulation :

The critical radius of insulation is given by

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$$Q_1 = h_o A \Delta t = 12.5 \times (2\pi \times 0.004 \times 1) \times (45 - 20) = 7.85 \text{ W/m}$$

In case of the sheathed (insulated) cable, the heat flow (per metre length) is given as

$$Q_2 = \frac{2\pi L (t_2 - 20)}{\dfrac{1}{h_o \cdot r_2} + \dfrac{\ln (r_2 / r_1)}{k}} = \frac{2\pi \times 1 (t_2 - 20)}{\dfrac{1}{2.5 \times 0.0144} + \dfrac{\ln (0.044 / 0.004)}{0.18}} = 0.495 (t_2 - 20)$$

Since the intensity of current flowing through the conductor remains unaltered, therefore,

$$Q_1 = Q_2 = (I^2 R)$$

$$7.85 = 0.495 (t_2 - 20)$$

or
$$t_2 = 20 + \frac{7.85}{0.495} = \mathbf{35.86°C} \qquad \textbf{(Ans.)}$$

2.9. HEAT CONDUCTION WITH INTERNAL HEAT GENERATION

Following are some of the cases where heat generation and heat conduction are encountered :

(*i*) Fuel rods – nuclear reactor;

(*ii*) Electrical conductors;

(*iii*) Chemical and combustion processes;

(*iv*) Drying and setting of concrete.

It is of paramount importance that the heat generation rate be controlled otherwise the equipment may fail (*e.g.*, some nuclear accidents, electrical fuses blowing out). Thus, in the design of the thermal systems temperature distribution within the medium and the rate of heat dissipation to the surroundings assumes ample importance / significance.

2.9.1. PLANE WALL WITH UNIFORM HEAT GENERATION

Refer to Fig. 2.91. Consider a plane wall of thickness L (small in comparison with other dimension) of uniform thermal conductivity k and in which heat sources are uniformly distributed in the whole volume. Let the wall surfaces are maintained at temperatures t_1 and t_2.

Let us assume that heat flow is one-dimensional, under steady state conditions, and there is a *uniform volumetric heat generation* within the wall.

Consider an element of thickness at a distance x from the left hand face of the wall.

Heat conducted in at distance x,

$$Q_x = -kA \frac{dt}{dx}$$

Heat generated in the element,

$$Q_g = A \,.\, dx \,.\, q_g$$

(where q_g = heat generated per unit volume per unit time in the element)

Heat conducted out at distance $(x + dx)$, $Q_{(x + dx)} = Q_x + \dfrac{d}{dx}(Q_x)\, dx$

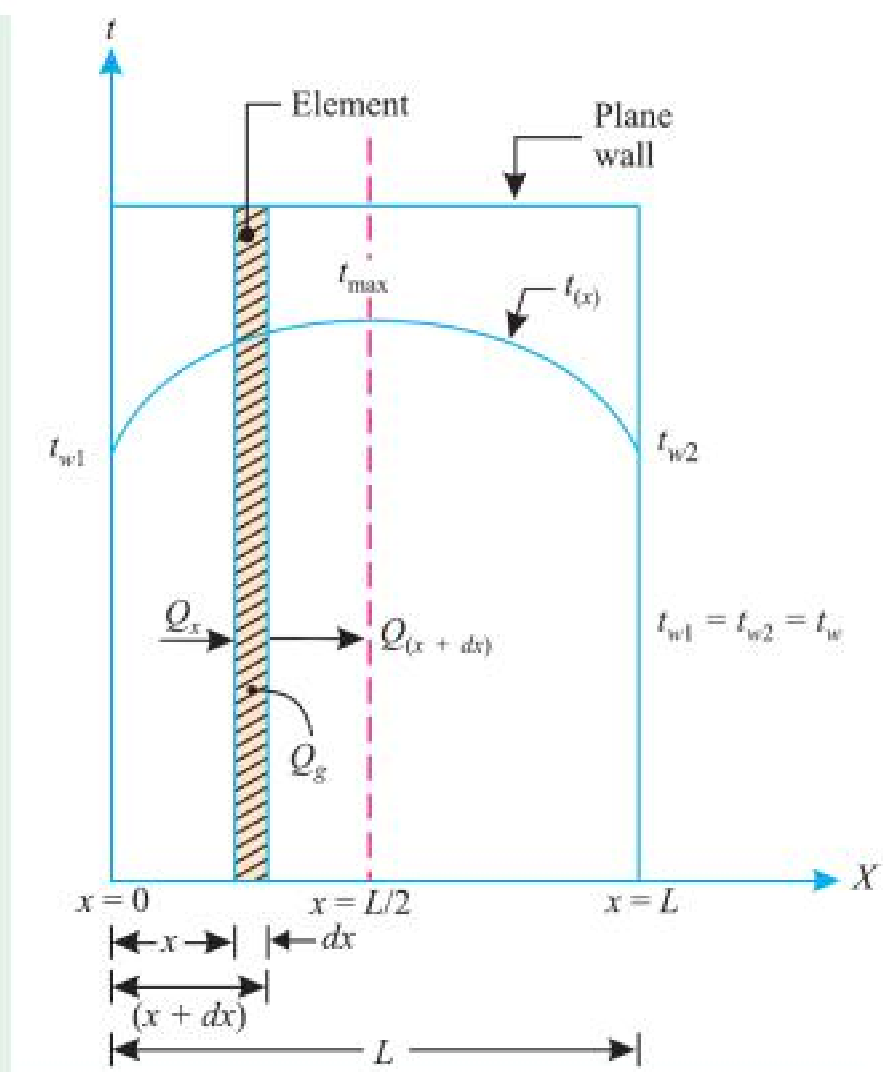

Fig. 2.91. Plane wall uniform heat generation. Both the surfaces maintained at a common temperature.

$$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = Z.\frac{x}{L}\left[1 - \frac{x}{L}\right] + \left[1 - \frac{x}{L}\right]$$

or, $$\frac{t - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{x}{L}\right]\left[\frac{Z\,x}{L} + 1\right] \quad ...(2.99)$$

In order to get maximum temperature and its location, differentiating Eqn. (2.99) w.r.t x and equating the derivative to zero, we have

$$\frac{dt}{d(x/L)} = \left(1 - \frac{x}{L}\right)Z + \left(\frac{Zx}{L} + 1\right)(-1) = 0$$

or, $$Z - \frac{Zx}{L} - \frac{Zx}{L} - 1 = 0$$

or, $$\frac{2\,Zx}{L} = Z - 1$$

or, $$\frac{x}{L} = \frac{Z - 1}{2Z} \quad ...(2.100)$$

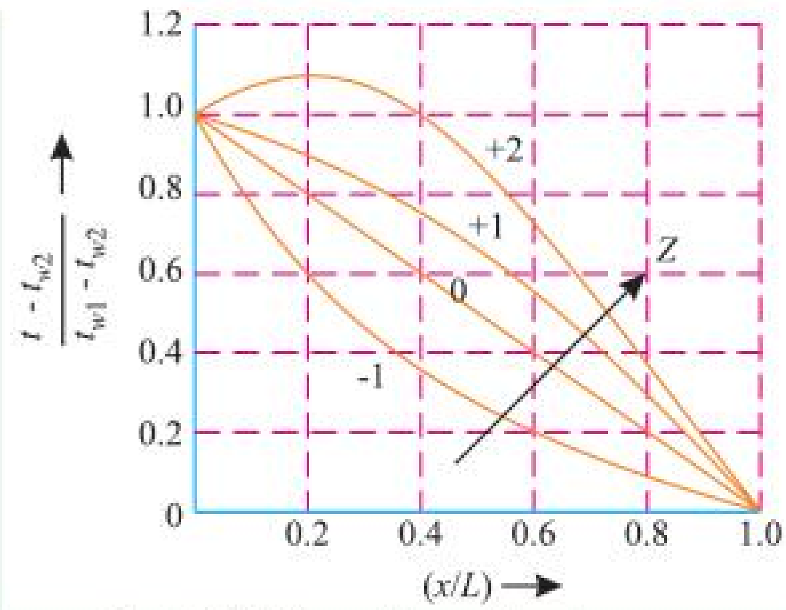

Fig. 2.94. Effect of factor Z on the temperature distribution in the plane wall.

Thus the maximum value of temperature occurs at $\frac{x}{L} = \frac{Z - 1}{2Z}$ and its value is given by:

$$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left[1 - \frac{Z - 1}{2Z}\right]\left[Z \times \left(\frac{Z - 1}{2Z}\right) + 1\right]$$

or, $$\frac{t_{max} - t_{w2}}{t_{w1} - t_{w2}} = \left(\frac{Z + 1}{2Z}\right)\left(\frac{Z + 1}{2}\right)$$

$$= \frac{(Z + 1)^2}{4Z} \quad ...(2.101)$$

Fig. 2.94 shows the effect of factor Z on the temperature distribution in the plane wall.The following points emerge :

- As the value of Z *increases* the slope of the curve changes; obviously the direction of heat flow can be reversed by an adequately large value of q_g.
- When Z = 0, the temperature distribution is *linear* (*i.e.*, no internal heat generation).
- When the value of Z is *negative*, q_g represents *absorption of heat* within the wall/body.

Case III. Current carrying electrical conductor :

When electrical current passes through a conductor, heat is generated (Q_g) in it and is given by

$$Q_g = I^2R, \text{ where } R = \frac{\rho L}{A}$$

where, I = Current flowing in the conductor,
R = Electrical resistance,
ρ = Specific resistance or resistivity,
L = Length of the conductor, and
A = Area of cross-section of the conductor.

Also, $$Q_g = q_g \times A \times L$$

$\therefore$ $$q_g \times A \times L = I^2 \times \frac{\rho L}{A} \quad \text{or,} \quad q_g = I^2 \times \frac{\rho L}{A} \times \frac{1}{AL} = \frac{I^2\rho}{A^2}$$

$$t = \left[\frac{q_g}{2k}(L - x) + \frac{(t_{w2} - t_{w1})}{L}\right]x + t_{w1} \quad \text{...[Fig. (2.98)]}$$

Substituting the values, we have

$$t = \left[\frac{30 \times 10^6}{2 \times 48}(0.025 - x) + \frac{(120 - 180)}{0.025}\right]x + 180$$

$$= [312500\ (0.025 - x) - 2400]x + 180$$

$$= [7812.5 - 312500x - 2400]x + 180$$

or, $t = 180 + 5412.5x - 312500\,x^2$ ...***Required temperature distribution.*** **(Ans.)**

(where temperature t is in °C and the distance x is in metres)

The temperature distribution is *parabolic*.

(*ii*) The value and position of the maximum temperature; t_{max}, x :

In order to determine the position of maximum temperature, differentiating the above expression and equating it to zero, we obtain

$$\frac{dt}{dx} = 5412.5 - 625000\,x = 0$$

$$\therefore \quad x = \frac{5412.5}{625000} = \mathbf{0.00866\,m\ or\ 8.66\,mm} \quad \textbf{(Ans.)}$$

The value of maximum temperature,

$$t_{max} = 180 + 5412.5 \times 0.00866 - 312500 \times 0.00866^2 = \mathbf{203.44°C} \quad \textbf{(Ans.)}$$

(*iii*) The flow of heat from each surface of the plate, q_1, q_2 :

The heat flow at the left face ($x = 0$)

$$q_1 = -k\,A\left(\frac{dt}{dx}\right)_{x=0}$$

$$= -48 \times 1 \times (5412.5 - 625000\,x)_{x=0}$$

$$= \mathbf{-259800\ W/m^2\ (Ans.)}$$

The negative signs signifies that the heat flow at the left face is in a direction *opposite* to that of measurement of the distance.

The heat flow at the right face,

$$q_2 = 48 \times 1 \times (5412.5 - 625000\,x)_{x=0.025}$$

$$= 48 \times 1(5412.5 - 625000 \times 0.025) = \mathbf{-490200\ W/m^2\ (Ans.)}$$

Check : The sum of q_1 and q_2 must be equal to total heat generated per unit length of plate.

Now $\quad q_1 + q_2 = 259800 + 490200 = 750000$ W/m²,

and $\quad Q_g = 30 \times 10^6 \times (0.025 \times 1) = 750000$ W/m²

i.e. $\quad q_1 + q_2 = Q_g$

Example 2.80. *A plane wall is 1m thick and it has one surface (x = 0) insulated while the other surface (x = L) is maintained at a constant temperature of 350°C. The thermal conductivity of wall is 25 W/m°C and a uniform heat generation per unit volume of 500 W/m³ exists throughout the wall. Determine the maximum temperature in the wall and the location of the plane where it occurs.*

(AMIE Summer, 1999)

Solution. Refer Fig. 2.97.

$L = 1$ m; $t_1 = 350$°C; $k = 25$ W/m°C,

Heat generated per unit volume

per unit time, $q_g = 500$ W/m³.

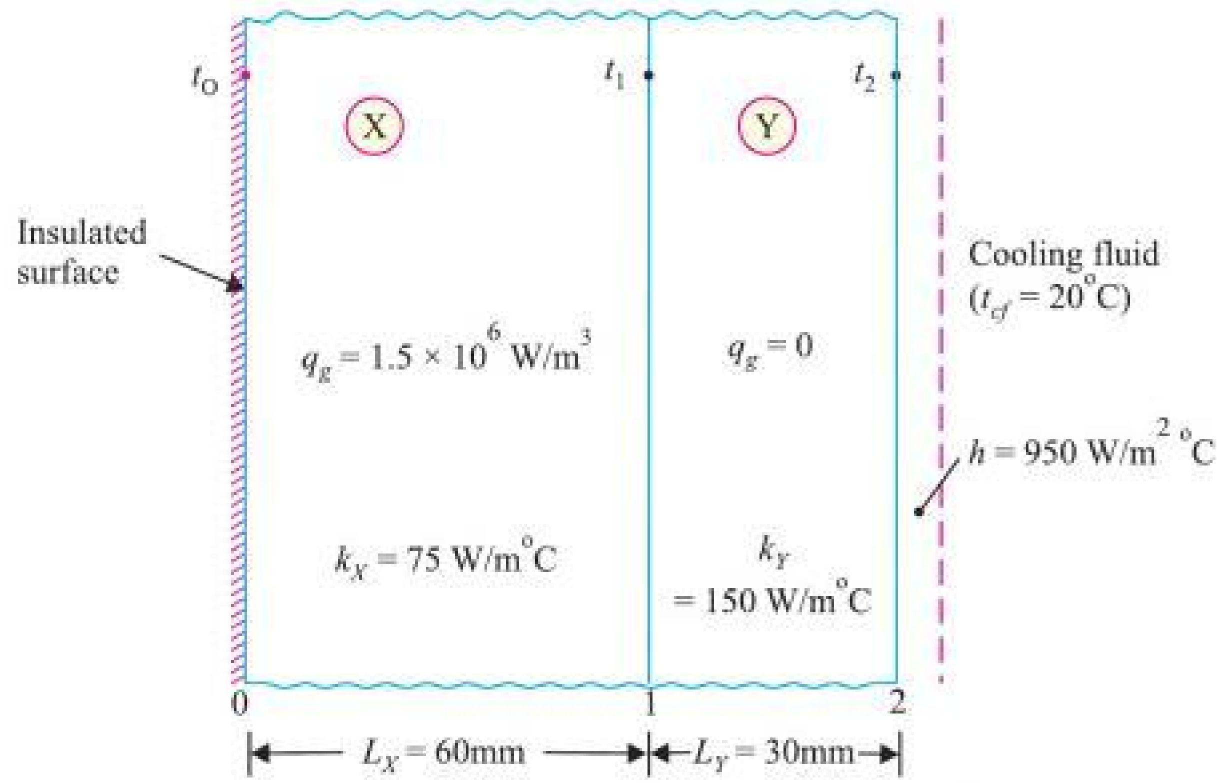

Fig. 2.99.

Cosidering heat flow through wall Y, we have

$$Q = \frac{k_Y \cdot A(t_1 - t_2)}{L_Y}$$

or,
$$90000 = \frac{150 \times 1 \times (t_1 - 114.7)}{0.03} = 5000(t_1 - 114.7)$$

or,
$$t_1 = \frac{90000}{5000} + 114.7 = 132.7°\text{C}$$

The temperature of the insulated surface of the wall X is given by,

Air force helicopters carry huge amounts of fuel in their tankers, which need to be safely protected from temperature variations.

2. $t = 150°C$ at $x = 0.05$ m

$$\therefore \quad C_2 = 150 + \frac{2 \times 10^6}{190} \times \frac{0.05^2}{2} = 163.16$$

∴ The temperature distribution in wall 'a' is given by

$$t + \frac{q_g}{k_a} \times \frac{x^2}{2} = 163.16 \qquad \text{[From eqn. } (ii)\text{]}$$

The temperature at insulated surface can be calculated by substituting $x = 0$ in the above eqn.

∴ $\boldsymbol{t_1 = 163.16° \text{ C.}}$ **(Ans.)**

Example 2.86. *A copper bar (conductor) 80 mm × 6 mm in cross-section (k = 370 W/m°C) is lying in an insulation trough so that the heat transfer from one face and both the edges is negligible. It is observed that when a current of 5000A flows through the conductor, the bare face has a constant temperature of 45°C. If the resistivity of copper is 2 × 10⁻⁸ Ωm, determine :*

(i) The maximum temperature which prevails in the bar and its location;

(ii) The temperature at the centre of the bar.

Solution. Refer Fig. 2.103.

Given : Cross-section of the conductor = 80 mm × 6 mm or 0.08 m × 0.006 m

Thermal conductivity of copper, $k = 370$ W/m°C

Resistivity of copper, $\rho = 2 \times 10^{-8}$ Ωm

Temperature of bare face, $t = 45°C$

Current flowing in the conductor, $I = 5000$ A

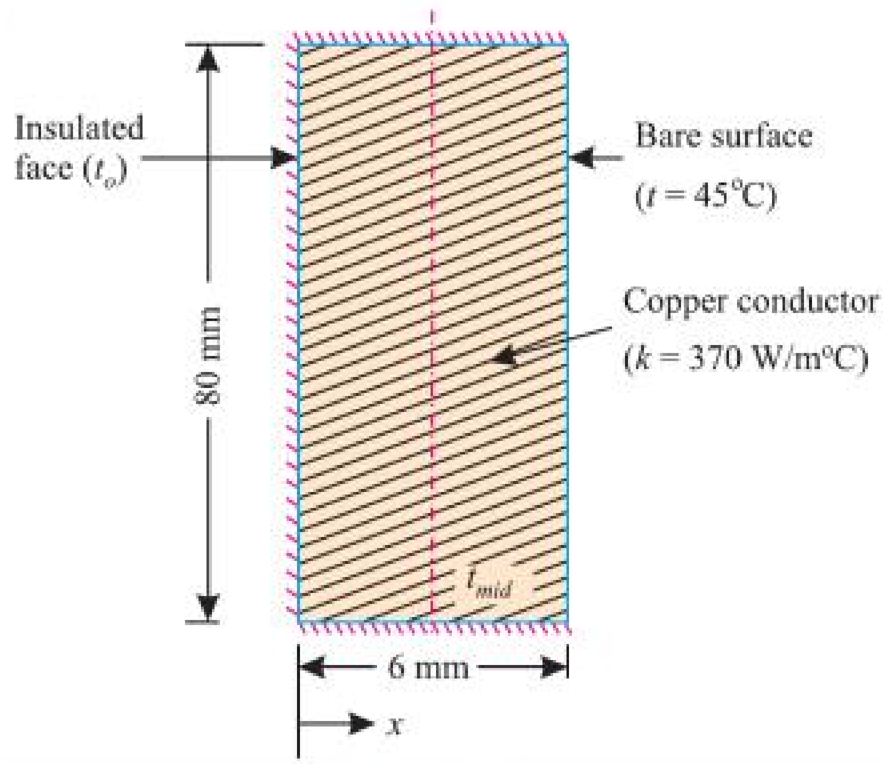

Fig. 2.103.

(*i*) The value of maximum temperature and its location :

In case of one-dimensional and steady state heat flow, the heat conduction equation may be written as :

$$\frac{d^2t}{dx^2} + \frac{q_g}{k} = 0 \qquad ...(i)$$

(where q_g = rate of heat generation per unit volume per unit time)

Integrating eqn. (*i*), twice, we have

$$\frac{dt}{dx} = -\frac{q_g}{k}x + C_1 \qquad ...(ii)$$

$$t = -\frac{q_g}{k} \cdot \frac{x^2}{2} + C_1 x + C_2 \qquad ...(iii)$$

(where C_1, C_2 = constants of integration).

The values of the constants C_1 and C_2 can be found out by using the following boundary conditions :

(*i*) At $x = 0$, $\frac{dt}{dx} = 0 \quad \therefore C_1 = 0$

(*ii*) At $x = 0.006$ m, $t = 45°C$

Amount of heat generated per unit time,

(ii) Location and magnitude of maximum temperature in the system.

Assume the flow of heat to be unidirectional and each electrode to be at a uniform temperature equal to that of the slab with which it is in contact.

Solution. Refer to Fig. 2.104.

$$L = 70 \text{ mm} = 0.07 \text{ m};\ k = 0.42 \text{ W/m°C};\ q_g = 40800 \text{ W/m}^3$$
$$h_1 = 12.5 \text{ W/m}^2\text{°C};\ h_2 = 14.5 \text{ W/m}^2\text{°C};\ t_a = 20\text{°C}.$$

(i) Surface temperatures t_{w1}, t_{w2} :

The temperature distribution, in case of dielectric heating, is given by

$$\theta = -\frac{q_g}{k}\cdot\frac{x^2}{2} + \frac{h_1\theta_1}{k}\cdot x + \theta_1$$
$$= -\frac{40800}{0.42}\cdot\frac{x^2}{2} + \frac{12.5\,\theta_1}{0.42}x + \theta_1$$

or, $\theta = -48571.4x^2 + 29.76\,\theta_1 x + \theta_1$ ...(*i*)

At $x = 0.07\text{m},\ \theta = \theta_2$

$\therefore$ $\theta_2 = -48571.4 \times (0.07)^2 + 29.76 \times 0.07\,\theta_1 + \theta_1$

or, $\theta_2 = -238 + 3.08\,\theta_1$ ...(*ii*)

Under steady state conditions,

Heat generated due to dielectric heating

= heat lost due to convection from the electrode surfaces

or, $q_g.A.L = h_1.A.\theta_1 + h_2.A.\theta_2$

or, $q_g.L = h_1.\theta_1 + h_2.\theta_2$

or, $40800 \times 0.07 = 12.5\,\theta_1 + 14.5\,\theta_2$

or, $196.96 = 0.862\,\theta_1 + \theta_2$

or, $\theta_2 = 196.96 - 0.862\theta_1$ ...(*iii*)

From eqns. (*ii*) and (*iii*), we have

$$-238 + 3.08\,\theta_1 = 196.96 - 0.862\,\theta_1$$

or, $$\theta_1 = \frac{(238 + 196.96)}{(3.08 + 0.862)} = 110.34\text{°C}$$

and $\theta_2 = -238 + 3.08 \times 110.34 = 101.85\text{°C}$ [From eqn. (*ii*)]

Thus, the electrode temperatures are :

$\theta_1 = t_{w1} - t_a \quad \therefore \quad t_{w1} = \theta_1 + t_a = 110.34 + 20 =$ **130.34°C (Ans.)**

$\theta_2 = t_{w2} - t_a \quad \therefore \quad t_{w2} = \theta_2 + t_a = 101.85 + 20 =$ **121.85°C (Ans.)**

(ii) Location and magnitude of maximum temperature; x, t_{max} :

The location of maximum temperature can be obtained by differentiating eqn. (*i*) w.r.t. x and equating the derivative to zero.

Thus, $$\frac{d\theta}{dx} = \frac{d}{dx}\left[-48571.4x^2 + 29.76\,\theta_1 \cdot x + \theta_1\right]$$

or, $$\frac{d\theta}{dx} = -48571.4 \times 2x + 29.76\,\theta_1 = 0$$

or, $$x = \frac{29.76\,\theta_1}{48571.4 \times 2} = \frac{29.76 \times 110.34}{48571.4 \times 2} = 0.0338\text{ m}$$ or 33.8 mm (from left hand electrode)

$\therefore$ $t_{max} = -48571.4 \times 0.0338^2 + 29.76 \times 110.4 \times 0.0338 + 110.4$

= **165.96°C (Ans.)**

$$q_g = \frac{Q_g}{AL} = I^2 \times \frac{\rho L}{A} \times \frac{1}{AL} = \rho\left(\frac{I}{A}\right)^2$$

$$= 70 \times 10^{-8}\left[\frac{300}{\pi \times 0.00125^2}\right]^2$$

$$= 26.14 \times 10^8 \text{ W/m}^3$$

Temperature at the surface of wire is given by

$$t_w = t_a + \frac{q_g}{2h} \cdot R \qquad \text{...[Eqn. (2.111)]}$$

or,
$$t_w = 50 + \frac{26.14 \times 10^8}{2 \times 4000} \times 0.00125 = \mathbf{458.44°\,C} \quad \textbf{(Ans.)}$$

Temperature at the centre of wire is given by

$$t_{max} = t_w + \frac{q_g}{4k} \cdot R^2 \qquad \text{[Eqn. (12.109)]}$$

or,
$$t_{max} = 458.44 + \frac{26.14 \times 10^8}{4 \times 20} \times (0.00125)^2 = \mathbf{509.5°\,C} \quad \textbf{(Ans.)}$$

Example 2.89. *A 3 mm diameter stainless steel wire (k = 20 W/m°C, resistivity, ρ = 10 × 10^{-8} Ωm) 100 metres long has a voltage of 100 V impressed on it. The outer surface of the wire is maintained at 100°C. Calculate the centre temperature of the wire. If the heated wire is submerged in a fluid maintained at 50°C, find the heat transfer coefficient on the surface of the wire.* **(M.U.)**

Solution. Radius of stainless steel wire, $R = \frac{3}{2} = 1.5 \text{ mm} = 0.0015 \text{ m}$

Length of the wire, $L = 100$ m

Voltage impressed $= 100$ V

Thermal conductivity, $k = 20$ W/m°C

Resistivity, $\rho = 10 \times 10^{-8}$ Ωm

The temperature of the outer surface of the wire,

$t_w = 100$°C

Fluid temperature, $t_a = 50$°C.

Centre temperature of the wire, t_{max} :

Electrical resistance of the wire, $R_e = \frac{\rho L}{A} = \frac{10 \times 10^{-8} \times 100}{\pi \times 0.0015^2} = 1.415\,\Omega$

Rate of heat generation, $Q_g = VI = \frac{V^2}{R_e} = \frac{100^2}{1.415} = 7067 \text{ W}$

∴ Rate of heat generation per unit volume

$$q_g = \frac{Q_g}{AL} = \frac{7067}{\pi \times 0.0015^2 \times 100} = 9.998 \times 10^6 \text{ W/m}^3$$

The centre temperature is given by

$$t_{max} = t_w + \frac{q_g}{4k} \cdot R^2 \qquad \text{...[Fig. (2.109)]}$$

$$= 100 + \frac{9.998 \times 10^6}{4 \times 20} \times 0.0015^2 = 100.28°\text{C}$$

At $r = R$, $t = t_s$ where t_s is surface temperature.

$$\therefore \qquad t_s = -\frac{q_0}{k}\left[\frac{R^2}{4} - \frac{R^4}{16R^2}\right] + C_2$$

or,
$$C_2 = t_s + \frac{q_0}{k} \times \frac{3}{16} R^2$$

From (*i*) as occurs at $r = 0$

$$t_{max} = C_2$$

or,
$$t_{max} = t_s + \frac{3}{16}\frac{q_0}{k} R^2 \text{ is the } \textbf{\textit{required expression.}} \quad \textbf{(Ans.)}$$

Substituting the numerical values,

$$t_{max} = 20 + \frac{3}{16} \times \frac{24 \times 10^6}{200} \times \left(\frac{40}{2 \times 100}\right)^2 = \mathbf{920° C.} \quad \textbf{(Ans.)}$$

Example 2.94. *A long hollow cylinder has inner and outer radii 50 mm and 150 mm resepctively. It generates heat at a rate of 1 kW/m³ (k = 0.5 W/m°C). If the maximum temperature occurs at radius of 100 mm and temperature of outer surface is 50°C, find :*

(i) Temperature at inner surface, and

(ii) Maximum temperature in the cylinder. **(P.U. 2001)**

Solution. Refer to Fig. 2.107.

$r_1 = 50$ mm $= 0.05$ m; $r_2 = 150$ mm $= 0.15$ m

q_g (rate of heat generation) $= 1$ kW/m³ $= 1000$ W/m³

$k = 0.5$ W/m°C; $t_2 = 50$°C.

t_1, t_{max} :

Consider an element of hollow cylinder at a radius r and thickness dr and length L.

Heat conducted at radius r,

$$Q_r = -k \times 2\pi r L \times \frac{dt}{dr}$$

Heat generated in the element,

$$Q_g = q_g \times 2\pi r L \times dr$$

Heat conducted at radius $(r + dr)$,

$$Q_{(r+dr)} = Q_r + \frac{d}{dr}(Q_r)\,dr$$

For steady state conduction of heat flow,

$$Q_r + Q_g = Q_{(r+dr)}$$

or,
$$Q_r + Q_g = Q_r + \frac{d}{dr}(Q_r)\,dr$$

or,
$$Q_g = \frac{d}{dr}(Q_r)\,dr$$

or,

$$q_g \times 2\pi r L \times dr = \frac{d}{dr}\left[-2\pi r L k \times \frac{dt}{dr}\right] dr$$

$$r q_g = -k\frac{d}{dr}\left(r\frac{dt}{dr}\right)$$

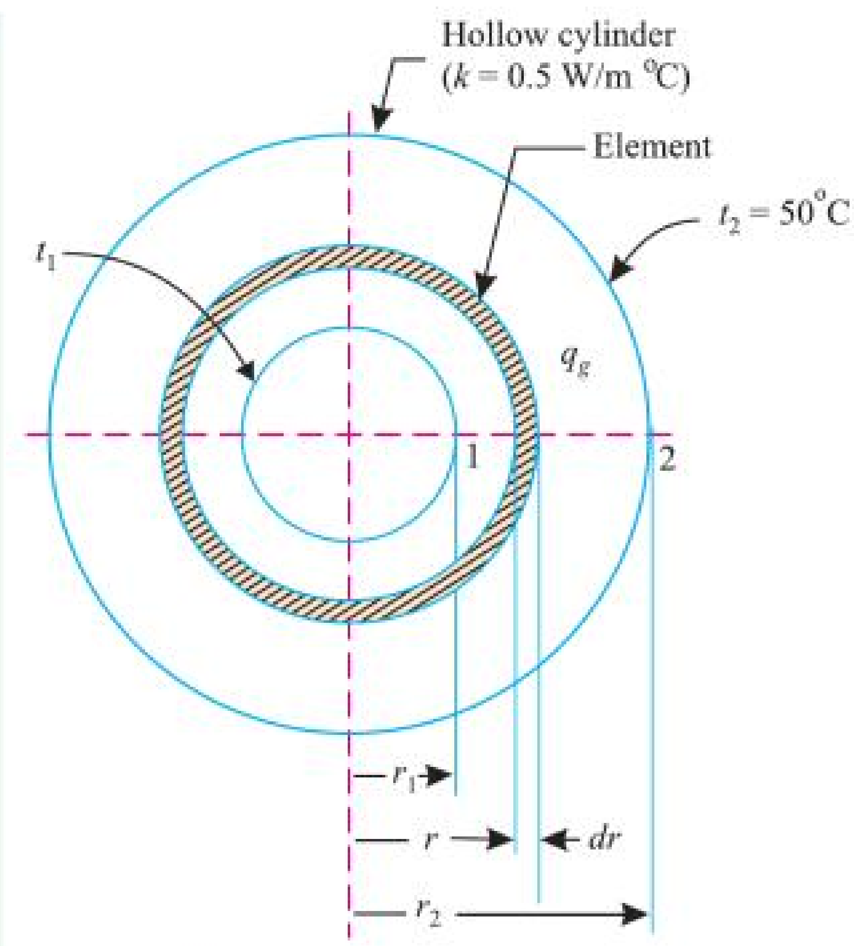

Fig. 2.107. Heat generation in hollow cylinder.

Integrating the above equation twice, we have

$$r \cdot \frac{dt}{dr} + \frac{q_g}{k} \cdot \frac{r^2}{2} = C_1$$

or,
$$\frac{dt}{dr} + \frac{q_g}{k} \cdot \frac{r}{2} = \frac{C_1}{r} \quad ...(i)$$

and,
$$t + \frac{q_g}{k} \cdot \frac{r^2}{4} = C_1 \ln(r) + C_2 \quad ...(ii)$$

(where C_1, C_2 = constants of integration).

Applying boundary condition; at $r = r_1$, $\frac{dt}{dr} = 0$, we have,

$$C_1 = \frac{q_g}{k} \cdot \frac{r_1^2}{2} \quad ...[\text{From Eqn. } (i)]$$

Applying the following boundary condition, we have,

(Heat conducted)$_{r = r_2}$ = Heat convected from the outer bondary to the surrounding fluid.

or,
$$-k \times (2\pi r_2 \times 1) \times \left(\frac{dt}{dr}\right)_{r = r_2} = h \times (2\pi r_2 \times 1) \times (t_2 - t_a)$$

(considering unit length of cylinder)

or,
$$-k\left[\frac{C_1}{r} - \frac{q_g}{k} \cdot \frac{r}{2}\right]_{r = r_2} = h(t_2 - t_a)$$

or,
$$-k\left[\frac{q_g}{2k} \cdot \frac{r_1^2}{r_2} - \frac{q_g r_2}{2k}\right] = h(t_2 - t_a)$$

or,
$$\frac{q_g r_2}{2}\left[1 - \left(\frac{r_1}{r_2}\right)^2\right] = h(t_2 - t_a) \quad ...(iii)$$

Substituting the value of C_1 in eqn. (ii), we have

$$t + \frac{q_g}{k} \cdot \frac{r^2}{4} = \frac{q_g}{k} \cdot \frac{r_1^2}{2} \ln(r) + C_2 \quad ...(iv)$$

Now, applying boundary condition, at $r = r_1$, $t = t_1$, we have

$$t_1 + \frac{q_g}{k} \cdot \frac{r_1^2}{4} = \frac{q_g}{k} \frac{r_1^2}{2} \ln(r_1) + C_2$$

$\therefore$
$$C_2 = t_1 + \frac{q_g r_1^2}{2k}\left[\frac{1}{2} - \ln(r_1)\right]$$

Inserting the values of C_1 and C_2 in eqn. (iv), we have

$$t + \frac{q_g}{k} \cdot \frac{r^2}{4} = \frac{q_g}{k} \cdot \frac{r_1^2}{2} \ln(r) + t_1 + \frac{q_g r_1^2}{2k}\left[\frac{1}{2} - \ln(r_1)\right]$$

or,
$$t - t_1 = \frac{q_g r_1^2}{2k} \ln(r) + \frac{q_g r_1^2}{4k} - \frac{q_g r_1^2}{2k} \ln(r_1) - \frac{q_g r^2}{4k}$$

or,
$$t - t_1 = \frac{q_g r_1^2}{4k}\left[1 - \left(\frac{r}{r_1}\right)^2\right] + \frac{q_g r_1^2}{2k} \ln(r/r_1) \quad ...(v)$$

At $r = r_2$, $t = t_2$; the eqn. (v) becomes

$$t + \frac{q_g}{k} \cdot \frac{x^2}{2} = C_1 x + C_2 \quad \text{...(iii)}$$

(where C_1, C_2 = constants of integration).

In order to evaluate C_1 and C_2, using the following boundary conditions, we have

(*i*) At $x = 0$, $t = t_1$ $\therefore$ $C_2 = t_1$

(*ii*) At $x = L$, $t = t_2$

$$\therefore \quad t_2 + \frac{q_g}{k} \cdot \frac{L^2}{2} = C_1 L + t_1 \quad \text{or} \quad C_1 = \frac{t_2 - t_1}{L} + \frac{q_g}{k} \cdot \frac{L}{2}$$

Substituting the values of C_1 and C_2 in eqn. (*iii*), we get

$$t = -\frac{q_g}{k} \cdot \frac{x^2}{2} + \left[\frac{t_2 - t_1}{L} + \frac{q_g}{k} \cdot \frac{L}{2}\right] x + t_1 \quad \text{...(iv)}$$

The maximum temperature occurs where $\frac{dt}{dx} = 0$

$$\therefore \quad \frac{dt}{dx} = -\frac{q_g}{k} x + C_1 = -\frac{q_g}{k} \cdot x \left[\frac{t_2 - t_1}{L} + \frac{q_g}{k} \cdot \frac{L}{2}\right] = 0 \quad \text{...from eqn. (ii)}$$

or,
$$\frac{q_g}{k} \cdot x = \frac{t_2 - t_1}{L} + \frac{q_g}{k} \cdot \frac{L}{2}$$

or,
$$x = \frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2} \quad \text{...(v)}$$

Inserting the value of x in eqn. (*iv*), we have

$$t_{\max} = -\frac{q_g}{2k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]^2 + \frac{t_2 - t_1}{L}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right] + \frac{q_g L}{2k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right] + t_1$$

$$= t_1 - \frac{q_g}{2k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]^2 + \left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]\left[\frac{q_g \cdot L}{2k} + \frac{t_2 - t_1}{L}\right]$$

$$= t_1 - \frac{q_g}{2k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]^2 + \frac{q_g}{k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]^2$$

or,
$$t_{max} = t_1 + \frac{q_g}{2k}\left[\frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2}\right]^2 \quad \text{...(vi)}$$

Substituting the proper values in eqn. (*vi*), we get

$$t_{max} = 100 + \frac{56588}{2 \times 40}\left[\frac{40(75 - 100)}{56588 \times 0.3} + \frac{0.3}{2}\right]^2 = \mathbf{105.8°C} \quad \textbf{(Ans.)}$$

The distance x at which t_{max} occurs is given by

$$x = \frac{k(t_2 - t_1)}{q_g \cdot L} + \frac{L}{2} \quad \text{...[Eqn. (v)]}$$

$$= \frac{40(75 - 100)}{56588 \times 0.3} + \frac{0.3}{2} = 0.09109 \text{ m or } \mathbf{91.09 \text{ mm}} \quad \textbf{(Ans.)}$$

(*ii*) The heat flux at each end :

The heat flow from the rod at $x = 0$,

$$Q_1 = -kA \left.\frac{dt}{dx}\right|_{x=0}$$

$$= -kA\left[-\frac{q_g \cdot x}{k} + \frac{t_2 - t_1}{L} + \frac{q_g}{k}\cdot\frac{L}{2}\right]_{x=0}$$

$$= -40 \times \frac{\pi}{4} \times 0.03^2\left[\frac{75-100}{0.3} + \frac{56588}{40} \times \frac{0.3}{2}\right] = \mathbf{-3.64\ W}$$

(–ve sign indicates that the heat flows in a direction *opposite* to that of x-direction). The heat flow from the rod at $x = L$ (=0.3m),

$$Q_2 = -k\,A\left|\frac{dt}{dx}\right|_{x=L}$$

$$= -kA\left[-\frac{q_g \cdot L}{k} + \frac{t_2 - t_1}{L} + \frac{q_g}{k}\cdot\frac{L}{2}\right]$$

$$= -kA\left[\frac{t_2 - t_1}{L} - \frac{q_g}{k}\cdot\frac{L}{2}\right]$$

$$= -40 \times \frac{\pi}{4} \times 0.03^2\left[\frac{75-100}{0.3} - \frac{56588}{40} \times \frac{0.3}{2}\right] = \mathbf{8.36\ W} \qquad \textbf{(Ans.)}$$

[Check : The sum of Q_1 and Q_2 should be equal to Q_g, *i.e.*, $Q_1 + Q_2 = 3.64 + 8.36 = 12\text{W}$ *i.e.*, Q_g]

Example 2.99. *The rate of heat generation per unit volume in a long cylinder of radius R is given by*

$$q_g = a + br^2$$

where a and b are constants and r is any radius. The cylinder is undergoing heat transfer with a medium at a temperature t_a and surface heat transfer coefficient is h. Find the steady state temperature distribution in the solid. **(M.U.)**

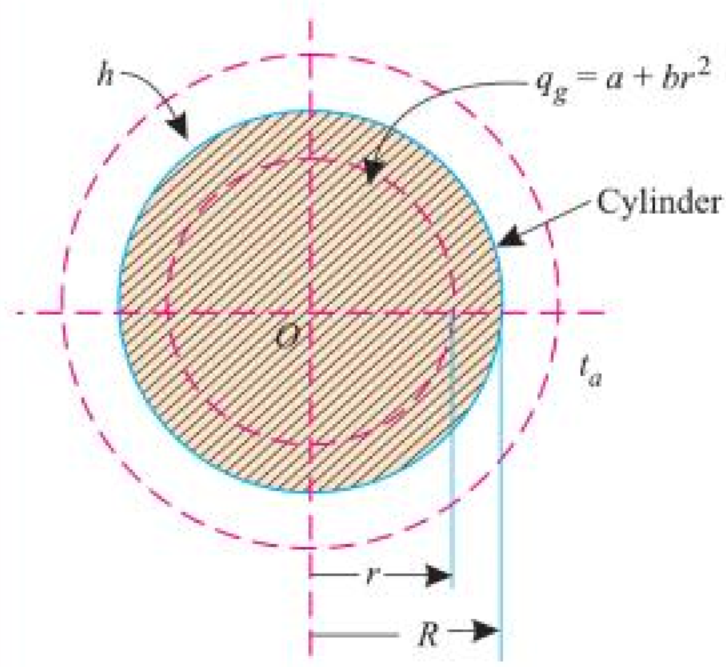

Fig. 2.112.

Solution. For the given problem, the controlling differential equation is given by

$$\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right) + \frac{q_g}{k}\cdot r = 0$$

...[Eqn. (2.105)]

Substituting the value of q_g, we have

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) + \frac{r}{k}(a + br^2) = 0$$

Integrating the above equation, we get

$$r\frac{dt}{dr} + \frac{1}{k}\left(\frac{ar^2}{2} + \frac{br^4}{4}\right) = C_1 \qquad ...(i)$$

or,
$$\frac{dt}{dr} + \frac{1}{k}\left(\frac{ar}{2} + \frac{br^3}{4}\right) = \frac{C_1}{r}$$

Integrating again, we get

$$t + \frac{1}{k}\left[\frac{ar^2}{4} + \frac{br^4}{16}\right] = C_1 \ln(r) + C_2 \qquad ...(ii)$$

(where C_1, C_2 = constants of integration).

The boundary conditions for finding C_1 and C_2 are :

Overground oil storage tanks.

(i) At $r = 0$, $\frac{dt}{dr} = 0$ $\therefore C_1 = 0$ [from eqn. (i)]

$$\therefore \quad \frac{dt}{dr} + \frac{1}{k}\left(\frac{ar}{2} + \frac{br^3}{4}\right) = 0 \quad ...(iii)$$

and
$$t + \frac{1}{k}\left[\frac{a\,r^2}{4} + \frac{br^4}{16}\right] = C_2 \quad ...(iv)$$

[From eqn. (ii)]

(ii) $(\text{Heat conducted})_{r=R} = (\text{Heat convected})_{r=R}$

or,
$$\left[-kA\cdot\left(\frac{dt}{dr}\right)\right]_{r=R} = \left[h\cdot A(t - t_a)\right]_{r=R}$$

$$\left[-k\cdot\frac{dt}{dr}\right]_{r=R} = \left[h(t - t_a)\right]_{r=R}$$

Substituting the values from eqns. (iii) and (iv) for $\frac{dt}{dr}$ and t in the above equation for $r = R$, we get

$$-k\left[-\frac{1}{k}\left(\frac{aR}{2} + \frac{bR^3}{4}\right)\right] = h\left[C_2 - \frac{1}{k}\left(\frac{aR^2}{4} + \frac{bR^4}{16}\right) - t_a\right]$$

$$\therefore \quad \frac{R}{2h}\left[a + \frac{bR^2}{2}\right] = C_2 - \frac{R^2}{4k}\left[a + \frac{bR^2}{4}\right] - t_a$$

or,
$$C_2 = t_a + \frac{R}{2h}\left[a + \frac{bR^2}{2}\right] + \frac{R^2}{4k}\left[a + \frac{b\,R^2}{4}\right]$$

Now, substituting this value in eqn. (iv), we obtain

$$t + \frac{r^2}{4k}\left[a + \frac{br^2}{4}\right] = t_a + \frac{R}{2h}\left[a + \frac{bR^2}{2}\right] + \frac{R^2}{4k}\left[a + \frac{bR^2}{4}\right]$$

$$\therefore \quad t - t_a = \frac{R}{2h}\left[a + \frac{bR^2}{2}\right] + \left[\frac{aR^2}{4k} + \frac{bR^4}{16k}\right] - \left[\frac{ar^2}{4k} + \frac{br^4}{16\,k}\right]$$

or,
$$t - t_a = \frac{R}{2h}\left[a + \frac{bR^2}{2}\right] + \frac{aR^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right] + \frac{bR^4}{16k}\left[1 - \left(\frac{r}{R}\right)^4\right]$$

...(Temperature distribution) **(Ans.)**

Example 2.100. *A copper conductor (k = 380 W/m°C, resistivity ρ = 2 × 10⁻⁸ W m) having inner and outer radii 1.0 cm and 2.25 cm respectively is carrying a current density of 4800 amperes/cm². The conductor is internally cooled and a constant temperature of 65°C is maintained at the inner surface and there is no heat transfer through insulation surrounding the conductor. Determine:*

(i) The maximum temperature of the conductor and the radius at which it occurs, and

(ii) The internal heat transfer rate.

Solution. Refer to Fig. 2.113.

r_1 = 1.0 cm; r_2 = 2.25 cm; J (current density) = 4800 amp/cm² or 4800 × 10⁴ amp/m²

k = 380 W/m°C; ρ = 2 × 10⁻⁸ Ωm; t_o = 65°C.

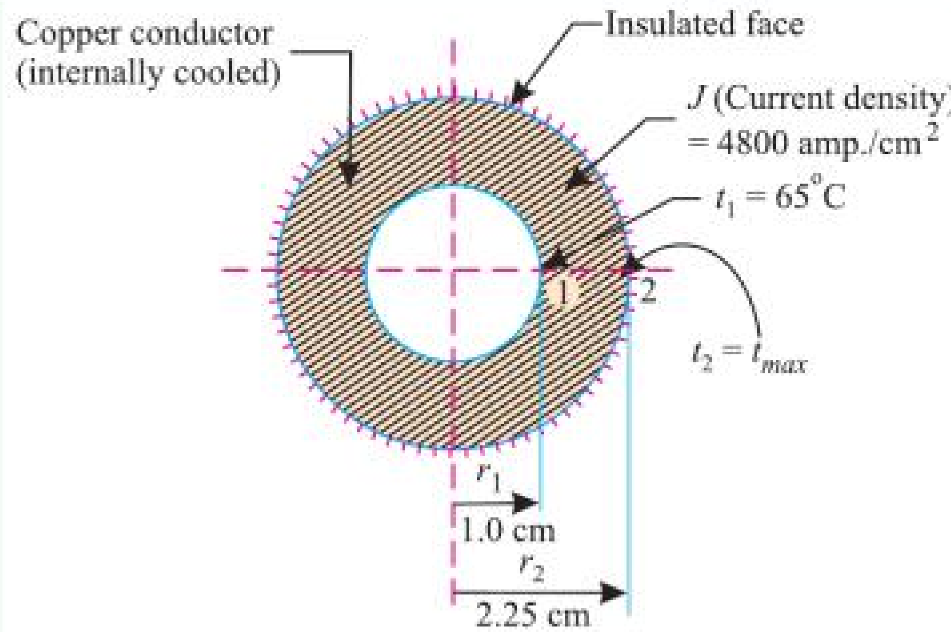

Fig. 2.113.

(i) The maximum temperature of the conductor and the radius at which it occurs; t_{max}, r :

Total volumetric heat generated

$$= I^2 R = I^2 \cdot \frac{\rho L}{A}$$

Heat generated per unit volume,

$$q_g = \frac{I^2 \cdot \frac{\rho L}{A}}{A \times L} = I^2 \cdot \frac{\rho L}{A} \times \frac{1}{AL} = \rho\left(\frac{I}{A}\right)^2$$

$$= \rho J^2 = 2 \times 10^{-8} \times [4800 \times 10^4]^2 = 46.08 \times 10^6 \text{ W/m}^3$$

The differential equation describing the temperature distribution through a cylindrical conductor is given by

$$\frac{d}{dr}\left(r\frac{dt}{dr}\right) + \frac{q_g}{k} \cdot r = 0$$

Integrating eqn. (*i*) twice, we get

$$r\frac{dt}{dr} + \frac{q_g}{k} \cdot \frac{r^2}{2} = C_1 \quad ...(ii)$$

or,
$$\frac{dt}{dr} + \frac{q_g}{k} \cdot \frac{r}{2} = \frac{C_1}{r}$$

and,
$$t + \frac{q_g}{k} \cdot \frac{r^2}{4} = C_1 \ln(r) + C_2$$

or,
$$t = -\frac{q_g}{k} \cdot \frac{r^2}{4} + C_1 \ln(r) + C_2 \quad ...(iii)$$

(where C_1, C_2 = constants of integration).

The values of C_1 and C_2 are found from the following boundary conditions :

(*i*) At $r = r_2$ = 2.25 cm (or 0.0225 m), $\frac{dt}{dr} = 0$ (since the face is insulated and there is no heat transfer).

$$\therefore \quad C_1 = \frac{q_g}{k} \cdot \frac{r^2}{2} = \frac{46.08 \times 10^6}{380} \times \frac{0.0225^2}{2} = 30.69. \quad \text{...[From eqn. (ii)]}$$

(*ii*) At $r = r_1 = 1.0$ cm (or 0.01 m), $t = t_1 = 65°C$

$$65 = -\frac{46.08 \times 10^6}{380} \times \frac{0.01^2}{4} + 30.69 \ln(0.01) + C_2$$

$$\therefore \quad C_2 = 65 + \frac{46.08 \times 10^6}{380} \times \frac{0.01^2}{4} - 30.69 \ln(0.01) = 209.36°C$$

Substituting the values of C_1 and C_2 in eqn. (*iii*) we get the temperature distribution through the conductor as

$$t = -\frac{46.08 \times 10^6}{4 \times 380} r^2 + 30.69 \ln(r) + 209.36$$

or,
$$t = -30315.8\, r^2 + 30.69 \ln(r) + 209.36 \quad \text{...(iv)}$$

It is evident from eqn. (*iv*), that the temperature distribution is *parabolic*.

Maximum temperature occurs at the insulated face (at $\mathbf{r = r_2 = 0.0225}$ **m**) and its value equals,

$$t_{max} = -30315.8 \times 0.0225^2 + 30.69 \ln(0.0225) + 209.36$$
$$= -15.35 - 116.44 + 209.36 = \mathbf{77.57°C} \quad \textbf{(Ans.)}$$

(*ii*) The internal heat transfer rate, Q :

$$Q = -kA\frac{dt}{dr}$$

But,
$$\frac{dt}{dr} = -\frac{q_g}{k} \cdot \frac{r}{2} + \frac{C_1}{r} \quad \text{[From eqn. (ii)]}$$

$$\therefore \quad \left.\frac{dt}{dr}\right|_{r=0.01} = -\frac{46.08 \times 10^6}{380} \times \frac{0.01}{2} + \frac{30.69}{0.01} = 2462.68$$

$\therefore \quad Q = -380 \times (2\pi \times 0.01 \times 1) \times 2462.68 = \mathbf{58800\ W/m}$

(– ve sign indicates that heat flow is radially *inwards*.)

[Check : Internal heat transfer $= q_g \times [\pi (0.0225^2 - 0.01^2) \times 1]$
$= 46.08 \times 10^6 [\pi (0.0225^2 - 0.01^2) \times 1] = \mathbf{58800\ W/m}$]

Example 2.101. *A hollow cylinder having inner and outer radii r_1 and r_2 respectively is developing heat uniformly, q_g per unit volume per unit time. The conductivity of the cylinder material is given by; $k = k_o (1 + \beta t)$. If the outside surface temperature is t_w, prove that the temperature distribution in cylinder is given by*

$$t = -\frac{1}{\beta} + \sqrt{\left(\frac{1}{\beta} + t_w\right)^2 - \frac{q_g r_1^2}{2\beta k_o}\left[\left(\frac{r}{r_1}\right)^2 + 2\ln(r_2/r) - \left(\frac{r_2}{r_1}\right)^2\right]}$$

Solution. Refer to Fig. 2.114.

Heat generation rate between $r = r_1$ and $r = r$ = heat conducted at $r = r$

$$(q_g)_r = q_g \times \pi (r^2 - r_1^2) \times 1$$
$$= -k_0 (1 + \beta t) 2\pi r \times 1 \times \frac{dt}{dr} \quad \text{(Considering unit length of the cylinder)}$$

$$\left(\frac{r^2 - r_1^2}{r}\right) \cdot \frac{q_g}{2k_o} \cdot dr = -(1 + \beta t)\, dt$$

$$-\left(r - \frac{r_1^2}{r}\right) \cdot \frac{q}{2k_o} \cdot dr = (1 + \beta t)\, dt$$

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or, $$\frac{dt}{dr} + \frac{q_o}{k}\left[\frac{r}{2} - \frac{r^3}{4R_{fr}^2}\right] = \frac{C_1}{r}$$

Integrating again, we obtain

$$t + \frac{q_o}{k}\left[\frac{r^2}{4} - \frac{r^4}{16R_{fr}^2}\right] = C_1 \ln(r) + C_2 \qquad ...(iii)$$

(where C_1, C_2 = constants of integration).

Using the following boundary conditions, we have

(*i*) At $r = 0$, $\frac{dt}{dr} = 0$ $\quad \therefore C_1 = 0$ [From eqn. (*ii*)]

(*ii*) At $r = 0$, $t = t_{max}$ $\quad \therefore C_2 = t_{max}$ [From eqn. (*iii*)]

Substituting these values of the constants in eqn. (*iii*), we get

$$t + \frac{q_o}{k}\left[\frac{r^2}{4} - \frac{r^4}{16R_{fr}^2}\right] = t_{max}$$

or, $$t - t_{max} = -\frac{q_o}{k}\left[\frac{r^2}{4} - \frac{r^4}{16R_{fr}^2}\right] \qquad ...(iv)$$

If t_w is the temperature at the outer surface of the rod *i.e.*, at $r = R_{fr}$, then

$$t_w - t_{max} = -\frac{q_o}{k_{fr}}\left[\frac{R_{fr}^2}{4} - \frac{R_{fr}^4}{16R_{fr}^2}\right] = -\frac{q_o}{k}\left[\frac{R_{fr}^2}{4} - \frac{R_{fr}^2}{16}\right]$$

or, $$t_w - t_{max} = -\frac{3q_0 R_{fr}^2}{16 k_{fr}} \qquad ...(2.118)$$

(where k_{fr} = thermal coductivity of fuel rod material).

Also, the rate of heat transfer at the surface of the rod,

$$Q = -k_{fr} A \left|\frac{dt}{dr}\right|_{r = R_{fr}}$$

$$= k_{fr} A \left[-\frac{q_o}{k_{fr}}\left\{\frac{r}{2} - \frac{r^3}{4R_{fr}^2}\right\}\right]_{r = R_{fr}}$$

[Substituting the value of $\frac{dt}{dr}$ from eqn. (*ii*)]

or, $$Q = -k_{fr} A\left[-\frac{q_0}{k_{fr}}\left\{\frac{R_{fr}}{2} - \frac{R_{fr}^3}{4R_{fr}^2}\right\}\right] = \frac{q_0 A R_{fr}}{4} \qquad ...(2.119)$$

This heat would be convected from the outside surface of the rod, under steady state conditions.

$$\therefore \quad \frac{q_o A R_{fr}}{4} = h A (t_w - t_a)$$

[where h = convective heat transfer coefficient and

t_a = ambient temperature.]

or, $$t_w = t_a + \frac{q_o A R_{fr}}{4hA} = t_a + \frac{q_o R_{fr}}{4h} \qquad ...(v)$$

Inserting this value of t_w in eqn. (2.118), we obtain

The maximum value of temperature (t_{max}) which occurs at the centre of the rod at (at $r = 0$), is given by

$$t_{\max} = t_w + \frac{q_o R_{fr}^2}{4}\left[\frac{3}{4k_{fr}} + \frac{1}{k_{cl}} \cdot \ln(R_{cl}/R_{fr})\right] \quad ...(2.121)$$

Example 2.102. *In a cylindrical fuel rod of nuclear reactor, the internal heat generation is given by*

$$q_g = q_o\left[1 - \left(\frac{r}{R_{fr}}\right)^2\right]$$

where R_{fr} is the radius of the fuel rod.

Calculate the temperature drop from the centre to the surface of a 25 mm diameter fuel rod having $k = 20$ W/m°C when the rate of heat generation from the surface = 0.25 MW/m².

(P.U. 1997)

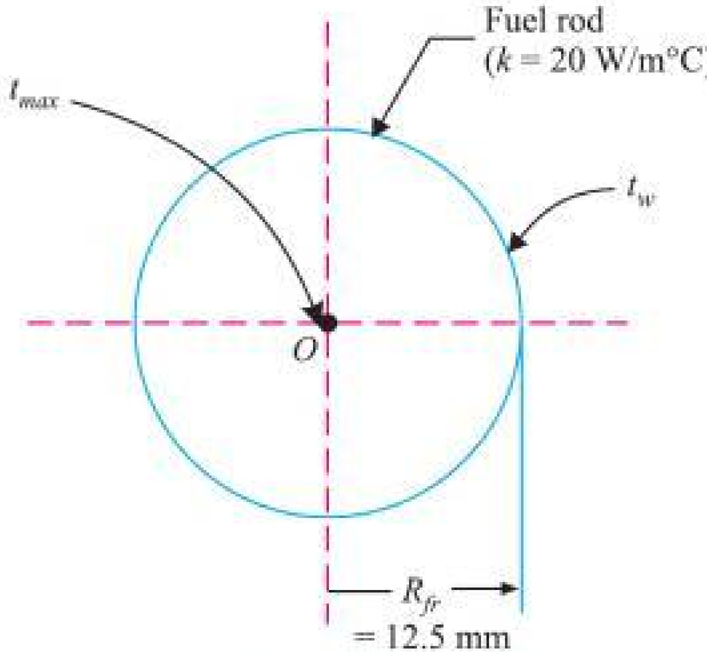

Fig. 2.118.

Solution. Radius of the fuel rod, $R_{fr} = \frac{25}{2} = 12.5\,\text{mm} = 0.0125\,\text{m}$

Thermal conductivity, $k_{fr} = 20$ W/m°C

The rate of heat generation from the surface = 0.25 MW/m²

Temperature drop from the centre to the surface of the rod :

Let, t_{max} = Maximum temperature at the centre of the rod ($r = 0$),

t_w = Temperature at the surface of the fuel rod,

q_o = Heat generation rate at the centre ($r = 0$), and

qg = Heat generation rate at a radius r.

Heat generation rate from the surface = Heat transfer rate from the surface of the rod,

$$Q = 0.25 \text{ MW/m}^2$$

Nuclear reactor.

$$= -k \times 4\pi R^2 \left[\frac{q_g}{6k}(-2r)\right]_{r=R} = k \times 4\pi R^2 \times \frac{q_g}{3k} \cdot R$$

or, $$Q = \frac{4}{3}\pi R^3 \times q_g$$

(= volume of sphere × heat generation capacity) ...(*iv*)

Thus heat conducted is equal to heat generated. Under steady state conditions the heat conducted (or generated) should be equal to the heat convected from the outer surface of the sphere.

i.e., $$q_g \times \frac{4}{3}\pi R^3 = h \times 4\pi R^2 (t_w - t_a)$$

or, $$t_w = t_a + \frac{q_g R}{3h} \quad ...(2.125)$$

Inserting this value of t_w in eqn. 2.122, we have

$$t = t_a + \frac{q_g R}{3h} + \frac{q_g}{6k}(R^2 - r^2) \quad ...(2.126)$$

The maximum temperature, $t_{max} = t_a + \frac{q_g}{3k} \cdot R + \frac{q_g}{6k} \cdot R^2$ (at $r = 0$) ...(2.127)

Example 2.104. *An approximately spherical shaped orange (k = 0.23 W/m°C), 90 mm in diameter, undergoes riping process and generates 5100 W/m³ of energy. If external surface of the orange is at 8°C, determine :*

(i) Temperature at the centre of the orange, and

(ii) Heat flow from the outer surface of the orange.

Solution. Outside radius of the orange, $R = \frac{90}{2} = 45$ mm = 0.045 m

Rate of heat generation, $q_g = 5100$ W/m³

The temperature at the outer surface of the orange, $t_w = 8$°C

(*i*) Temperature at the centre of the orange, t_{max} :

$$t_{max} = t_w + \frac{q_g}{6k} R^2 \quad ...[\text{Eqn. } (2.123)]$$

or, $$t_{max} = 8 + \frac{5100}{(6 \times 0.23)} \times (0.045)^2 = \mathbf{15.48°C} \quad \textbf{(Ans.)}$$

(*ii*) Heat flow from the outer surface of the orange, Q :

Heat conducted = Heat generated

∴ $$Q = q_g \times \frac{4}{3}\pi R^3$$

or, $$Q = 5100 \times \frac{4}{3}\pi \times (0.045)^3 = \mathbf{1.946\ W} \quad \textbf{(Ans.)}$$

Example 2.105. *Write down Fourier equation for heat conduction in spherical coordinate system. Hence, deduce an expression for steady state heat conduction in radial direction through a solid sphere of radius R with a uniform volumetric heat generation of q_g W/m³ at the centre. Assume thermal conductivity of the material of the cylinder to be constant.* **(AMIE Summer, 2000)**

Solution. For constant thermal conductivity k, the general heat conduction equation in spherical coordinates is given as

$$Q_{(x+dx)} = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx} \qquad ...(ii)$$

Heat convected out of the element between the planes x and $(x + dx)$,

$$Q_{conv} = h\,(P\,.\,dx)\,(t - t_a)$$

Applying an energy balance on the element, we can write

$$Q_x = Q_{(x+dx)} + Q_{conv.}$$

$$-k\,A_{cs}\left[\frac{dt}{dx}\right]_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x+dx} + h(P.dx)(t - t_a) \qquad ...(2.128)$$

Making a Taylor's expansion of the temperature gradient at $(x + dx)$ in terms of that at x, we get

$$\left(\frac{dt}{dx}\right)_{x+dx} = \left(\frac{dt}{dx}\right)_x + \frac{d}{dx}\left(\frac{dt}{dx}\right)_x dx + \frac{d^2}{dx^2}\left(\frac{dt}{dx}\right)\frac{(dx)^2}{2!} + ...$$

Substituting this in eqn. (2.128), we have

$$-k\,A_{cs}\left[\frac{dt}{dx}\right]_x = -k\,A_{cs}\left[\frac{dt}{dx}\right]_x - kA_{CS}\left[\frac{d^2t}{dx^2}\right]_x dx - k\,A_{cs}\left[\frac{d^3t}{dx^3}\right]\frac{(dx)^2}{2!} + ..+ h(P.dx)(t - t_a)$$

Neglecting higher terms as $dx \to 0$, we have

$$-k\,A_{cs}\left[\frac{dt}{dx}\right] = -k\,A_{cs}\left[\frac{dt}{dx}\right] - k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx + h(P.dx)(t - t_a)$$

$$k\,A_{cs}\left[\frac{d^2t}{dx^2}\right]dx - h(P.dx)(t - t_a) = 0$$

Dividing both sides by $A_{cs}\,dx$, we get,

$$k\frac{d^2t}{dx^2} - \frac{hP}{A_{cs}}(t - t_a) = 0$$

or,
$$\frac{d^2t}{dx^2} - \frac{h\,P}{k\,A_{cs}}(t - t_a) = 0 \qquad ...(2.129)$$

Eqn. (2.129) is further simplified by transforming the dependent variable by defining the *temperature excess* θ as,

$$\theta_{(x)} = t_{(x)} - t_{(a)}$$

As the ambient temperature t_a is constant, we get by differentiation

$$\frac{d\theta}{dx} = \frac{dt}{dx}; \qquad \frac{d^2\theta}{dx^2} = \frac{d^2t}{dx^2}$$

Thus, $$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad ...(2.130)$$

where $$m = \sqrt{\frac{h\,P}{k\,A_{cs}}}$$

Eqns. (2.129) and (2.130) represent a general form of the energy equation for one-dimensional heat dissipation from an extended surface (fin). The parameter m, for a given fin, is constant provided the convective film coefficient h is constant over the whole surface and the thermal conductivity k is constant within the temperature range considered. Then the general solution of this linear and homogeneous second order differential equation is of the form :

$$\theta = C_1 e^{mx} + C_2\,e^{-mx} \qquad ...(2.131)$$

or, $$[t - t_a = C_1 e^{mx} + C_2\,e^{-mx}]$$

$\therefore$ $$Q_{fin} = 330 \times 1.767 \times 10^{-6} \times 12.71\,(190 - 20) = 1.26 \text{ W}$$

$\therefore$ Total energy required for two wires = 2 × 1.26 = **2.52 W (Ans.)**

Example 2.107. *It is required to heat oil to about 300°C for frying purpose. A laddle is used in the frying. The section of the handle is 5 mm × 18 mm. The surroundings are at 30°C. The conductivity of the material is 205 W/m°C. If the temperature at a distance of 380 mm from the oil should not reach 40°C, determine the convective heat transfer coefficient.*

Solution. Refer to Fig. 2.124. t_o = 300°C; b = 18 mm = 0.018 m; y = 5 mm = 0.005 m; l = 380 mm = 0.38 m; k = 205 W/m°C; t_a = 30°C.

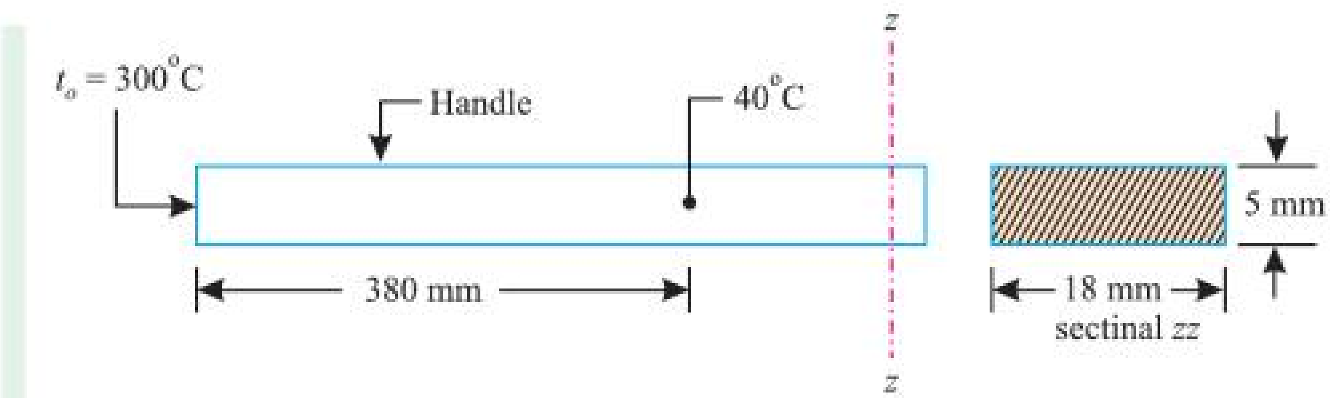

Fig. 2.124.

Convective heat transfer coefficient, h :

Assumming the *fin* to be *long one*, we have

$$\frac{t - t_a}{t_o - t_a} = e^{-mx} \qquad \text{...[Eqn. (2.132)]}$$

or, $$\frac{t_o - t_a}{t - t_a} = e^{mx}$$

or, $$\frac{300 - 30}{40 - 30} = e^{m \times 0.38} \qquad (\because\; x = 380 \text{ mm} = 0.38 \text{ m})$$

or, $$e^{0.38m} = 27 \quad \text{or} \quad m = 8.673$$

But, $$m = \sqrt{\frac{hP}{kA_{cs}}} = 8.673$$

or, $$\frac{hP}{kA_{cs}} = 75.22$$

or, $$\frac{h \times [0.018 + 0.005) \times 2]}{205 \times (0.018 \times 0.005)} = 75.22$$

or, $$h = \frac{75.22 \times 205 \times (0.018 \times 0.005)}{(0.018 + 0.005) \times 2} = \mathbf{30.17\ W/m^2{}^\circ C} \quad \textbf{(Ans.)}$$

Example 2.108. *A temperature rise of 60°C in a circular shaft of 60 mm diameter is caused by the amount of heat generated due to friction in the bearing mounted on the crankshaft. The thermal conductivity of the shaft material is 50 W/m°C and the heat transfer coefficient is 6.5 W/m²°C.*

(i) Develop an expression for the temperature distribution;

(ii) Determine amount of heat transferred through the shaft.

Assume that the shaft is a rod of infinite length.

Solution. Temperature rise, θ_o = 60°C

Diameter of the shaft, d = 60 mm = 0.06 m

Thermal conductivity of material, k = 50 W/m°C

Heat transfer coefficient, h = 6.5 W/m² °C

Solving eqns. (*i*) and (*ii*), we have

$$C_2 = \theta_o - C_1 \qquad \text{...[From eqn. }(i)]$$

$$C_1 e^{ml} - (\theta_o - C_1)\, e^{-ml} = 0$$

or, $$C_1 e^{ml} - \theta_o\, e^{-ml} + C_1\, e^{-ml} = 0$$

or, $$C_1\, (e^{ml} + e^{-ml}) = \theta_o\, e^{-ml}$$

or, $$C_1 = \theta_o \left[\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right]$$

∴ $$C_2 = \theta_o - \left[\theta_o \left\{\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right\}\right] \qquad \text{...[From eqn. }(i)]$$

or, $$C_2 = \theta_o \left[1 - \frac{e^{-ml}}{e^{ml} + e^{-ml}}\right] = \theta_o \left[\frac{e^{ml}}{e^{ml} + e^{-ml}}\right]$$

Inserting the values of C_1 and C_2 in eqn. (2.131), we have

$$\theta = \theta_o \left[\frac{e^{-ml}}{e^{ml} + e^{-ml}}\right] e^{mx} + \theta_o \left[\frac{e^{ml}}{e^{ml} + e^{-ml}}\right] e^{-mx}$$

or, $$\frac{\theta}{\theta_o} = \left[\frac{e^{m(x-l)} + e^{m(l-x)}}{e^{ml} + e^{-ml}}\right] = \left[\frac{e^{m(l-x)} + e^{[-m(l-x)]}}{e^{ml} + e^{-ml}}\right]$$

The above expression, in terms of hyperbolic functions, can be expensed as

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh\{m(l-x)\}}{\cosh(ml)} \qquad \text{..(2.134)}$$

...Expression for temperature distribution

$$\left[\because \cosh\{m(l-x)\} = \frac{e^{m(l-x)} + e^{[-m(l-x)]}}{2}, \text{ and } \cosh(ml) = \frac{e^{ml} + e^{-ml}}{2}\right]$$

The rate of heat flow from the fin is given by

$$Q_{fin} = -k\,A_{cs}\left[\frac{dt}{dx}\right]_{x=0}$$

Now, $$t - t_a = (t_o - t_a)\left[\frac{\cosh\{m(l-x)\}}{\cosh(ml)}\right] \qquad \text{...[From eqn. (2.134)]}$$

$$\frac{dt}{dx} = (t_o - t_a)\left[\frac{\sinh\{m(l-x)\}}{\cosh(ml)}\right](-m)$$

$$\left[\because \frac{d}{dx}[\cosh(mx)] = m \sinh(mx)\right]$$

$$\left[\frac{dt}{dx}\right]_{x=0} = -m\,(t_o - t_a)\,\tanh(ml)$$

∴ $$Q_{fin} = kA_{cs}\, m\,(t_o - t_a)\tanh(ml) \qquad \text{...(2.135)}$$

(Substituting for m)

or, $$Q_{fin} = \sqrt{PhkA_{cs}}\,(t_o - t_a)\tanh(ml) \qquad \text{...[2.135 }(a)]$$

Example 2.111. *Aluminium fins of rectangular profile are attached on a plane wall with 5 mm spacing. The fins have thickness y = 1 mm, length l = 10 mm, and the thermal conductivity, k = 200 W/m K. The wall is maintained at a temperature 200°C, and the fins dissipate heat by convection into the ambient air at 40°C, with heat transfer coefficient h = 50 W/m²K. Determine the heat loss.*

(AMIE Winter, 1998)

(*i*) *Case I.* $n = 6$ and $l = 10$ cm $= 0.1$ m

$$ml = 4.47 \times 0.1 = 0.447$$

$\therefore$ $$Q_1 = 6\,[200 \times 2 \times 10^{-4} \times 4.47 \times (230 - 30) \tanh(0.447)] = 89.99 \text{ W}$$

(*ii*) *Case II.* $n = 12$ and $l = 5$ cm $= 0.05$ m $\quad ml = 4.47 \times 0.05 = 0.2235$

$\therefore$ $$Q_2 = 12\,[200 \times 2 \times 10^{-4} \times 4.47\,(230 - 30) \tanh(0.2235)] = 94.34 \text{ W}$$

This shows that the rate of heat transfer is higher in second case, therefore, this arrangement **(Case II) is better.**

Example 2.116. *One end of a long rod, 35 mm in diameter, is inserted into a furnace with the other end projecting in the outside air. After the steady state is reached, the temperature of the rod is measured at two points 180 mm apart and found to be 180°C and 145°C. The atmospheric air temperature is 25°C. If the heat transfer coefficient is 65 W/m² °C, calculate the thermal conductivity of the rod.*

Solution. Diameter of the rod, $d = 35$ mm $= 0.035$ m

The atmospheric air temperature, $t_a = 25$°C

Heat transfer coefficient; $h = 65$ W/m²°C

The starting point $x = 0$ is considered at the first point where the temperature is measured; $x = l$ is considered at the outer point. *Assume* that the *end of the fin is insulated.*

For *insulated end*, we have

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh\{m(l - x)\}}{\cosh ml} \qquad \text{...[Eqn. (2.134]}$$

At $x = l$, this equation, reduces to

or, $$\frac{\theta_l}{\theta_o} = \frac{1}{\cosh ml} \qquad ...(i)$$

Here, $\theta_l = 145 - 25 = 120$°C and $\theta_o = t_o - t_a = 180 - 25 = 155$°C

$\therefore$ $$\frac{120}{155} = \frac{1}{\cosh ml}$$

or, $$\cosh ml = \frac{155}{120} = 1.292 \quad \text{or} \quad ml = 0.747$$

or, $$m = \frac{0.747}{l}$$

But, $$m = \sqrt{\frac{hP}{kA_{cs}}}$$

$\therefore$ $$\sqrt{\frac{hP}{kA_{cs}}} = \frac{0.747}{l}$$

or, $$\frac{hP}{kA_{cs}} = \frac{0.747^2}{l^2}$$

or, $$\frac{h}{k} \cdot \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{0.747^2}{l^2} \quad \text{or} \quad \frac{h}{k} \times \frac{4}{d} = \frac{0.558}{l^2}$$

or, $$k = \frac{4hl^2}{0.558\,d} = \frac{4 \times 65 \times 0.18^2}{0.558 \times 0.035} = \mathbf{431.34 \text{ W/m°C}} \qquad \textbf{(Ans.)}$$

[where, $l = 180$ mm $= 0.18$ m (*given*)]

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Solution. Consider tank surface of 1m × 1 m as shown in Fig. 2.130.

Let, Q_1 = The rate of heat transfer from the tank surface when fins are *not* fitted,

Q_2 = The amount of heat which will be dissipated per unit time *after fitting the fins*, and [= 1.7 Q_1 .... given]

l = Length of each fin.

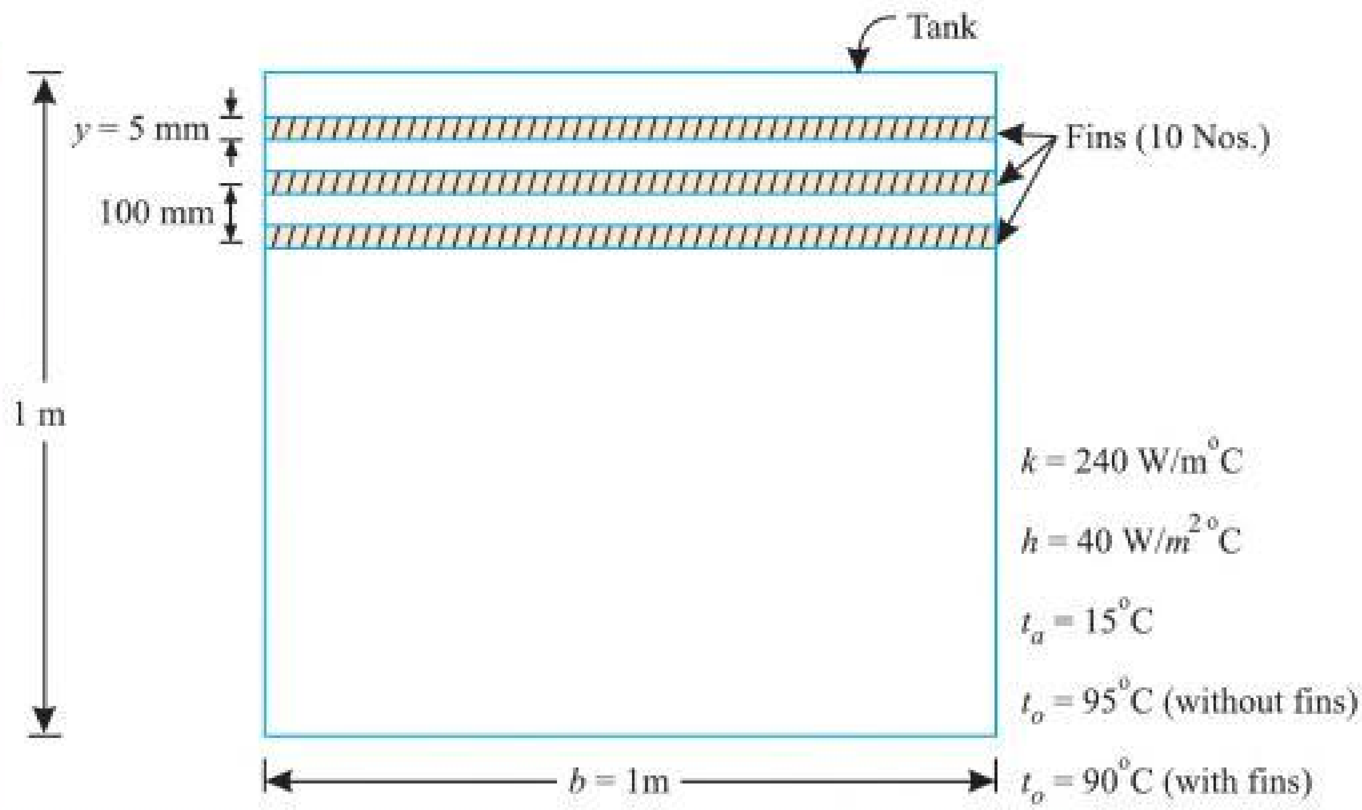


Fig. 2.130.

Now,
$$Q_1 = hA\,(t_o - t_a) = 40 \times 1 \times (95 - 15) = 3200 \text{ W}$$
$$Q_2 = 1.7\,Q_1 = 1.7 \times 3200 = 5440 \text{ W}$$

Metal tanks are used in automobile manufacturing facility.

$$= \theta_o \left[1 - \frac{\left(1 - \dfrac{h}{km}\right)e^{-ml}}{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})} \right]$$

$$= \theta_o \left[\frac{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml}) - e^{-ml} + \dfrac{h}{km}e^{-ml}}{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})} \right]$$

$$= \theta_o \left[\frac{e^{ml} + e^{-ml} + \dfrac{h}{km}e^{ml} - \dfrac{h}{km}e^{-ml} - e^{-ml} + \dfrac{h}{km}\cdot e^{-ml}}{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})} \right]$$

or, $$C_2 = \frac{\theta_o \left[1 + \dfrac{h}{km}\right] e^{ml}}{\left[(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})\right]}$$

Substituting these values of constants C_1 and C_2 in eqn. (2.131), we get

$$\theta = C_1 e^{mx} + C_2\, e^{-mx} \qquad \text{...[Eqn (2.129)]}$$

$$\theta = \left[\frac{\theta_o \left(1 - \dfrac{h}{km}\right)e^{-ml}}{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})} \right] e^{mx} + \left[\frac{\theta_o \left(1 + \dfrac{h}{km}\right)e^{ml}}{(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})} \right] e^{-mx}$$

or, $$\frac{\theta}{\theta_o} = \frac{[e^{m(l-x)} + e^{-m(l-x)}] + \dfrac{h}{km}[e^{m(l-x)} - e^{-m(l-x)}]}{[(e^{ml} + e^{-ml}) + \dfrac{h}{km}(e^{ml} - e^{-ml})]}$$

or, $$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh[m(l-x)] + \dfrac{h}{km}[\sinh\{m(l-x)\}]}{\cosh(ml) + \dfrac{h}{km}[\sinh(ml)]} \qquad \text{...(2.136)}$$

The rate of heat flow from the fin is given by

$$Q_{fin} = -k\, A_{cs} \left[\frac{dt}{dx}\right]_{x=0}$$

Now, $$t - t_a = (t_o - t_a)\left[\frac{\cosh\{m(l-x)\} + \dfrac{h}{km}[\sinh m(l-x)]}{\cosh(ml) + \dfrac{h}{km}\{\sinh(ml)\}} \right]$$

Differentiating the above expression w.r.t. x, we get

$$\frac{dt}{dx} = (t_o - t_a)\left[\frac{-m\sinh\{m(l-x)\} - m\left[\dfrac{h}{km}\{\cosh[m(l-x)]\}\right]}{\cosh(ml) + \dfrac{h}{km}\{\sinh(ml)\}} \right]$$

(i) Temperature at the end of the fin,

(ii) Temperature at the middle of the fin, and

(iii) Heat dissipated by the fin (per metre width).

Solution. Refer to Fig. 2.134.

l = 45 mm = 0.045 m; b = 1m; y = 5mm = 0.005m; k = 55 W/m°C; h = 145 W/m²°C; t_o = 125°C; t_a = 25°C.

(*i*) Temperature at the end of the fin, t_l :

Assuming heat loss by convection from the end of the fin; under this condition, temperature at the end of the fin is given by

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \left[\frac{\cosh\{m(l-x)\} + \frac{h}{km}[\sinh\{m(l-x)\}]}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]}\right]_{x=l} \quad \text{...[Refer Eqn. 2.136]}$$

or,
$$\frac{t_l - t_a}{t_o - t_a} = \frac{1}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]}$$

where,
$$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h \times (2b + 2y)}{k \times (b \times y)}} = \sqrt{\frac{h}{k} \times \frac{2}{y}} = \sqrt{\frac{145}{55} \times \frac{2}{0.005}} = 32.47$$

$(\because 2y << 2b)$

or,
$$ml = 32.47 \times 0.045 = 1.461$$

$\therefore$
$$\frac{t_l - 25}{125 - 25} = \frac{1}{\cosh(1.461) + \frac{145}{55 \times 32.47}[\sinh(1.461)]} = 0.41$$

or, $t_l = 25 + 0.41\,(125 - 25) =$ **66°C (Ans.)**

(*ii*) Temperature at the middle of the fin, $t_{l/2}$:

$$\frac{\theta}{\theta_0} = \frac{t - t_a}{t_o - t_a} = \left[\frac{\cosh\{m(l-x)\} + \frac{h}{km}[\sinh\{m(l-x)\}]}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}}\right]_{x=\frac{l}{2}}$$

or,
$$\frac{t_{l/2} - t_a}{t_o - t_a} = \left[\frac{\cosh\left(\frac{ml}{2}\right) + \frac{h}{km}\left\{\sinh\left(\frac{ml}{2}\right)\right\}}{\cosh(ml) + \frac{h}{km}\{\sinh(ml)\}}\right]$$

or,
$$\frac{t_{l/2} - 25}{125 - 25} = \left[\frac{\cosh(0.7305) + \frac{145}{55 \times 32.47}\{\sinh(0.7305)\}}{\cosh(1.461) + \frac{145}{55 \times 32.47}\{\sinh(1.461)\}}\right]$$

$$= \left[\frac{1.2789 + 0.0647}{2.2711 + 0.1655}\right] = 0.5514$$

$\therefore$ $t_{l/2} = 25 + 0.5514\,(125 - 25) =$ **80.14°C (Ans.)**

For a straight rectangular fin of thickness of y and width b,

$$\frac{P}{A_{cs}} = \frac{2(b+y)}{b \cdot y} \simeq \frac{2}{y}$$

$$\therefore \qquad \varepsilon_{fin} = \sqrt{\frac{2k}{hy}} \qquad \text{...(2.144)}$$

From the relation for fin effectiveness, following results can be inferred :

1. Fin effectiveness $\sqrt{\frac{Pk}{hA_{cs}}}$ should be *greater than unity* if the rate of heat transfer from the primary surface is to be improved. It has been observed that use of fins on surfaces is justified only if $\frac{Pk}{hA_{cs}} > 5$.
2. If the ratio of P (perimeter) and A_{cs} (cross-sectional area) is increased the effectiveness of fin is improved. Due to this reason, thin and closely spaced fins are preferred; *the lower limit on the distance between two adjacent fins (pitch) is governed by the thickness of boundary layer that develops on the surface of the fin.*
3. Use of fins is only justified where h is small; finning is hardly justified unless $h < 0.25\left[\frac{kP}{A}\right]$. If the value of h is large (as experienced in boiling, condensation and high velocity fluids), the fins may actually produce a reduction is heat transfer.
4. It is also apparent that the use of fins will be more effective with materials of large thermal conductivities [Although copper is superior to aluminium regarding thermal conductivity, yet fins are generally made of aluminium since it (aluminium) is cheaper in cost and lighter in weight].

Relation between η_{fin} *and* ε_{fin} :

The performance parameters (*i.e.* η_{fin} and ε_{fin}), in case of a fin *insulated at the tip*, are related to each other by the following expressions :

Efficiency of fin,
$$\eta_{fin} = \frac{\sqrt{PhkA_{cs}}\,(t_0 - t_a)\tanh(ml)}{hPl\,(t_0 - t_a)} \qquad \text{...(i)}$$

Effectiveness of fin,
$$\varepsilon_{fin} = \frac{\sqrt{PhkA_{cs}}\,(t_0 - t_a)\tanh(ml)}{hA_{cs}\,(t_o - t_a)} \qquad \text{...(ii)}$$

Dividing eqn. (*ii*) by eqn. (*i*), we have

$$\frac{\varepsilon_{fin}}{\eta_{fin}} = \frac{Pl}{A_{cs}} \qquad \text{...(2.145)}$$

or,
$$\varepsilon_{fin} = \eta_{fin}\frac{Pl}{A_{cs}} = \eta_{fin} \times \frac{\text{Surface area of the fin}}{\text{Cross-sectional area of the fin}} \qquad \text{...[2.145 (a)]}$$

It is evident from the above equations that *an increase in fin effectiveness can be obtained by increasing the length of the fin but it decreases the efficiency of the fin on the other hand.*

Example 2.126. *A longitudinal copper fin (k = 380 W/m°C) 600 mm long and 5 mm diameter is exposed to air stream at 20°C. The convective heat transfer coefficient is 20W/m²°C. If the fin base temperature is 150°C, determine :*

(*i*) *The heat transferred, and*

(*ii*) *The efficiency of the fin.* **[P.U., 1997]**

(*ii*) Expression for efficiency of this fin and its value for the given data :

Let the suffix *sf* stands for solid fin and suffix *hf* for hollow fin.

Refer to Fig. 2.137. Consider an element of thickness dx at a distance x from the base of the composite fin. Then,

Heat conducted in, $Q_x = \left(-k_{sf}\, A_{sf}\, \frac{dt}{dx}\right) + \left(-k_{hf} \cdot A_{hf}\, \frac{dt}{dx}\right)$

Heat conducted out,

$$Q_{(x+dx)} = \left[-k_{sf}\, A_{sf}\, \frac{dt}{dx} + \frac{d}{dx}\left\{-k_{sf}\, A_{sf}\, \frac{dt}{dx}\right\} dx\right] + \left[-k_{hf}\, A_{hf}\, \frac{dt}{dx} + \frac{d}{dx}\left(-k_{hf}\, A_{hf}\, \frac{dt}{dx}\right) dx\right]$$

Heat transfer from the surface,

$$Q_{conv.} = h\,(P \times dx)\,(t - t_a)$$

For steady state : $Q_x = Q_{(x+dx)} + Q_{conv.}$

or, $k_{sf}\, A_{sf}\, \frac{d^2 t}{dx^2} + k_{hf}\, A_{sf} \cdot \frac{d^2 t}{dx^2} = hP(t - t_a)$

or, $(k_{sf}\, A_{sf} + k_{hf}\, A_{hf})\, \frac{d^2 t}{dx^2} = hP\theta$ $\qquad (\because \theta = t - t_a)$

or, $(k_{sf}\, A_{sf} + k_{hf}\, A_{hf})\, \frac{d^2 \theta}{dx^2} = hP\theta$

or, $$\frac{d^2\theta}{dx^2} = \frac{hP\theta}{k_{sf}\, A_{sf} + k_{hf}\, A_{hf}}$$

or, $$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

where, $$m^2 = \frac{hP}{k_{sf}\, A_{sf} + k_{hf}\, A_{hf}}$$

Efficiency of the fin is given by

$$\eta_{fin} = \frac{\tanh(ml)}{ml} \qquad \text{...[Eqn. (2.139)]}$$

Now $$m^2 = \frac{12 \times (\pi \times 0.01)}{15 \times \left(\frac{\pi}{4} \times 0.003^2\right) + 45 \times \frac{\pi}{4}(0.01^2 - 0.003^2)} = \frac{0.377}{0.000106 + 0.0032} = 114.03$$

or, $m = 10.68$

$\therefore$ $$\eta_{fin} = \frac{\tanh(10.68 \times 0.1)}{(10.68 \times 0.1)} = 0.74 \text{ or } \mathbf{74\%} \qquad \textbf{(Ans.)}$$

2.10.2.5. Design of Rectangular fins

The design of fins is considered to be *optimum* when the *fins*

(*i*) offer minimum resistance to flow of fluid,

(*ii*) are easy to manufacture,

(*iii*) require minimum cost of manufacture, and

(*iv*) are light in weight.

$$\frac{1}{h} = \frac{(y/2)}{k} \qquad ...(2.154)$$

where, $\frac{1}{h}$ = *Surface convection resistance per unit area*, and

$(y/2)/k$ = Conduction resistance for a plane wall whose thickness is one-half the fin thickness.

- The *heat flow rate will be minimum when these resistances are equal.*
- The *heat flow rate will be minimum when Biot number (B_i) is equal to unity and will increase when $B_i < 1$.*

Thus the conditions for *fins to be effective* are :

1. Thermal conductivity (k) should be *large*.
2. Heat transfer coefficient (h) should be *small*.
3. Thickness of the fin (y) should be *small*.

It is *advantageous to use a large number of fins of smaller thickness*.

2.10.3. HEAT FLOW THROUGH "STRAIGHT TRIANGULAR FIN"

The tapered fin is of paramount practical importance since it yields the maximum heat flow per unit weight. Fig. 2.138. shows a straight triangular fin.

Let, l = Length of the fin, between the base and the origin/tip,

b = Width of the fin (per-pendicular to the paper),

y = Thickness at the base of the fin (increas-ing uniformly from 0 at the tip to y at the base),

t_o = Temperature of the base of the fin,

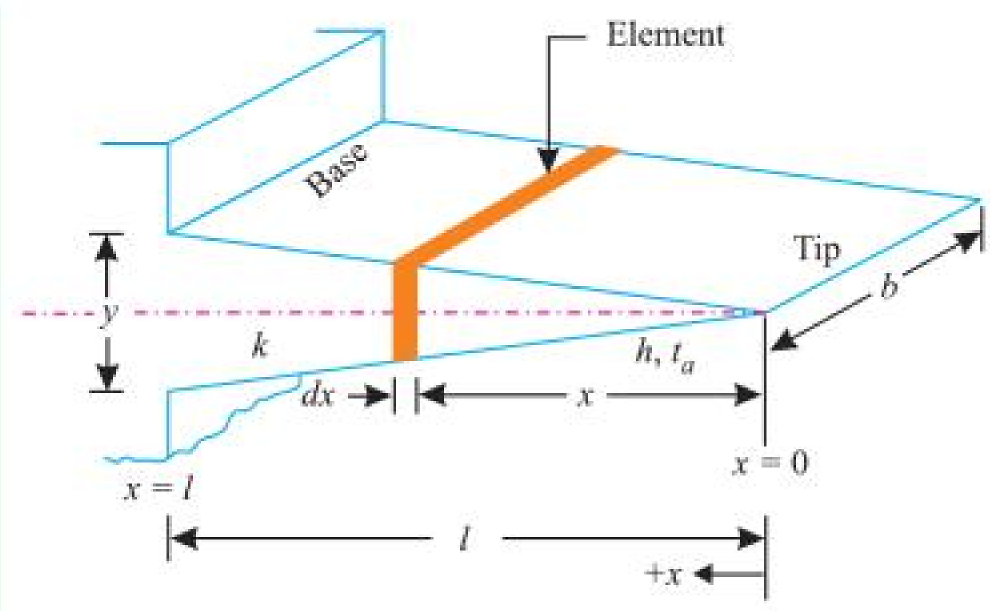


Fig. 2.138.

t_a = Temperature of the ambient/surrounding fluid,

k = Thermal conductivity of the material of the fin (constant), and

h = Heat transfer coefficient (convective).

Assume the fin to be *sufficiently thin i.e.*, ($y << l$), so that one-dimensional heat conduction can be considered.

Consider a small element of thickness dx at a distance x from the origin.

$$\text{Area of cross-section, } A_{cs} = \left[\frac{x}{l} \times y\right] \times b \qquad ...(2.155)$$

Perimeter, $P = 2b$ [neglecting the effect of the edges]

Applying energy balance on the element, we get

$$Q_x = Q_{(x+dx)} + Q_{conv}.$$

$$Q_x = Q_x + \frac{d}{dx}(Q_x)dx + hPdx\,(t - t_a)$$

temperature shown by the thermometer will not be the true temperature of the fluid. This error may be calculated by assuming the well to be a spine protruding from the wall of a pipe in which fluid is flowing. It may be assumed, for simplicity, that there is no flow of heat from the tip of the well (*i.e.*, the tip of the well is *insulated*). The temperature distribution at any distance x measured from pipe wall along the temperature well is given by

$$\frac{\theta_x}{\theta_o} = \frac{t_x - t_f}{t_o - t_f} = \frac{\cosh[m(l - x)]}{\cosh(ml)} \quad \text{...[Eqn. (2.134)]}$$

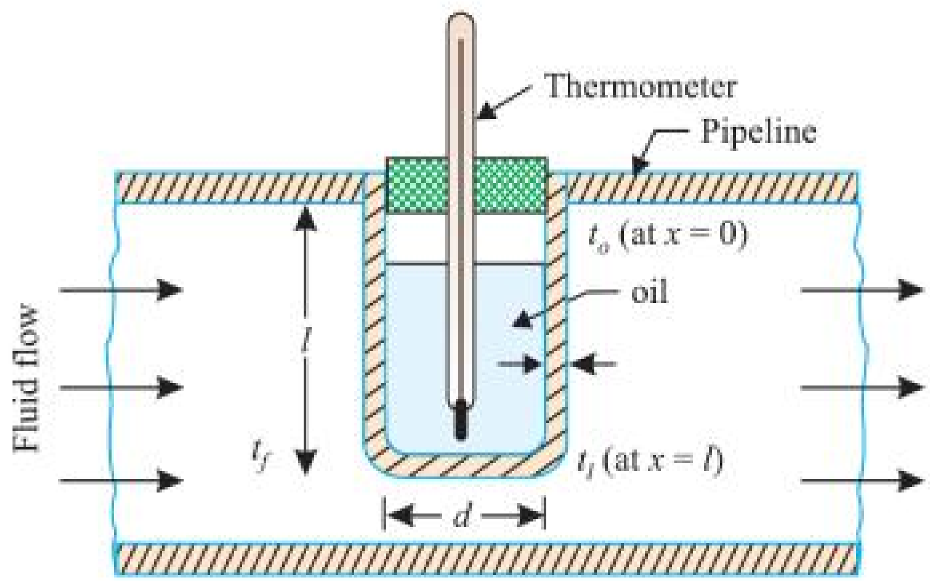


Fig. 2.139. Thermometer well.

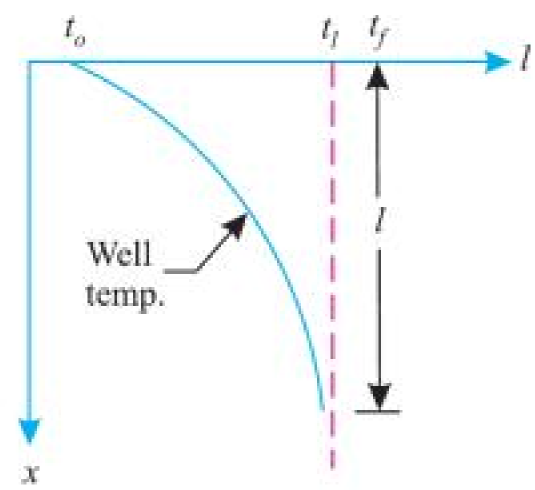


Fig. 2.140. Temperature varitation in well.

At $x = l$, we have

$$\frac{t_l - t_f}{t_o - t_f} = \frac{\cosh[m(l - l)]}{\cosh(ml)} = \frac{1}{\cosh(ml)} \quad \text{...(2.164)}$$

[Thermometeric error]

(where, t_l = Temperature recorded by the thermometer at the bottom of the well.)

Now, perimeter of the well, $P = \pi(d + 2\delta) \simeq \pi d$,

and cross-sectional area, $A_{cs} = \pi\, d\, \delta$

$$\therefore \quad \frac{P}{A_{cs}} = \frac{\pi d}{\pi d \delta} = \frac{1}{\delta}$$

Then, $$m = \sqrt{\frac{hP}{k A_{cs}}} = \sqrt{\frac{h}{k\delta}}$$

Thus, the temperature measured by the thermometer is *not* affected by the diameter of the well.

From the Eqn. (2.164) it is obvious that in order to reduce the temperature measurement error, ml should be large necessitating the following :

(*i*) Large value of h (heat transfer coefficient).

(*ii*) Small value of of k (thermal conductivity).

(*iii*) Long and thin well, the pocket (protruding small tube) may be placed obliquely/inclined, if necessary, to provide a longer insertion of thermometer.

Example 2.130. *A mercury thermometer placed in oil well is required to measure temperature of compressed air flowing in a pipe. The well is 140 mm long and is made of steel (k = 50 W/m°C) of 1 mm thickness. The temperature recorded by the well is 100°C while pipe wall temperature is 50°C. Heat transfer coefficient between the air and well wall is 30 W/m²°C. Estimate true temperature of air.* **(M.U.)**

Length of the pocket, l :

The temperature recorded by the thermometer (t_l) is found from the relation

$$\frac{t_l - t_f}{t_o - t_f} = \frac{1}{\cosh(ml)} \qquad \text{...[Eqn. (2.164)]}$$

where, t_o = Wall temperature, and

t_f = Steam temperature.

The error in measurement is indicated by $(t_f - t_l)$.

Now, $$t_f - t_l = (1.5\%)\, t_f = \frac{1.5}{100} t_f$$

$\therefore$ $$t_l = t_f - \frac{1.5}{100} t_f = 0.985\, t_f$$

$\therefore$ $$\frac{0.985 t_f - t_f}{t_o - t_f} = \frac{1}{\cosh(ml)} \qquad \text{...(1)}$$

where, $$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h}{k} \times \frac{\pi d_o}{\frac{\pi}{4}(d_o^2 - d_i^2)}}$$

$$= \sqrt{\frac{93}{52.3} \times \frac{4d_o}{d_o^2 - d_i^2}} = \sqrt{\frac{93}{52.3} \times \frac{4 \times 0.017}{(0.017^2 - 0.015^2)}} = 43.5$$

Substituting the value in eqn. (1), we get

$$\frac{0.985 \times 320 - 320}{120 \times 320} = \frac{1}{\cosh(ml)} = \frac{1}{\cosh(43.5l)}$$

or, $$\cosh(43.5l) = \frac{120 \times 320}{0.985 \times 320 - 320} = 41.67$$

or, $$\cosh(43.5l) = 4.423$$

or, l = 0.1016 or **101.6 mm** **(Ans.)**

As $l > D$ (95 mm), the pocket should be fitted **inclined**.

2.10.5. HEAT TRANSFER FROM A BAR CONNECTED TO THE TWO HEAT SOURCES AT DIFFERENT TEMPERATURES

Consider the system shown in Fig. 2.143.

Let, l = Length of the bar connecting two heat sources,

A_{cs} = Cross-sectional area of the bar (constant),

P = Perimeter of the bar,

t_1 = Temperature of heat source-1

t_2 = Temperature of heat source-2

t_a = Temperature of air surrounding the bar,

h = Heat transfer coefficient on the surface of the bar,

k = Thermal conductivity of the bar,

$\theta_1 = t_1 - t_a$, and

$\theta_2 = t_2 - t_a$.

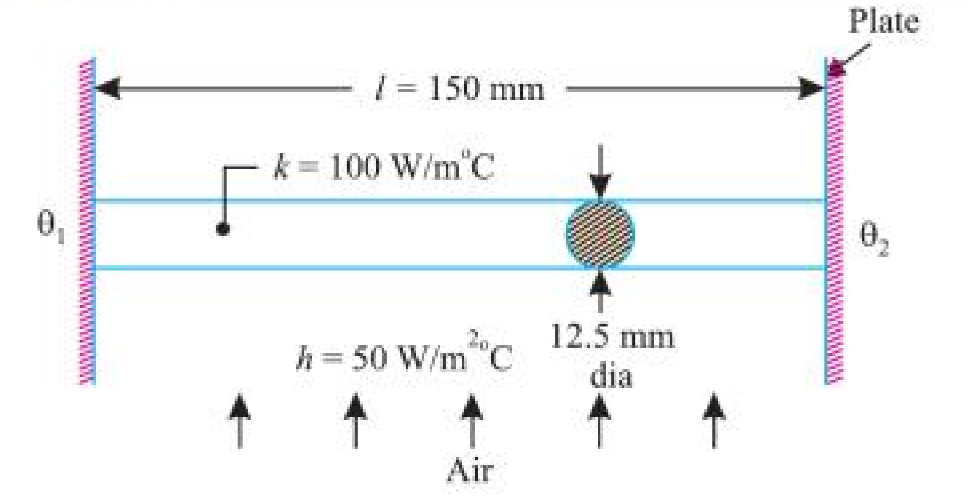


Fig. 2.144.

$$\theta = \frac{\theta_1 \sinh \{m(l-x) + \theta_2 \sinh (mx)}{\sinh (ml)} \quad \text{...[Eqn. (2.168)]}$$

At, $x = \frac{l}{2}$, the value of θ is given by

$$\theta\Big|_{x=\frac{l}{2}} = 2\theta_1 \left[\frac{\sinh (ml/2)}{\sinh (ml)}\right] \quad ...(i)$$

where, $m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{h \times \pi d}{k \times \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 50}{100 \times 0.0125}} = 12.65$

∴ $ml = 12.65 \times 0.15 = 1.9$

Substituting the proper values in eqn. (*i*), we have

$$\theta\Big|_{x=\frac{l}{2}} = 2 \times 40 \left[\frac{\sinh (1.9/2)}{\sinh (1.9)}\right] = 80 \times \frac{1.0995}{3.2682} = \mathbf{26.9^\circ C} \quad \textbf{(Ans.)}$$

(*ii*) The heat lost from the rod, Q :

$$Q = \sqrt{hPkA_{cs}}\,(\theta_1 + \theta_2)\left[\frac{\cosh (ml) - 1}{\sinh (ml)}\right] \quad \text{...[Eqn. (2.169)]}$$

$$= \sqrt{h \times (\pi d) k \times \left(\frac{\pi}{4} d^2\right)} \times 2\theta_1 \left[\frac{\cosh (ml) - 1}{\sinh (ml)}\right]$$

$$= \sqrt{hk\pi^2 d^3} \times \theta_1 \times \left[\frac{\cosh (ml) - 1}{\cosh (ml)}\right]$$

$$= \sqrt{50 \times 100 \times \pi^2 \times 0.0125^3} \times 40 \times \left[\frac{\cosh (1.9) - 1}{\sinh (1.9)}\right]$$

$$= 0.31 \times 40 \times \left[\frac{3.4177 - 1}{3.2682}\right] = \mathbf{9.17\ W} \quad \textbf{(Ans.)}$$

Example 2.135. *A 25 mm diameter rod of 360 mm length connects two heat sources maintained at 127°C and 227°C respectively. The curved surface of the rod is losing heat to the surrounding air at 27°C. The heat transfer coefficient is 10 W/m²°C. Calculate the loss of heat from the rod if it is made of (i) copper (k = 335 W/m°1C), (ii) steel (k = 40 W/m°C).* **(N.U. 1998)**

Solution. Refer to Fig. 2.145. *Given :* $d = 25$ mm = 0.025 m; $l = 360$ mm = 0.36 m; $\theta_1 = t_1 - t_a = 127 - 27 = 100°C$; $\theta_2 = t_2 - t_a = 227 - 27 = 200°C$; $k_{copper} = 335$ W/m°C, $k_{steel} = 40$ W/m°C; $h = 10$ W/m^2°C.

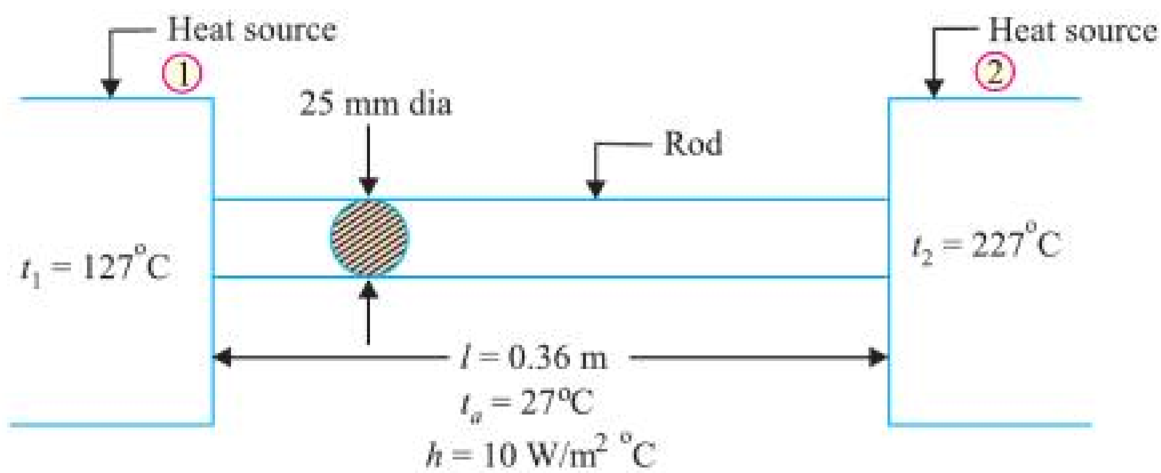


Fig. 2.145.

(*i*) Loss of heat from the copper rod :

The loss of heat is given by,

$$Q = \sqrt{hPkA_{cs}}\,(\theta_1 + \theta_2)\left[\frac{\cosh(ml) - 1}{\sinh(ml)}\right] \quad \text{...[Eqn. (2.169)]}$$

where,
$$m = \sqrt{\frac{hP}{kA_{cs}}}\ \frac{h \times \pi d}{k \times \frac{\pi}{4} d^2} = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 10}{335 \times 0.025}} = 2.185$$

and,
$$ml = 2.185 \times 0.36 = 0.7866,$$
$$P = \pi d = \pi \times 0.025 \text{ m}$$
$$A_{cs} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.025^2 \text{ m}^2$$

Substituting the values in the above equation, we get

$$Q = \sqrt{10 \times (\pi \times 0.25) \times 335 \times \left(\frac{\pi}{4} \times 0.025^2\right)} \times (100 + 200)\left[\frac{\cosh(0.7866) - 1}{\sinh(0.7866)}\right]$$

$$= 1.136 \times 300 \times \frac{1.328 - 1}{0.87} = \mathbf{128.48\ W} \quad \textbf{(Ans.)}$$

(*ii*) Loss of heat from steel rod :

$$m = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 10}{40 \times 0.025}} = 6.32$$
$$ml = 6.32 \times 0.36 = 2.27$$
$$hPkA_{cs} = 10 \times \pi d \times 40 \times \frac{\pi}{4} d^2$$
$$= 10 \times \pi \times 0.025 \times 40 \times \frac{\pi}{4} \times 0.025^2 = 0.0154$$

Substituting the values in eqn. (2.169) above, we get

$$Q = \sqrt{0.0154} \times (100 + 200)\left[\frac{\cosh(2.27) - 1}{\sinh(2.27)}\right]$$

or,
$$Q = 0.124 \times 300 \times \left[\frac{4.89 - 1}{4.79}\right] = \mathbf{30.21\ W} \quad \textbf{(Ans.)}$$

Example 2.136. *One end of the copper rod (k = 380 W/m°C) 300 mm long is connected to wall which is maintained at 300°C. The other end is firmly connected to a wall which is maintained at 100°C. Air is blown across the rod so that heat transfer coefficient of 20 W/m²°C is maintained. The diameter of the rod is 15 mm and temperature of air is 40°C. Determine :*

(i) The net heat transferred to the air in watts;

(ii) The heat conducted to the other end which is at 100°C. **[MU, 1999]**

Solution. Refer to Fig. 2.146. *Given :* $l = 0.3$ m; $d = 15$ mm $= 0.015$ m, $t_1 = 300°C$; $t_2 = 100°C$, $t_a = 40°C$; $h = 20$ W/m²°C; $k = 380$ W/m°C.

(*i*) The net rate of heat transfer to the air, *Q* :

$$\theta_1 = t_1 - t_a = 300 - 40 = 260°C; \; \theta_2 = t_2 - t_a = 100 - 40 = 60°C$$

The rate of heat flow is given by

$$Q = \sqrt{hPkA_{cs}}\;(\theta_1 + \theta_2)\left[\frac{\cosh(ml) - 1}{\sinh(ml)}\right] \quad \text{...[Eqn. (2.169)]}$$

where,
$$m = \sqrt{\frac{hP}{kA_{cs}}} = \frac{h \times \pi d}{k \times \frac{\pi}{4} d^2} = \sqrt{\frac{4h}{kd}}$$

$$= \sqrt{\frac{4 \times 20}{380 \times 0.015}} = 3.746$$

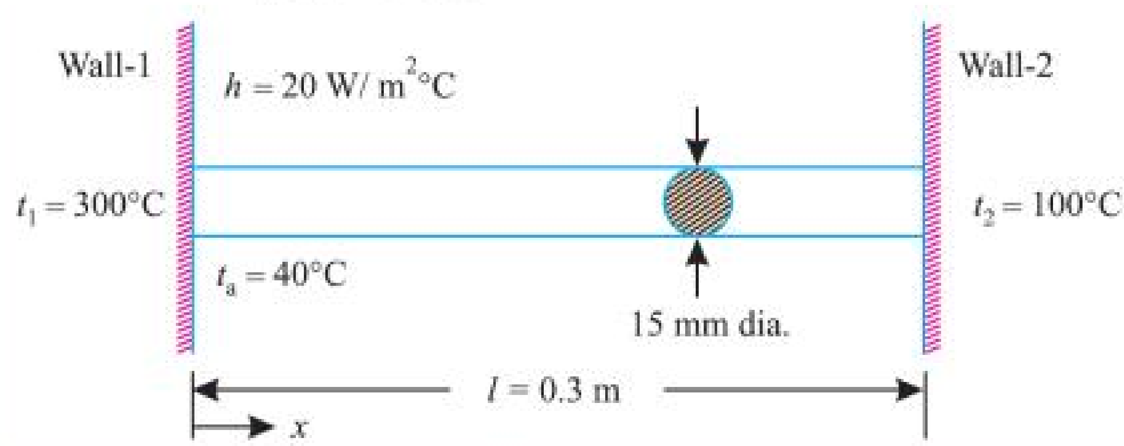


Fig. 2.146.

$$\therefore \quad ml = 3.746 \times 0.3 = 1.12$$

$$hPkA_{cs} = 20 \times \pi d \times 380 \times \frac{\pi}{4} d^2$$

$$= 20 \times \pi \times 0.015 \times 380 \times \frac{\pi}{4} \times 0.015^2 = 0.0633$$

Substituting the proper values in the equation, we get

$$\boldsymbol{Q} = \sqrt{0.0633} \times (260 + 60)\left[\frac{\cosh(1.12) - 1}{\sinh(1.12)}\right]$$

$$= 0.2516 \times 320\left[\frac{1.6956 - 1}{1.3693}\right] = \mathbf{40.9\ W} \quad \textbf{(Ans.)}$$

(*ii*) The heat conducted to the other end which is at 100°C $[Q_{cond.}]_{x=l}$:

$$[Q_{cond.}]_{x=l} = [Q_{cond.}]_{x=0} - Q \quad \text{...}(i)$$

The temperature distribution is given by

$$\theta = \frac{\theta_1 \sinh[m(l - x)] + \theta_2 \sinh(mx)}{\sinh(ml)} \quad \text{...[Eqn. (2.168)]}$$

$$\therefore \qquad \frac{d\theta}{dx} = \left[\frac{-\theta_1 \cdot \cosh m(l-x) + \theta_2 \cdot m \cosh(mx)}{\sinh(ml)}\right]$$

and, $$\left.\frac{d\theta}{dx}\right|_{x=0} = \left[\frac{-\theta_1 \cdot m \cosh(ml) + \theta_2 \cdot m}{\sinh(ml)}\right]$$

$$\therefore \qquad [Q_{cond.}]_{x=0} = -kA_{cs} \left.\frac{d\theta}{dx}\right|_{x=0}$$

or, $$[Q_{cond.}]_{x=0} = kA_{cs}\, m \left[\frac{\theta_1 \cosh(ml) - \theta_2}{\sinh(ml)}\right] \qquad \left[\text{substituting for } \left.\frac{d\theta}{dx}\right|_{x=0}\right]$$

Substituting the proper values in eqn. (*i*), we get

$$[Q_{cond.}]_{x=l} = kA_{cs}\, m \left[\frac{\theta_1 \cosh(ml) - \theta_2}{\sinh(ml)}\right] - Q$$

$$= 380 \times \left(\frac{\pi}{4} \times 0.015^2\right) \times 3.746 \left[\frac{260 \cosh(1.12) - 60}{\sinh(1.12)}\right] - 40.9$$

or, $$[Q_{cond.}]_{x=l} = 0.2515 \left[\frac{260 \times 1.6956 - 60}{1.3693}\right] - 40.9 = \mathbf{29.05\ W} \quad \textbf{(Ans.)}$$

Example 2.137. *Two ends of a fin of cross-sectional area 200 mm² and length 1m are maintained at 127°C and 227°C. Perimeter is 20 mm. It loses heat from the surface due to natural convection to the surrounding at 27°C with a surface heat transfer coefficient of 5 W/m²°C. Find the minimum temperature in the fin and its location. Thermal conductivity of fin material = 45 W/m°C.*

Solution. Refer Fig. 2.147. A_{cs} = 200 mm²; l = 1m; P = 20 mm = 0.02 m; $\theta_1 = t - t_a$ = 127 – 27 = 100°C; $\theta = t_2 - t_a$ = 227 – 27 = 200°C; h = 5 W/m²°C; k = 45 W/m°C.

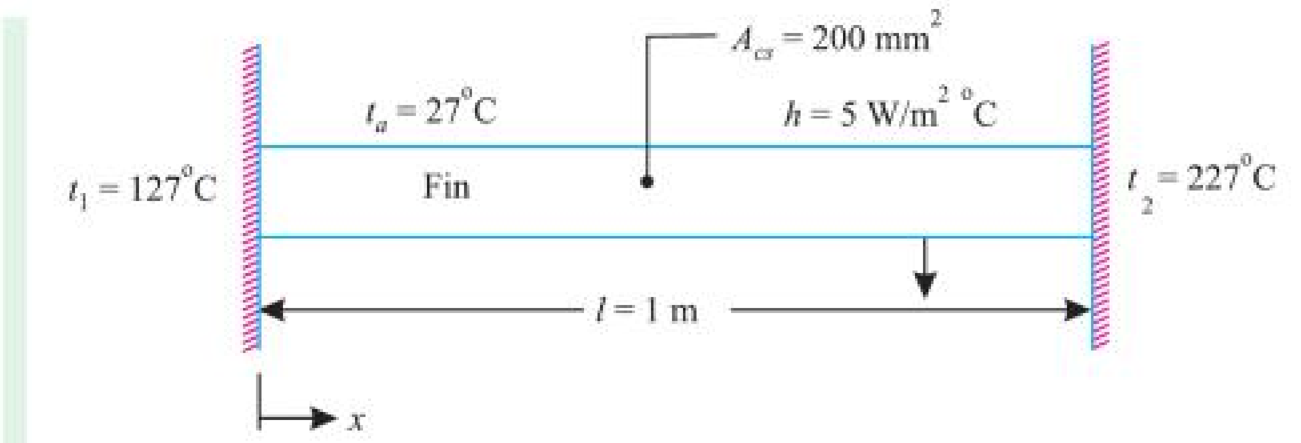


Fig. 2.147.

The temperature distribution for such configuration is given by

$$\theta = (t - t_a) = \frac{\theta_1 \sinh[m(l-x)] + \theta_2 \sinh(mx)}{\sinh(ml)} \qquad \text{...[Eqn. (2.168)]}$$

For finding the *position of the minimum temperature*, the required condition is

$$\frac{d\theta}{dx} = 0$$

or, $$\theta_1 \cosh\{m(l-x)\} = \theta_2 \cosh(mx) \qquad \text{[Refer eqn. (2.170)]}$$

or, $$\frac{\cosh(mx)}{\cosh\{m(l-x)\}} = \frac{\theta_1}{\theta_2} = \frac{100}{200} = 0.5$$

where, $$m = \sqrt{\frac{hP}{kA_{cs}}} = \sqrt{\frac{5 \times 0.02}{45 \times 200 \times 10^{-6}}} = 3.33$$

Solar heat collector.

(Heat lost from the absorber plate by convection to cover) + (heat lost by the absorber plate by radiation to the cover) = (Heat lost by the cover to air by convection) + (heat lost by the cover to the air by radiation)

i.e., $h_{cp}(70 - t_c) + h_{rp}(70 - t_c) = h_{cc}(t_c - 10) + h_{rc}(t_c - 10)$

where, t_p = Absorber plate temperature,

t_a = Ambient temperature,

t_c = Glass cover temperature,

h_{cp}, h_{rp} = Convection and radiation heat transfer co-efficients from the absorber plate to cover respectively, and

h_{cc}, h_{rc} = Convection and radiation heat transfer co-efficients from the glass cover to the surrounding air respectively.

$$\therefore \quad (h_{cp} + h_{rp})(70 - t_c) = (h_{cc} + h_{rc})(t_c - 10)$$

$$(3+6)(70 - t_c) = (25+5)(t_c - 10)$$

$$630 - 9t_c = 30t_c - 300$$

$$\therefore \quad t_c = 23.85°\text{C}$$

Total heat lost by the collector plate

= Heat lost by the absorber (collector) plate to the cover plate + heat lost through insulation by conduction.

$$= (h_{cp} + h_{rp}) \times 1 \times (t_p - t_c) + \frac{k \times 1 \times (t_p - t_a)}{(5/100)}$$

$$= (3 + 6) \times 1 \times (70 - 23.85) + \frac{0.05 \times 1 \times (70 - 10)}{0.05}$$

$= 415.35 + 60 =$ **475.35 W/m²** **(Ans.)**

D. Sphere :

$$t = t_w + \frac{q_g}{6k}(R^2 - r^2) \quad ...(i)$$

$$t_{max} = t_w + \frac{q_g}{6k} R^2 \quad ...(ii)$$

$$\frac{t - t_w}{t_{max} - t_w} = 1 - \left(\frac{r}{R}\right)^2 \quad ...(iii)$$

(Temperature distribution in dimensionless form)

$$t_w = t_a + \frac{q_g R}{3h} \quad ...(iv)$$

$$t = t_a + \frac{q_g R}{3h} + \frac{q_g}{6k}(R^2 - r^2)$$

$$t_{max} = t_a + \frac{q_g}{3h} . R + \frac{q_g}{6k} . R^2$$

Considering h and t_a

12. Heat transfer from extended surfaces (Fins) :

A. Rectangular fin :

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad ...(i)$$

or $$[(t - t_a) = C_1 e^{mx} + C_2 e^{-mx}]$$

Case I. *Heat dissipation from an infinitely long fin ($l \to \infty$) :*

$$\theta = \theta_0 e^{-mx} \quad ...(ii)$$

or $$[(t - t_a) = (t_o - t_a) e^{-mx}]$$

$$Q_{fin} = k A_{cs} m (t_o - t_a) \quad ...(iii)$$

[An infinitely long fin is one for which $ml \to \infty$, and this condition may be approached when $ml > 5$]

Case II. *Heat dissipation from a fin insulated at the tip :*

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh[m\{l - x\}]}{\cosh(ml)} \quad ...(iv)$$

$$Q_{fin} = kA_{cs} m (t_o - t_a) \tanh(ml) \quad ...(v)$$

Case III. *Heat dissipation from a fin losing heat at the tip :*

$$\frac{\theta}{\theta_o} = \frac{t - t_a}{t_o - t_a} = \frac{\cosh\{m(l - x)\} + \frac{h}{km}[\sinh\{m(l - x)\}]}{\cosh(ml) + \frac{h}{km}[\sinh(ml)]} \quad ...(vi)$$

$$Q_{fin} = kA_{cs} m (t_o - t_a) \left[\frac{\tanh(ml) + \frac{h}{km}}{1 + \frac{h}{km} . \tanh(ml)}\right] \quad ...(vii)$$

where $$m = \sqrt{\frac{hP}{kA_{cs}}}$$

[A_{cs} = cross-sectional area ($b \times y$); P = perimeter of the fin {= $2(b + y)$}]

[t_o = temperature at the base of the fin; t_a = temperature of ambient/surrounding fluid].

Efficiency of fin : It is defined as the ratio of the actual heat transferred by the fin to the maximum heat transferable by fin, if entire fin area were at the base temperature.

Effectiveness of fin : It is the ratio of the fin heat transfer rate to the heat transfer rate that would exist without a fin.

20. A spherical shaped vessel of 1.2 m diameter is 100 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 200°C. Thermal conductivity of material is 0.3 kJ/mh°C. [**Ans.** 2262 kJ/h]

HEAT GENERATION

21. A voltage of 10 V is impressed on a stainless steel wire ($k = 22.5$ W/m°C), resistivity $\rho = 70 \times 10^{-6}\ \Omega$ cm) of 3.2 mm diameter and 300 mm length. The outer surface temperature of the wire is maintained at 93°C. What is the temperature at the centre of the wire? [**Ans.** 138.4°C]

22. A current of 200 amperes is passed through a stainless steel wire ($k = 20$ W/m°C, resistivity $\rho = 70 \times 10^{-6}\ \Omega$ cm). If the wire is submerged in liquid at 110°C and the heat transfer coefficient on wire surface is 4000 W/m^2°C, calculate the centre line temperature of the wire. [**Ans.** 230.7°C]

23. A plane wall ($k = 15$ W/m°C) 100 mm thick generates heat at the rate of 4×10^4 W/m^3 when an electric current is passed through it. The convective heat transfer coefficient between each face of the wall and the ambient air is 50 W/m^2°C. If the ambient air temperature is 20°C, determine the surface temperature and maximum temperature in the wall. [**Ans.** 60°C, 63.33°C]

24. An electric current of 34000 amperes is passed through a flat steel plate ($k = 54$ W/m°C, resistivity $\rho = 12\ \mu\Omega$cm) 12.5 mm thick and 100 mm wide. The temperature on the two surfaces of the plate are 95°C and 80°C. Determine :

(*i*) The maximum temperature and its position, and

(*ii*) The total amount of heat generated and flow of heat from each surface of the plate.

[**Ans.** (*i*) 120°C, 5.52 mm; (*ii*) 111 kW, 49 kW, 62 kW]

25. The temperatures on the two surfaces of a 20 mm thick steel plate ($k = 50$W/m°C), having a uniform volumetric heat generation of 40×10^6 W/m^3, are 160°C and 100°C. Neglecting the end effects, determine the following :

(*i*) The position and value of the maximum temperature, and

(*ii*) The flow of heat from each surface of the plate.

[**Ans.** (*i*) 6.25 mm, 175.6°C; (*ii*) $- 250 \times 10^3$ W/m^2, $+ 550 \times 10^3$ W/m^2]

26. A plane wall 80 mm thick ($k = 0.15$ W/m°C) is insulated on one side while the other side is exposed to environment at 90°C. The rate of heat generation within the wall is 12×10^4 W/m^3. If the convective heat transfer coefficient between the wall and the environment is 560 W/m^2°C, determine the maximum temperature to which the wall will be subjected. [**Ans.** 2667°C]

27. A 60 mm thick slab of insulating material ($k = 0.407$ W/m°C) is placed between and is in contact with two parallel electrodes, and is then subjected to dielectric heating (high frequency) at a uniform rate of 40694.4 W/m^3. On the attainment of steady state conditions, the coefficients of combined radiation and convection for the exposed surfaces are 12.21 W/m^2°C and 13.96 W/m^2°C respectively. If the ambient temperature is 25°C, determine :

(*i*) Surface temperature;

(*ii*) Location and magnitude of maximum temperature in the system.

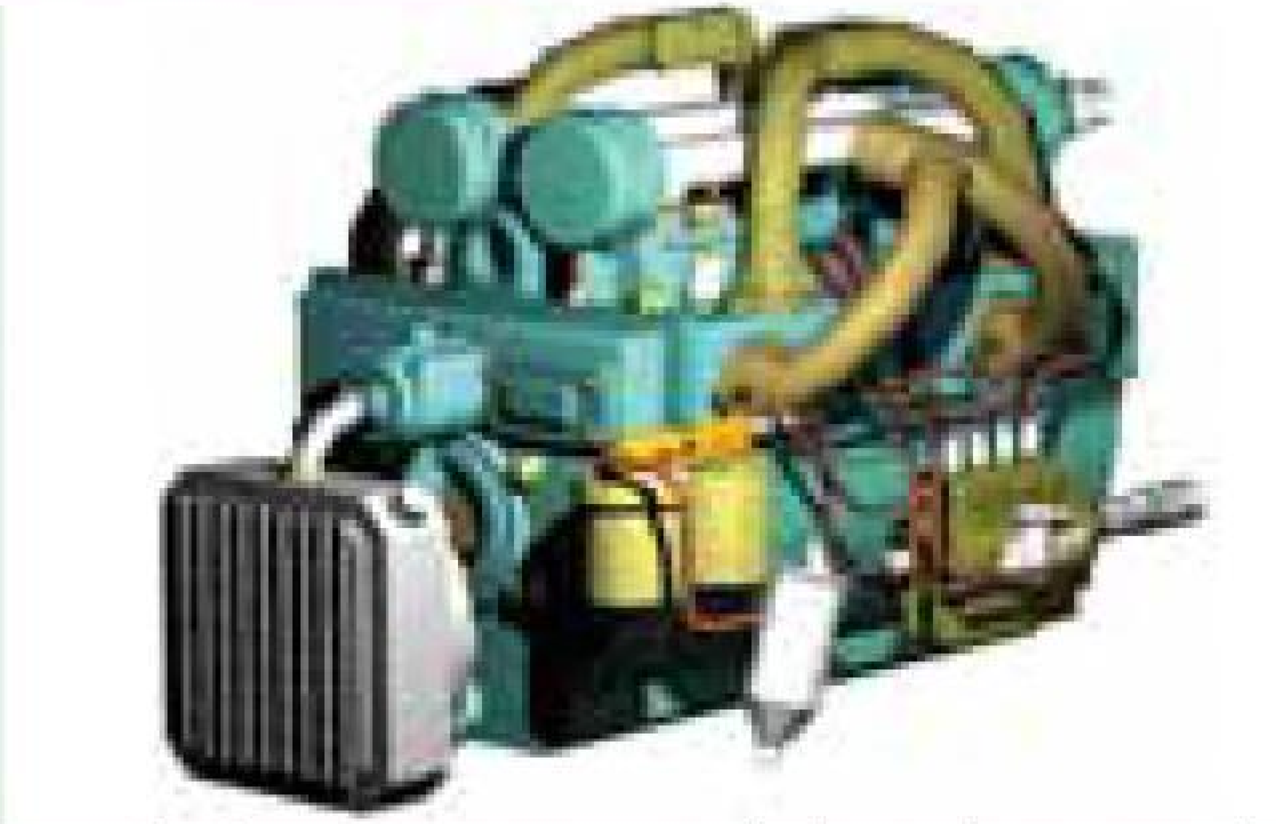
Assume the flow of heat to be unidirectional and each electrode to be at a uniform temperature equal to that of the slab with which it is in contact. [**Ans.** (*i*) 121°C, 115°C; (*ii*) 29 mm, 164°C (app.)]

28. A stainless steel rod of 2 cm diameter is carrying an electrical current of 1000 amperes. The thermal and electrical conductivities of the rod are 16 W/m°C and 1.5×10^4 (ohm cm)$^{-1}$ respectively. What is the temperature at the centre of the rod if the outer surface temperature is 400°C? [**Ans.** 400.105°C]

(M.U.)

29. The electric cable of thermal conductivity $k = 20$ W/m°C, 3 mm in diameter and one metre long has current flow = 200 amperes, ρ (resistivity) = 70 $\mu\Omega$cm. The wire is submerged in a liquid at 100°C and the surface heat transfer coefficient (h) is 4000 W/m^2°C. Calculate the centre temperature of the wire.

[**Ans.** 372°C]



Internal combustion engine is an innovative technology to substitute pistons and crank gear with oscillating flaps.

4. *Numerical methods* :

These methods can be applied to any mutli-dimensional problem.

An analytical approach to a multi-dimensional problems requires a prior knowledge of Fourier'series, Bessel's functions, Legendre polynomials, Laplace transform methods and complex variable theory. The other three methods do not require advanced knowledge of mathematics and are more useful for engineering calculations.

3.2. TWO DIMENSIONAL STEADY-STATE CONDUCTION

In a two-dimensional heat flow, the temperature is a function of two-coordinates; the heat flow through a corner, where two walls meet, is an *example* of such a system.

3.2.1. ANALYTICAL METHOD

3.2.1.1. Two-dimensional steady state heat conduction in rectangular plates

Refer to Fig. 3.1. Consider a thin rectangular plate ($\frac{\partial t}{\partial z}$ is negligible and temperature is a function of x and y) having no heat source. Assume that the three edges of this plate are maintained at a constant temperature and faces of the plate are adiabatic. Let the thermal conductivity of the plate (k) is uniform (*i.e.*, independent of both temperature and direction).

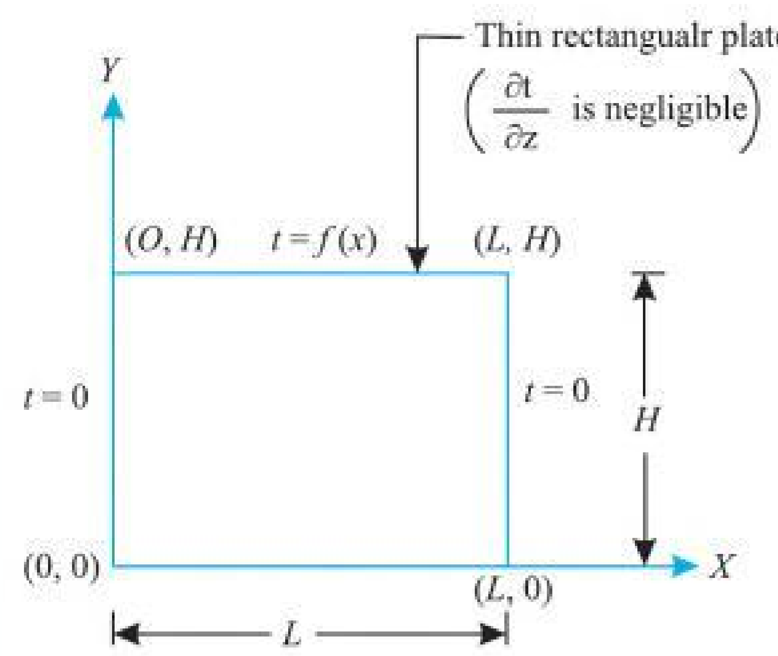


Fig. 3.1. Rectangular plate with temperature-specified boundary conditions.

The controlling differential equation for two dimensional steady state heat conduction is given by

$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0$, where t is a function of

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Substituting the solution into the controlling equation, we get

$$\frac{1}{X}\cdot\frac{d^2X}{dx^2} = -\frac{1}{Y}\cdot\frac{d^2Y}{dy^2} = \pm\lambda^2$$

The sign of λ^2 should be so chosen that the homogeneous Y-direction results in a characteristic value problem; a +ve value of λ^2 will satisfy all the values mentioned in the boundary conditions.

Thus the required equations are

$$\frac{d^2X}{dx^2} - \lambda^2 X = 0 \qquad ...(iii)$$

$$\frac{d^2Y}{dy^2} + \lambda^2 Y = 0 \qquad ...(iv)$$

The solutions of eqns. (*ii*) and (*iii*) respectively are:

$$X = A\,e^{\lambda x} + B\,e^{-\lambda x}$$

$$Y = C\cos(\lambda y) + D\sin(\lambda y)$$

$$\therefore \quad \theta = (Ae^{\lambda x} + Be^{-\lambda x})\,[C\cos(\lambda y) + D\sin(\lambda y)] \qquad ...(v)$$

From boundary condition (*i*), we have

$$0 = (Ae^{\lambda\infty} + Be^{-\lambda\infty})\,[C\cos(\lambda y) + D\sin(\lambda y)]$$

$$\therefore \quad A = 0, \text{ and}$$

$$\theta = Be^{-\lambda x}\,[C\cos(\lambda y) + D\sin(\lambda y)] \qquad ...(vi)$$

From boundary condition (*iv*), we get

$$0 = C.Be^{-\lambda x}$$

$$\therefore \quad C = 0, \text{ and eqn. } (vi) \text{ reduces to}$$

$$\theta = B\,e^{-\lambda x}\,[D\sin(\lambda y)]$$

$$\text{or,} \quad \theta = Ee^{-\lambda x}\sin(\lambda y)$$

$$(\text{where } E = B.D) \qquad ...(vii)$$

From boundary condition (*iii*), we get

$$0 = Ee^{-\lambda x}\sin(\lambda L)$$

Since $E \neq 0$, $\sin(\lambda L) = 0$; this expression is satisfied.

For $\lambda = 0, \dfrac{\pi}{L}, \dfrac{2\pi}{L}$ ...; or in general

$$\lambda_n = \frac{n\pi}{L}; \text{ where } n = 0, 1, 2, 3, ...$$

$$\therefore \quad \theta = E\exp(-\lambda_n x)\sin(\lambda_n y) \qquad ...(viii)$$

For each integer n there exist a different solution and each solution has a separate integration constant E_n. The general solution will be the sum of these individual solutions.

$$\theta = \sum_{n=1}^{\infty} E_n \exp(-\lambda_n x)\sin(\lambda_n y) \qquad ...(ix)$$

For $n = 0$, $\lambda_n = 0$ and as such no contribution is made by the first term; eqn. (*ix*) can be written as

1. ***Boundary conditions :***
   (*i*) Isotherms must be perpendicular to insulated boundaries.
   (*ii*) Heat flow lines must be perpendicular to isothermal boundaries.
   (*iii*) Heat flow line leading to a corner of an isothermal boundary must bisect an angle between the surfaces of the boundary at the corner.
2. Heat flow lines and isotherms must cut each other at right angles at all intersection points.
3. Heat flow lines and isotherms must form a network of curvili -near squares.
4. Diagonals of the curvilinear square must bisect each other at right angles and also must bisect the corners.

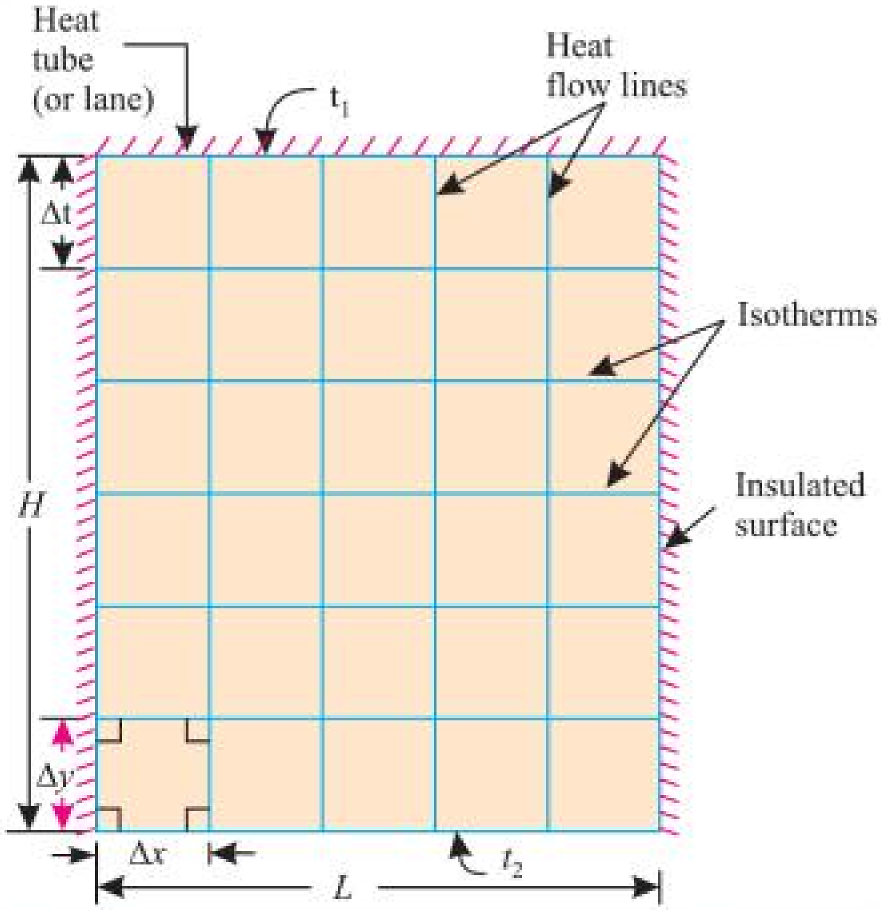

Fig. 3.5. Isotherms and heat flux lines in a parallelopiped.

Fig. 3.5 shows a parallelopiped whose two faces are maintained at constant temperatures of t_1 and t_2 respectively and other faces perfectly insulated. The face at t_1 is divided into segments of Δx each to represent heat flow tubes. Similarly lines representing isotherms can be described at intervals of Δy maintaining $\Delta y = \Delta x$ and the condition of orthogonality of isotherms and heat flow lines.

Fig. 3.6 shows another two-dimensional system whose inside and outside surfaces are maintained at temperatures t_1 and t_2 respectively. For the given system, due to symmetry, only one eight of the configuration is considered for the flux plot. In the Fig. 3.6 is also shown diagrammatically a potential fluid element for curvilinear square analysis of the two-dimensional heat flow. Assuming *unit depth* of the material, the heat flow through this curvilinear section is given by Fourier's law,

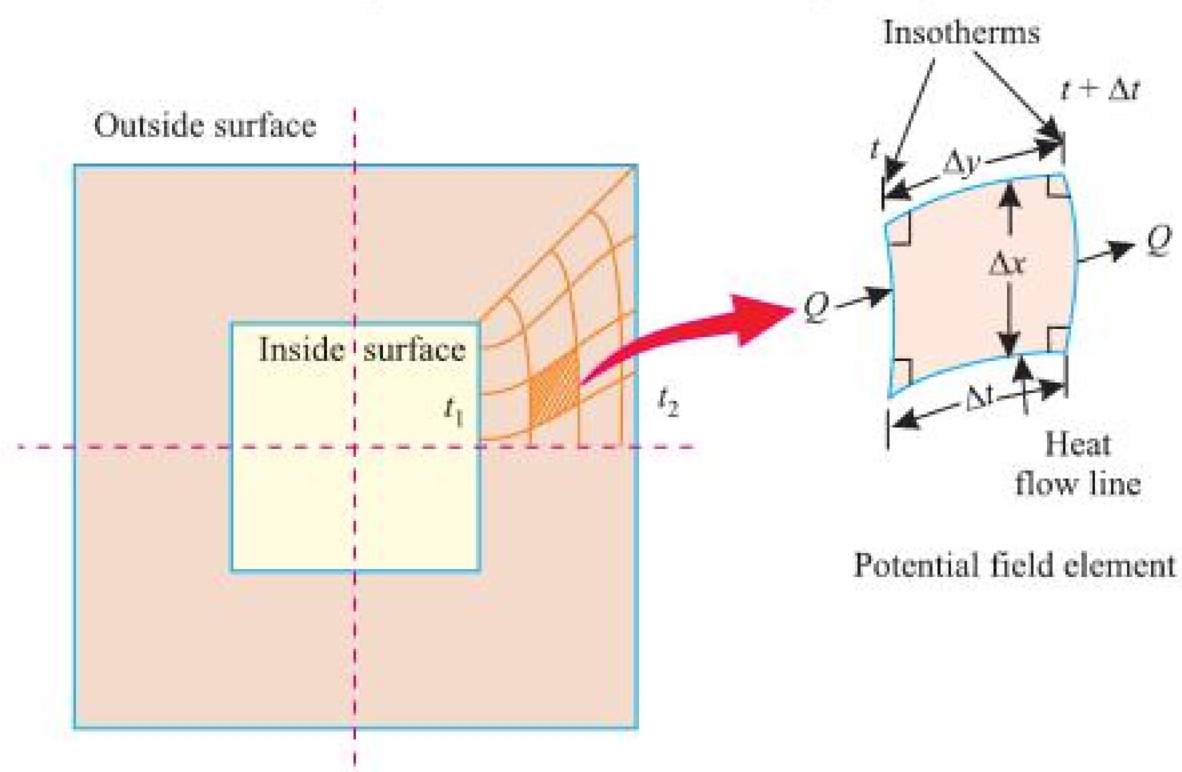

Fig. 3.6. Sketch showing potential field element for curvilinear square analysis of two-dimentional heat flow.

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Example 3.3. *A long pipe of 0.6 m outside diameter is buried in earth with axis at a depth of 1.8 m. The surface temperatures of pipe and earth are 95° C and 25° C respectively. Calculate the heat loss from the pipe per metre length. The conductivity of earth is 0.51 W/m° C.*

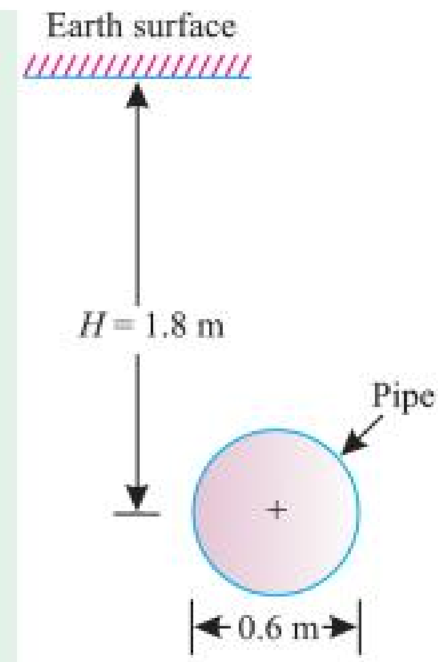

Fig. 3.8.

Solution. Refer to Fig. 3.8. $r = \frac{0.6}{2} = 0.3$ m; $L = 1$ m; $H = 1.8$ m; $k = 0.51$ W/m° C.

t_p (surface temperature of pipe) = 95°C; t_e (surface temperature of earth) = 25° C

Heat loss from the pipe per metre length, Q:

$$Q = k\, S_{fc}\,(t_p - t_e) \qquad \text{...[Refer to eqn. (3.17)]}$$

where, S_{fc} (shape factor) $= \frac{2\pi L}{\ln(2H/r)} = \frac{2\pi \times 1}{\ln[(2 \times 1.8)/0.3]} = 2.528$ m

$$\therefore \quad Q = 0.51 \times 2.528 \times (95 - 25) = \textbf{90.25 W (Ans.)}$$

Example 3.4. *A 1.6 m diameter sphere is buried in soil with centre at a depth of 5.5 m. Heat is generated in the sphere at a rate of 580 W. If the conductivity of the soil is 0.51 W/m°C and the soil surface is at 6° C, calculate the surface temperature of the sphere under steady state condition.*

Solution. Refer to Fig. 3.9. $r = \frac{1.6}{2} = 0.8$ m; $H = 5.5$ m; $k = 0.51$ W/m°C; $Q_g = 580$ W

Temperature of soil surface = 6°C

Surface temperature of the sphere, t :

Heat generated = Heat conducted away

or,
$$Q_g = kS_{fc}\,(t - 6)$$

where,
$$S_{fc} = \frac{4\pi r}{1 - (r/2H)} = \frac{4\pi \times 0.8}{1 - [0.8/(2 \times 5.5)]} = 10.84 \text{ m}$$

Substituting the values in the above equation, we get

$$580 = 0.51 \times 10.84\,(t - 6) \qquad \text{or} \qquad t = \textbf{110.9°C (Ans.)}$$

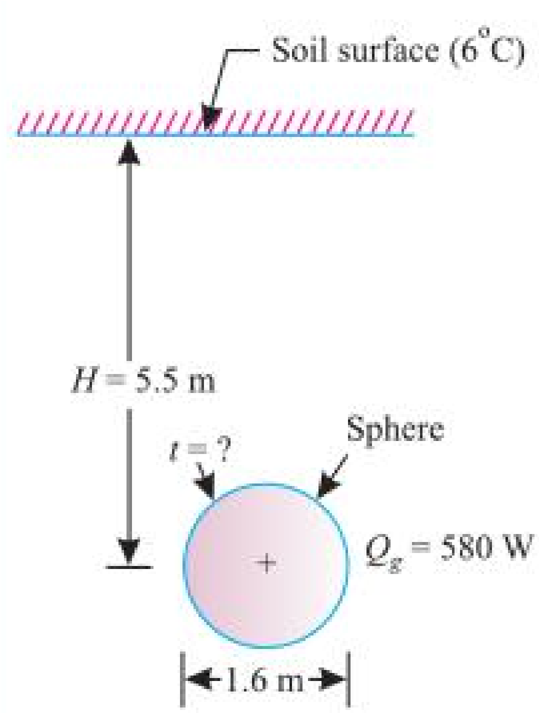

Fig. 3.9.

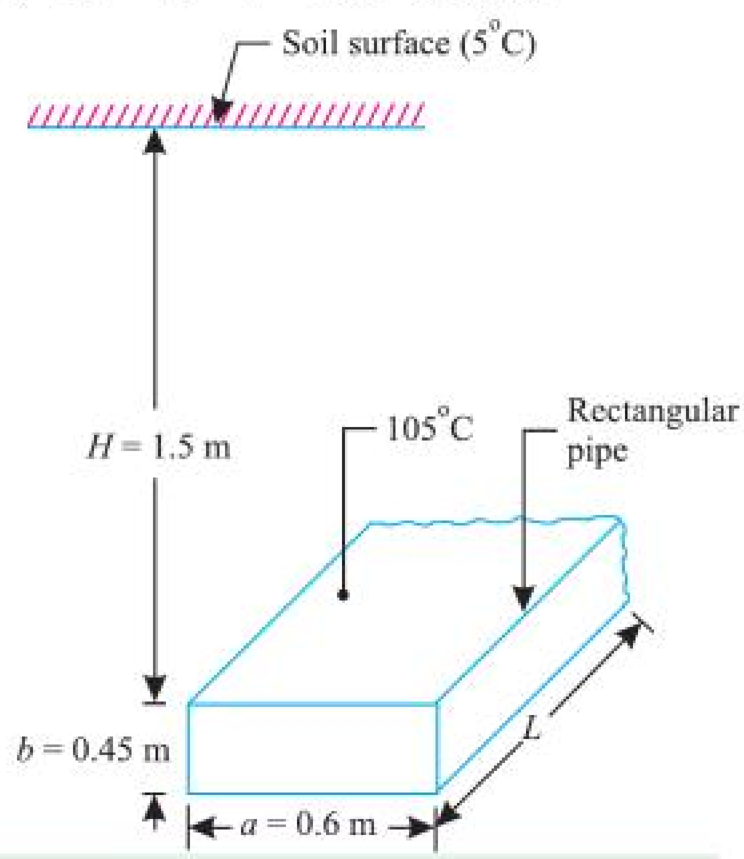

Fig. 3.10.

the nodes are designated by point (m, n); m location indicates x increment and n location indicates y increment. It is required to establish the temperatures at these nodal points. Thus, instead of the continuous distribution, there will be a stepped distribution of temperature through the conduction region. It is considered that the temperature at any node represents the temperature in the region $\pm\frac{\Delta x}{2}$ and $\pm\frac{\Delta y}{2}$ around the node. In the numerical method, *finite differences* are used to approximate differential increments in temperature and are substituted in equation. *The smaller the increment we choose, the closer will be the stepped temperature distribution to actual one.*

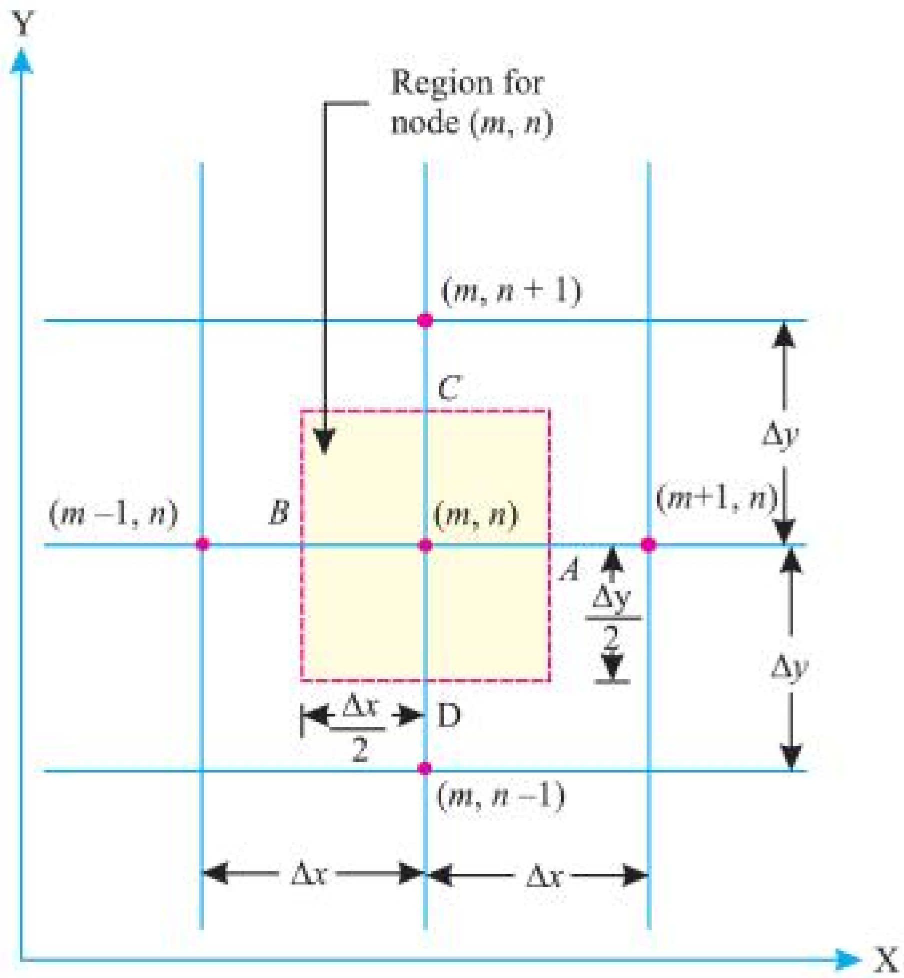

Fig. 3.13. Nodal point in conduction region.

The temperature gradients at points A, B, C and D may be written as:

$$\left.\frac{\partial t}{\partial x}\right|_{m+\frac{1}{2},\,n} \approx \frac{t_{m+1,\,n} - t_{m,\,n}}{\Delta x}$$

$$\left.\frac{\partial t}{\partial x}\right|_{m-\frac{1}{2},\,n} \approx \frac{t_{m,\,n} - t_{m-1,\,n}}{\Delta x}$$

$$\left.\frac{\partial t}{\partial y}\right|_{m,\,n+\frac{1}{2}} \approx \frac{t_{m,\,n+1} - t_{m,\,n}}{\Delta y}$$

$$\left.\frac{\partial t}{\partial y}\right|_{m,\,n-\frac{1}{2}} \approx \frac{t_{m,\,n} - t_{m,\,n-1}}{\Delta y} \qquad ...(3.19)$$

$$\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,\,n} \approx \frac{\left.\frac{\partial t}{\partial x}\right|_{m+\frac{1}{2},\,n} - \left.\frac{\partial t}{\partial x}\right|_{m-\frac{1}{2},\,n}}{\Delta x} = \frac{t_{m+1,\,-n} + t_{m-1,\,n} - 2t_{m,\,n}}{(\Delta x)^2}$$

$$\left.\frac{\partial^2 t}{\partial y^2}\right|_{m,n} = \frac{\left.\frac{\partial \tau}{\partial \psi}\right|_{\mu,\,\nu+\frac{1}{2}} \;TM\; \left.\frac{\partial \tau}{\partial \psi}\right|_{\mu,\,\nu^{TM}\frac{1}{2}}}{\Delta \psi} = \frac{\tau_{\mu,\,\nu+1} + \tau_{\mu,\,\nu^{TM}1} \;TM\; 2\tau_{\mu,\,\nu}}{(\Delta \xi)^2}$$

Substituting the values of $\frac{\partial^2 t}{\partial x^2}$ and $\frac{\partial^2 t}{\partial y^2}$ in the two-dimensional steady state equation [Eqn. (3.1)], we get

$$\frac{t_{m+1,n} + t_{m-1,n} - 2t_{m,n}}{(\Delta x)^2} + \frac{t_{m,n+1} + t_{m,n-1} - 2t_{m,n}}{(\Delta y)^2} = 0 \qquad ...(3.20)$$

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CHAPTER

4

Conduction–Unsteady– State (Transient)

4.1. INTRODUCTION

If the temperature of a body does not *vary with time*, it is said to be in a *steay state*. But if there is an *abrupt change* in its surface temperature, it (body) attains an equilibrium temperature or a steady state after some period. During this period the temperature *varies with time* and the body is said to be in an *unsteady or transient state*. The term transient or unsteady designates a phenomenon which is time dependent. The steady state is thus the *limit* of transient temperature distribution for large values of time.

Conduction of heat in unsteady state refers to the transient conditions wherein the heat flow and the temperature distribution at any point of the system vary continuously with time. Transient conditions occur in:

(*i*) Cooling of I.C. engines;
(*ii*) Automobile engines;
(*iii*) Heating and cooling of metal billets;
(*iv*) Cooling and freezing of food;
(*v*) Heat treatment of metals by quenching;
(*vi*) Starting and stopping of various heat exchange units in power installation;
(*vii*) Brick burning;
(*viii*) Vulcanization of rubber etc.

The temperature field in any transient problem, in general, is given by

$$t = f(x, y, z, \tau)$$

During an unsteady state the change in temperature may follow a periodic or non-periodic variation.

(*i*) Non-periodic variation. In a non-

The power on exponential, *i.e.*, $\frac{hA_s}{\rho Vc}\tau$ can be arranged in dimensionless form as follows.

$$\frac{hA_s}{\rho Vc}\tau = \left(\frac{hV}{kA_s}\right)\left(\frac{A_s^2\, k}{\rho V^2 c}\tau\right) = \left(\frac{h\,L_c}{k}\right)\left(\frac{\alpha\tau}{L_c^2}\right) \qquad ...(4.5)$$

where $\alpha = \left[\frac{k}{\rho c}\right]$ = Thermal diffusivity of the solid

$$L_c = \text{Characteristic length} = \frac{\text{Volume of the solid } (V)}{\text{Surface area of the solid } (A_s)}$$

The values of characteristic length (L_c), for simple geometric shapes, are given below:

Flat plate : $L_c = \frac{V}{A_s} = \frac{LBH}{2BH} = L/2$ = semi-thickness

where L, B and H are thickness, width and height of the plate.

Cylinder (*long*) : $L_c = \frac{\pi R^2 L}{2\pi RL} = \frac{R}{2}$ where, R = radius of the cylinder.

Sphere: $L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$ where, R = radius of the sphere.

Cube: $L_c = \frac{L^3}{6L^2} = \frac{L}{6}$ where, L = Side of the cube.

Further, from eqn. (4.5):

(*i*) The non-dimensional factor $\frac{hL_c}{k}$ is called the **Biot member B_i,**

i.e. $$B_i = \frac{hL_c}{k} = \text{Biot number.}$$

It gives an indication of the *ratio of internal* (*conduction*) *resistance to surface* (*convection*) *resistance*. When the value of B_i is small, it indicates that the system has a small internal (conduction) resistance, *i.e.*, relatively small temperature gradient or the existence of practically uniform temperature within the system. The convective resistance then predominates and the transient phenomenon is controlled by the convective heat exchange.

If $B_i < 0.1$, the lumped heat capacity approach can be used to advantage with simple shapes such as plates, cylinders, spheres and cubes. The error associated is around 5%.

(*ii*) The non-dimensional factor $\frac{\alpha\tau}{L_c^2}$ is called the **Fourier number, F_0.**

i.e. $$F_0 = \frac{\alpha\tau}{L_c^2} = \text{Fourier number}$$

It signifies the *degree of penetration of heating or cooling effect* through a solid.

Using non-dimensional terms, eqn. (4.4) takes the form of

$$\frac{\theta}{\theta_i} = \frac{t - t_a}{t_i - t_a} = e^{-BiF_0} \qquad ...(4.6)$$

The graphical representation of eqn. (4.5) for different solids (Infinite plates, infinite cylinders and infinite square rods and cubes and spheres) is shown in Fig. 4.4.

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Substituting the value, we get

$$\frac{t-50}{250-50} = \exp\left[-\frac{200}{8954 \times 0.01667 \times 383} \times 300\right] = 0.35$$

$$\left(\because \frac{A_s}{V} = \frac{L}{L_c} = \frac{1}{0.01667}\right)$$

$\therefore$ $t = (250 - 50) \times 0.35 + 50 = \mathbf{120°C}$ **(Ans.)**

Example 4.4. *An average convective heat transfer coefficient for flow of 90°C air over a flat plate is measured by observing the temperature time history of a 40 mm thick copper slab (ρ = 9000 kg/m^3, c = 0.38 kJ/kg°C, k = 370 W/m°C) exposed to 90°C air. In one test run, the initial temperature of the plate was 200°C, and in 4.5 minutes the temperature decreased by 35°C. Find the heat transfer coefficient for this case. Neglect internal thermal resistance.*

Solution. *Given* : t_a = 90°C; L = 40 mm or 0.04 m; ρ = 9000 kg/m^3; c = 0.38 kJ/kg°C; t_i = 200°C; t = 200 – 35 = 165°C; τ = 4.5 min = 270s

Characteristic length, $L_c = \frac{L}{2} = \frac{0.04}{2} = 0.02$ m

$$\frac{hA_s}{\rho Vc} = \frac{h}{\rho (V/A_s)c} = \frac{h}{\rho c L_c} = \frac{h}{9000 \times (0.38 \times 1000) \times 0.02} = 1.462 \times 10^{-5}\ h$$

Now, $$\frac{t-t_a}{t_i-t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad \text{...[Eqn. (4.4)]}$$

or, $$\frac{165-90}{200-90} = e^{-(1.462 \times 10^{-5} h) \times (270)} = e^{-0.003947h} = \frac{1}{e^{0.003947\,h}}$$

or, $$0.682 = \frac{1}{e^{0.003947\,h}} \text{ or } e^{0.003947h} = 1.466$$

or, $$0.003947\ h = ln\ 1.466 = 0.3825$$

$\therefore$ $$h = \frac{0.3825}{0.003947} = \mathbf{96.9\ W/m^2\,°C}\ \textbf{(Ans.)}$$

Example 4.5. *The heat transfer coefficients for the flow of air at 28°C over a 12.5 mm diameter sphere are measured by observing the temperature-time history of a copper ball of the same dimension. The temperature of copper ball (c = 0.4 kJ/kg K and ρ = 8850 kg/m^3) was measured by two thermocouples, one located in the centre and other near the surface. Both the thermocouples registered the same temperature at a given instant. In one test the initial temperature of the ball was 65°C and in 1.15 minute the temperature decreased by 11°C. Calculate the heat transfer coefficient for this case.* **(AMIE Winter, 2001)**

Solution. *Given* : t_a = 28°C; R (sphere) = $\frac{12.5}{2}$ = 6.25 mm = 0.00625 m; c = 0.4 kJ/kg°C; ρ = 8850 kg/m^3; t_i = 65°C; t = 65 – 11 = 54°C; τ = 1.15 min = 69 s.

Heat transfer coefficient, *h*:

Biot number, $$B_i = \frac{hL_c}{k} = \frac{h \cdot (R/2)}{k}$$

Since *heat transfer coefficient has to be calculated, so assume that the internal resistance is negligible and* B_i *is less than 0.1.*

Using eqn. (4.4), we have

$$\frac{\theta}{\theta_i} = \frac{t-t_a}{t_i-t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] = e^{-B_i F_0}$$

$$Nu = 2 + \left[0.4 \times (6510)^{0.5} + 0.06\ (6510)^{2/3}\right] \times (0.709)^{0.4} \times \left(\frac{18.16 \times 10^{-6}}{19.78 \times 10^{-6}}\right)^{0.25}$$

$$= 2 + [32.27 + 20.92] \times 0.87 \times 0.979 = 47.3$$

or, $$Nu = \frac{hD}{k} = 47.3$$

$\therefore$ $$h = \frac{k}{D} \times 47.3 = \frac{0.0258}{0.01} \times 47.3 = 122\ \text{W/m}^2\text{°C}$$

The time taken to cool from 75°C to 35°C may be found from the following relation :

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc} \cdot \tau\right]$$

$$\frac{35 - 23}{75 - 23} = \exp\left[-\frac{122 \times 4\pi R^2}{\rho \times \frac{4}{3}\pi R^3 \times c} \cdot \tau\right]$$

$$0.2308 = \exp\left(-\frac{122 \times 3}{8933 \times 0.005 \times 380} \cdot \tau\right) = e^{-0.02156\tau}$$

or, $$e^{0.02156\,\tau} = \frac{1}{0.2308} = 4.333$$

or, $$0.2156\,\tau = 1.466$$

or, $$\tau = \frac{1.466}{0.2156} = \mathbf{68\ s}\ \textbf{(Ans.)}$$

Example 4.10. *An egg with mean diameter of 40 mm and initially at 20°C is placed in a boiling water pan for 4 minutes and found to be boiled to the consumer's taste. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5°C. Take the following properties for egg:*

k = 10 W/m°C, ρ = 1200 kg/m^3, c = 2 kJ/kg°C and h (heat transfer coefficient) = 100 W/m^2°C.

Use lump theory. **(N.M.U.)**

Solution. *Given* : $R = \frac{40}{2} = 20$ mm = 0.02 m; t_i = 20°C; $\tau = 4 \times 60 = 240$ s; k = 10 W/m°C; ρ = 1200 kg/m^3; c = 2 kJ/kg°C; h = 100 W/m^2°C.

For using the lump theory, the required condition is $B_i < 0.1$

$$B_i = \frac{h L_c}{k}$$ where L_c is the characteristic length which is given by,

$$L_c = \frac{V\ \text{(volume)}}{A_s\ \text{(surface area)}} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

$\therefore$ $$B_i = \frac{h}{k} \times \frac{R}{3} = \frac{100 \times 0.02}{10 \times 3} = 0.067$$

As $B_i < 0.1$, we can use lump theory.

The temperature variation with time is given by :

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \qquad ...(1)$$

$$\frac{hA_s}{\rho Vc} = \left(\frac{h}{\rho c}\right)\left(\frac{A_s}{V}\right) = \left(\frac{100}{1200 \times 2000}\right)\left(\frac{3}{R}\right)$$

or, $0.01143 = \dfrac{1}{e^{\tau}}$ or $e^{\tau} = 87.49$

$$\tau = ln\,(87.49) = \mathbf{4.47s}\ \textbf{(Ans.)}$$

Example 4.13. *A thermocouple junction is in the form of 8 mm diameter sphere. Properties of material are:*

$c = 420$ J/kg°C; $\rho = 8000$ kg/m^3; $k = 40$ W/m°C and $h = 40$ W/m^2°C.

This junction is initially at 40°C and inserted in a stream of hot air at 300°C. Find:

(i) Time constant of the thermocouple;

(ii) The thermocouple is taken out from the hot air after 10 seconds and kept in still air at 30°C. Assuming the heat transfer coefficient in air 10 W/m^2°C, find the temperature attained by the junction 20 seconds after removing from hot air. **(P.U.)**

Solution. *Given* : $R = \dfrac{8}{2} = 4$ mm $= 0.004$ m; $c = 420$ J/kg°C; $\rho = 8000$ kg/m^3; $k = 40$ W/m°C; $h = 40$ W/m^2°C (gas stream); $h = 10$ W/m^2°C (air).

(*i*) Time constant of the thermocouple, τ*:

$$\tau^* = \frac{\rho V c}{h A_s} = \frac{\rho \times \left(\frac{4}{3}\pi R^3\right) \times c}{h \times 4\pi R^2} = \frac{\rho R c}{3h}$$

$$\tau^* = \frac{8000 \times 0.004 \times 420}{3 \times 40} = \mathbf{112\ s}$$ ...when thermocouple is in *gas stream*. **(Ans.)**

(*ii*) The temperature attained by the junction, t:

Given: $t_i = 40$°C; $t_a = 300$°C; $\tau = 20$ s

The temperature variation with respect to time during *heating* (when dipped in gas stream) is given by

$$\frac{t - t_a}{t_i - t_a} = \exp\left(-\frac{hA_s}{\rho V c}\tau\right)$$

or, $$\frac{t - 300}{40 - 300} = \exp\left(-\frac{\tau}{\tau^*}\right) = e^{(-10/112)} = \frac{1}{e^{(10/112)}} = 0.9146$$

or, $$t = 300 + 0.9146\,(40 - 300) = 62.2\text{°C}$$

The temperature variation with respect to time during cooling (when exposed to air) is given by

$$\frac{t - t_a}{t_i - t_a} = e^{\tau/\tau^*}$$

where, $$\tau^* = \frac{\rho R c}{3h} = \frac{8000 \times 0.004 \times 420}{3 \times 10} = 448\,\text{s}$$

$$\frac{t - 30}{62.2 - 30} = e^{(-20/448)} = \frac{1}{e^{(20/448)}} = 0.9563$$

or, $$t = 30 + 0.9563\,(62.2 - 30) = \mathbf{60.79\text{°C}}\ \textbf{(Ans.)}$$

Example 4.14. *A very thin glass walled 3 mm diameter mercury thermometer is placed in a stream of air, where heat transfer coefficient is 55 W/m^2°C, for measuring the unsteady temperature of air. Consider cylindrical thermometer bulb to consist of mercury only for which $k = 8.8$ W/m°C and $\alpha = 0.0166$ m^2/h. Calculate the time required for the temperature change to reach half its final value.*

Solution. *Given* : $R = \dfrac{3}{2} = 1.5$ mm $= 0.0015$ m; $h = 55$ W/m^2°C; $k = 8.8$ W/m°C; $\alpha = 0.0166$ m^2/h.

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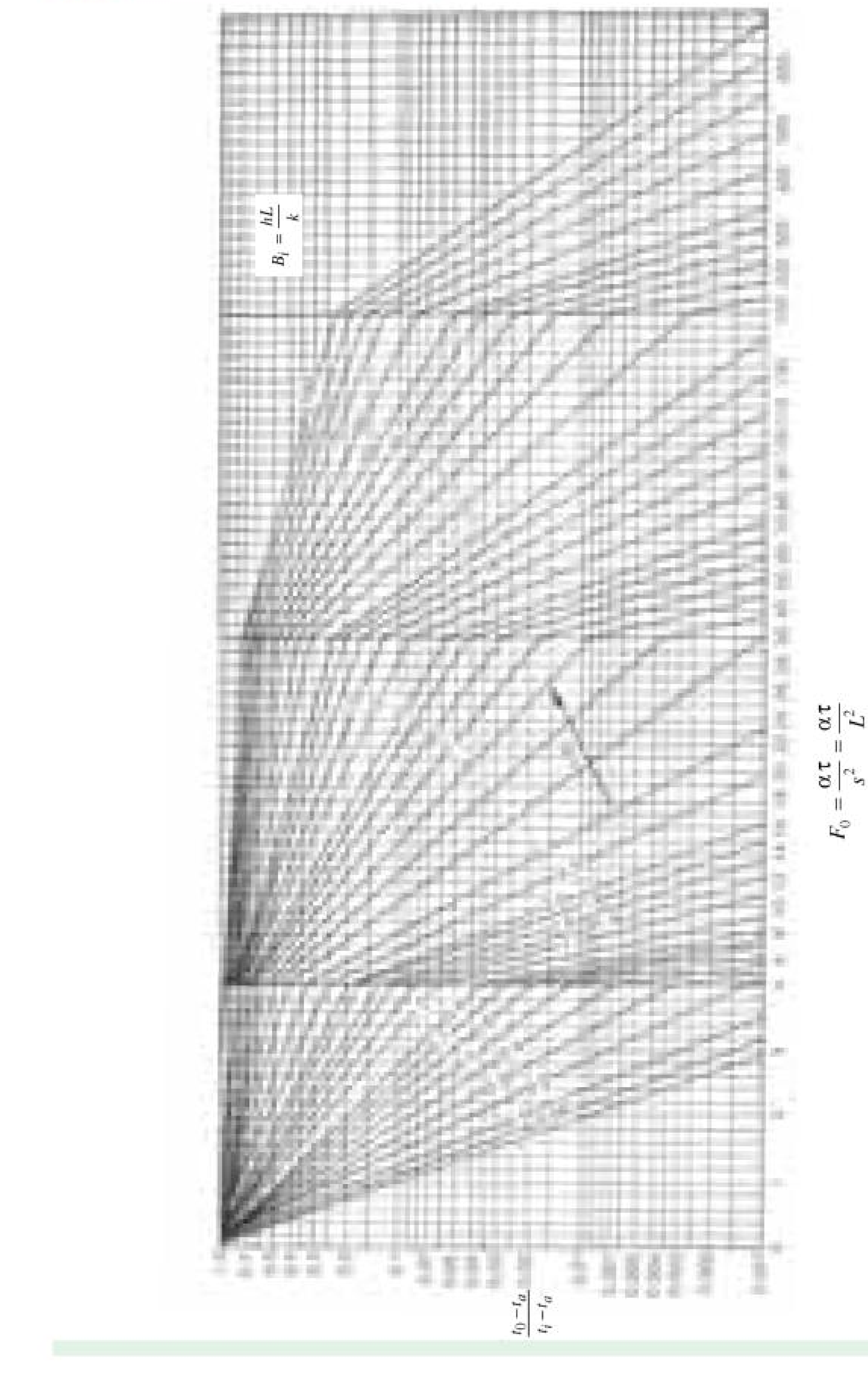

Fig. 4.8. Heisler chart for temperature history at the centre of a plate of thickness 2 L or $(x/L) = 0$

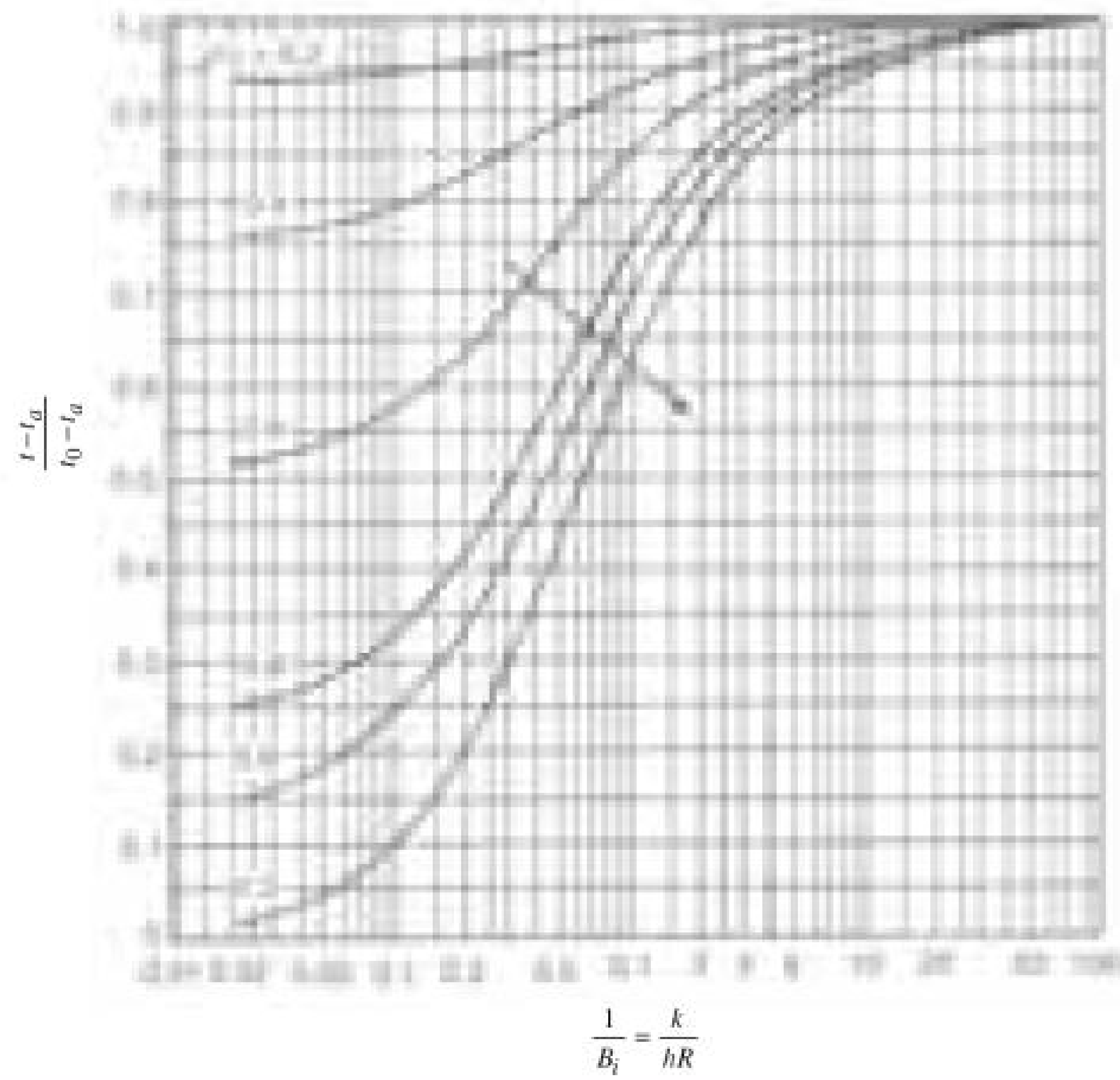

Fig. 4.13. Heisler position-correction factor chart for temperature history in sphere.

be neglected. Thus the plate *cannot* be considered as a lumped system. Further, as the $B_i < 100$. *Heisler charts can be used* to find the solution of the problem.

Corresponding to the following parameteric values, from Heisler charts, (Fig. 4.8), we have

$$F_o = 3.424; \frac{1}{B_i} = \frac{1}{0.165} = 6.06 \text{ and } \frac{x}{L} = 0 \text{ (midplane)}$$

$$\frac{t_o - t_a}{t_i - t_a} = 0.6 \qquad \text{(From Heisler charts)}$$

Substituting the values, we have

$$\frac{t_o - 50}{440 - 50} = 0.6$$

or, $t_o = 50 + 0.6\,(440 - 50) =$ **284°C (Ans.)**

Temperature inside the plate 15 mm from the midplane, *t*:

The distance 15 mm from the midplane implies that

$$\frac{x}{L} = \frac{15}{30} = 0.5$$

Corresponding to $\frac{x}{L} = 0.5$ and $\frac{1}{B_i} = 6.06$, from Fig. 4.11, we have

$$\frac{t - t_a}{t_i - t_a} = 0.97$$

The parametric values for the spherical apple are :

$$\frac{1}{B_i} = \frac{1}{0.441} = 2.267$$

$$F_o = \frac{\alpha\tau}{R^2} = \left(\frac{k}{\rho c}\right)\frac{\tau}{R^2} = \left(\frac{0.58}{990 \times 4170}\right) \times \left(\frac{7200}{0.06^2}\right) = 0.281$$

$$\frac{r}{R} = 0 \text{ (midplane or centre of the apple)}$$

Corresponding to the above values, from the chart for a sphere (Fig. 4.13), we read

$$\frac{t_o - t_a}{t_i - t_a} = 0.75$$

or, $$\frac{t_o - t_a}{25 - 6} = 0.75$$

or, $$t_o = 6 + 0.75\,(25 - 6) = \mathbf{20.25°C\ (Ans.)}$$

4.5. TRANSIENT HEAT CONDUCTION IN SEMI-INFINITE SOLIDS (H OR $B_i \to \infty$)

A solid which extends itself infinitely in all directions of space is termed as an *infinite solid*. If an infinite solid is split in the middle by a plane, each half is known as *semi-infinite solid*. In a semi-infinite body, at any instant of time, there is always a point where the effect of heating (or cooling) at one of its boundaries is not felt at all. At the *point* the temperature remains unaltered. The transient temperature change in a plane infinitely thick wall is similar to that of a semi-infinite body until enough time has passed for the surface temperature effect to penetrate through it.

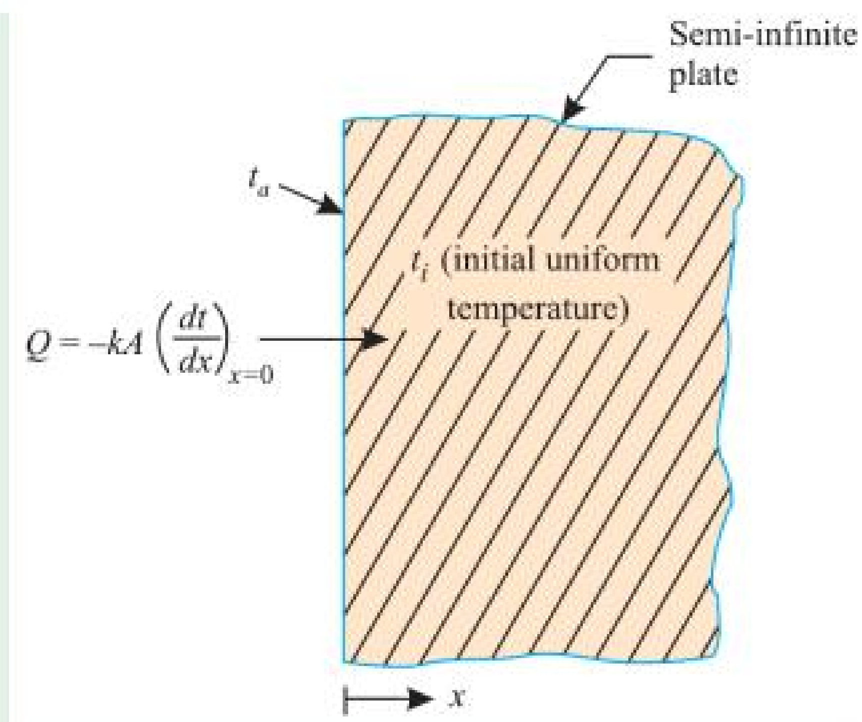

Fig. 4.14. Transient heat flow in a semi-infinite plate.

As shown in the Fig. 4.14, consider a semi-infinite plate, a plate bounded by a plane $x = 0$ and extending to infinity in the +ve X-direction. The entire body is initially at uniform temperature t_i including the surface at $x = 0$. The surface temperature at $x = 0$ is suddenly raised to t_a for all times greater than $\tau = 0$. The governing equation is :

$$\frac{d^2t}{dx^2} = \frac{1}{\alpha}\frac{dt}{d\tau}$$

The boundary conditions are :

(*i*) $t(x, 0) = t_i$; (*ii*) $t(0, \tau) = t_a$ for $\tau > 0$; (*iii*) $t(\infty, \tau) = t_i$ for $\tau > 0$.

The solution of the above differential equation, with these boundary conditions, for temperature distribution at any time τ at a plane parallel to and at a distance x from the surface is given by :

$$\frac{t(x, \tau) - t_a}{t_i - t_a} = erf(z) = erf\left(\frac{x}{2\sqrt{\alpha\tau}}\right) \quad ...(4.14)$$

where, $$z = \frac{x}{2\sqrt{\alpha\tau}}$$

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or, $$\tau = \frac{x^2}{4\alpha \times 0.81}$$

$$= \frac{0.012^2}{4 \times 1.2 \times 10^{-5} \times 0.81} = \mathbf{3.7\,s} \quad \textbf{(Ans.)}$$

Example 4.21. *It is proposed to bury water pipes underground in wet soil which is initially at 5.4°C. The temperature of the surface of soil suddenly drops to – 6°C and remains at this value for 9.5 hours. Determine the minimum depth at which the pipes be laid if the surrounding soil temperature is to remain above 0°C (without water getting frozen). Assume the soil as semi-infinite solid.*

For wet soil take α (thermal diffusitivity) = 2.75 × 10^{-3} m²/h.

Solution. *Given* : $t_i = 5.4°C$, $t_a = -6°C$, $t = 0°C$, $\alpha = 2.75 \times 10^{-3}$ m²/h

Minimum Depth, *x* :

The temperature, at critical depth, will just reach 0°C after 9.5 hours,

Now, $$\frac{t - t_a}{t_i - t_a} = erf\left[\frac{x}{2\sqrt{\alpha\tau}}\right]$$

or, $$\frac{0-(-6)}{5.4-(-6)} = 0.526 = erf\left[\frac{x}{2\sqrt{\alpha\tau}}\right]$$

or, $$\frac{x}{2\sqrt{\alpha\tau}} = 0.50$$ ...From table 4.1 or Fig. 4.15

or, $$x = 0.5 \times 2\sqrt{\alpha\tau}$$

or, $$x = 0.5 \times 2\sqrt{2.75 \times 10^{-3} \times 9.5} = \mathbf{0.162\ m} \quad \textbf{(Ans.)}$$

Example 4.22. *A 60 mm thick mild steel plate (α = 1.22 × 10^{-5} m²/s) is initially at a temperature of 30°C. It is suddenly exposed on one side to a fluid which causes the surface temperature to increase to and remain at 110°C. Determine :*

(i) The maximum time that the slab be treated as a semi-infinite body;

(ii) The temperature at the centre of the slab 1.5 minutes after the change in surface temperature.

Solution. *Given* : L = 60 mm = 0.06 m, $\alpha = 1.22 \times 10^{-5}$ m²/s, t_i = 30°C, t_a = 110°C, τ = 1.5 minutes = 90s.

(*i*) The maximum time that the slab be treated as a semi-infinite body, τ_{max} :

The general criterion for the infinite solution to apply to a body of finite thickness subjected to one-dimensional heat transfer is

$$\frac{L}{2\sqrt{\alpha\tau}} \geq 0.5$$ (where, L = Thickness of the body)

or, $$\frac{L}{2\sqrt{\alpha\tau_{max}}} = 0.5 \quad \text{or} \quad \frac{L^2}{4\alpha\tau_{max}} = 0.25$$

or, $$\tau_{max} = \frac{L^2}{4\alpha \times 0.25} = \frac{0.06^2}{4 \times 1.22 \times 10^{-5} \times 0.25} = \mathbf{295.1\ s} \quad \textbf{(Ans.)}$$

(*ii*) The temperature at the centre of the slab, *t* :

At the centre of the slab, x = 0.03 m; τ = 90s

$$\frac{t - t_a}{t_i - t_a} = erf\left[\frac{x}{2\sqrt{\alpha\tau}}\right]$$

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(ii) Time required to attain a temperature of 350°C, t:

Since, t = 350°C, hence

$$\frac{350-480}{20-480} = erf\left(\frac{\alpha\tau}{R^2}\right)$$

$$0.2826 = erf\left(\frac{\alpha\tau}{R^2}\right)$$

∴ $$\frac{\alpha\tau}{R^2} = 0.23$$

or, $$\tau = \frac{0.23 \times R^2}{\alpha}$$

$$= \frac{0.23 \times 0.3^2}{1.12 \times 10^{-4}} = \mathbf{184.8\ s} \quad \textbf{(Ans.)}$$

4.6. SYSTEMS WITH PERIODIC VARIATION OF SURFACE TEMPERATURE

The periodic type of heat flow occurs in cyclic generators, in reciprocating I.C. engines and in the earth as the result of daily cycle of the sun. These periodic changes, in general, are not simply sinusoidal but rather complex. However, these comoplex changes can be approximated by a number of sinusoidal components.

Let us consider a thick plane wall (one-dimensional case) whose surface temperature alters according to a sine function as shown in Fig. 4.17. The surface temperature oscillates about the mean temperature level t_m according to the following relation :

$$\theta_{s,\tau} = \theta_{s,a} \sin(2\,\pi n\tau)$$

where, $\theta_{s,\tau}$ = Excess over the mean temperature $(= t_{s,\tau} - t_m)$;

$\theta_{s,a}$ = Amplitude of temperature excess, *i.e.*, the maximum temperature excess at the surface;

n = Frequency of temperature wave.

The temperature excess at any depth x and time τ can be expressed by the relation,

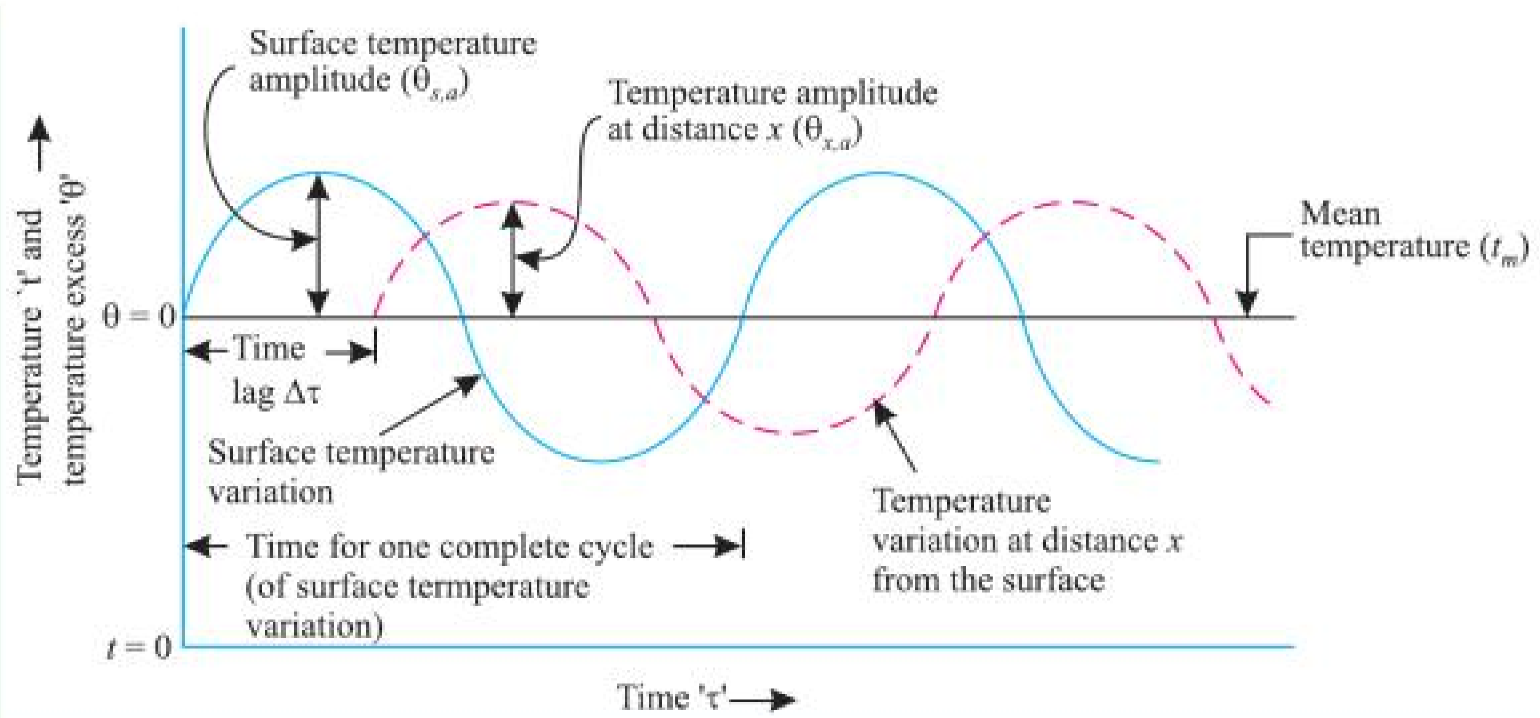

Fig. 4.17. Temperature curves for periodic variation of surface temperature.

$$\theta_{x,\tau} = \theta_{s,a} \exp\left[-x\sqrt{\pi n/\alpha}\right] \sin\left[2\pi n\tau - x\sqrt{\frac{\pi n}{\alpha}}\right] \qquad ...(4.21)$$

Take: h = 20 W/m²°C; k (steel) = 40 W/m°C; ρ (steel) = 7800 kg/m³; c (steel) = 460 J/kg°C. **(M.U.)**

Solution: *Given* : $R = \frac{100}{2} = 50$ mm = 0.05 m; $t_i = 900°C$; $t_a = 30°C$; $h = 20 W/m^2 °C$; k (steel) = 40 W/m°C; ρ (steel) = 7800 kg/m³; c (steel) = 460 J/kg°C; τ = 30s.

(*i*) Temperature of ball after 30 seconds, *t* :

Characteristic length $$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3} = \frac{0.05}{3} = 0.01667 \text{ m}$$

Biot number, $$B_i = \frac{h\,L_c}{k} = \frac{20 \times 0.01667}{40} = 0.008335$$

Since B_i is less than 0.1, hence lumped capacitance method (Newtonian heating or cooling) may be applied for the solution of the problem.

The time versus temperature distribution is given by (Eqn. 4.4)

$$\frac{t - t_a}{t_i - t_a} = \exp\left[-\frac{hA_s}{\rho Vc}\tau\right] \quad ...(1)$$

Now $$\frac{hA_s}{\rho Vc}\tau = \left(\frac{h}{\rho c}\right)\left(\frac{A_s}{V}\right)\tau = \left(\frac{20}{7800 \times 460}\right)\left(\frac{1}{0.01667}\right)(30) = 0.01$$

∴ $$\frac{t - 30}{900 - 30} = e^{-0.01} = \frac{1}{e^{0.01}} = 0.99$$

or, $$t = 30 + 0.99\,(900 - 30) = \mathbf{891.3°C} \quad \textbf{(Ans.)}$$

(*ii*) The rate of cooling (°C/min) after 30 seconds :

The rate of cooling means we have to find out $\frac{dt}{d\tau}$ at the required time.

Now differentiating eqn. (1), we get

$$\frac{1}{t_i - t_a} \times \frac{dt}{d\tau} = -\left(\frac{hA_s}{\rho Vc}\right)\exp\left[-\frac{hA_s}{\rho Vc}\tau\right]$$

Now substituting the proper values in the above equation, we have

$$\frac{1}{(900 - 30)} \cdot \frac{dt}{d\tau} = -\left(\frac{20}{7800 \times 460} \times \frac{1}{0.01667}\right) \times 0.99 = -\,3.31 \times 10^{-4}$$

∴ $$\frac{dt}{d\tau} = (900 - 30)\,(-\,3.31 \times 10^{-4}) = -\,0.288°C/s$$

or, $$\frac{dt}{d\tau} = -\,0.288 \times 60 = \mathbf{-\,17.28°C/min} \quad \textbf{(Ans.)}$$

Example 4.31. *A thin copper plate 20 mm thick is initially at 150°C. One surface is in contact with water at 30°C (h_w = 100 W/m² °C) and other surface is exposed to air at 30°C (h_a = 20 W/m² °C). Determine the time required to cool the plate to 90°C.*

Take the following properties of the copper :

ρ = 8800 kg/m³, c = 400 J/kg°C and k = 360 W/m°C

Solution. *Given* : L = 20 mm = 0.02 m; t_i = 150°C; t_a = 30°C; h_w = 100 W/m² °C; h_a = 20 W/m² °C; t = 90°C; ρ = 8800 kg/m³; c = 400 J/kg°C; k = 360 W/m°C.

The instantaneous heat flow rate at a given X-location within the semi-infinite body at a specified time is given by

$$Q_i = -kA\,(t_i - t_a)\,\frac{e^{[-x^2/(4\alpha\tau)]}}{\sqrt{\pi\alpha\tau}} \qquad ...(ii)$$

The heat flow rate at the surface ($x = 0$) is given by

$$Q_{surface} = \frac{-kA\,(t_i - t_a)}{\sqrt{\pi\alpha\tau}} \qquad ...(iii)$$

The heat flow rate (Q') is given by

$$Q' = -1.13\,kA\,(t_i - t_a)\sqrt{\frac{\tau}{\alpha}} \qquad ...(iv)$$

THEORETICAL QUESTIONS

1. What is meant by transient heat conduction?
2. What is lumped capacity?
3. What are the assumptions for lumped capacity analysis?
4. What are Fourier and Biot numbers? What is the physical significance of these numbers?
5. Define a semi-infinite body.
6. What is an error function? Explain its significance in a semi-infinite body in transient state.
7. What are Heisler charts?
8. Explain the significance of Heisler charts in solving transient conduction problems.

UNSOLVED EXAMPLES

1. A copper slab (ρ = 9000 kg/m^3, c = 380 J/kg°C, k = 370 W/m°C) measuring 400 mm × 400 mm × 5 mm has a uniform temperature of 250°C. Its temperature is suddenly lowered to 30°C. Calculate the time required for the plate to reach the temperature of 90°C. Assume convective heat transfer coefficient as 90 W/m^2 °C. (**Ans.** t = 123.75 s)
2. An aluminium alloy plate 0.2 m^2 surface area (both sides), 4 mm thick and at 200°C is suddenly quenched into liquid oxygen which is at –183°C. Find the time required for the plate to reach the temperature of –70°C.

   Take : ρ = 2700 kg/m^3; c_p = 890 J/kg°C and h = 500 W/m^2 °C. (**Ans.** 23.45 s)
3. A sphere of 200 mm diameter made of cast iron initially at uniform temperature of 400°C is quenched into oil. The oil bath temperature is 40°C. If the temperature of sphere is 100°C after 5 minutes, find heat transfer coefficient on the surface of the sphere.

   Take : c_p (cast iron) = 0.32 kJ/kg°C; ρ (cast iron) = 7000 kg/m^3

   Neglect internal thermal resistance. (**Ans.** 134 kW/m^2 °C)
4. An average convective heat transfer coefficient for flow of 100°C air over a flat plate is measured by observing the temperature-time history of a 30 mm thick copper slab (ρ = 9000 kg/m^3, c = 0.38 kJ/kg°C, k = 370 W/m°C) exposed to 100°C air. In one test run, the initial temperature of the plate was 210°C, and in 5 minutes the temperature decreased by 40°C. Find the heat transfer coefficient for this case. Neglect internal thermal resistance. (**Ans.** 77.24 W/m^2 °C)
5. A cylindrical steel ingot 150 mm in diameter and 400 mm long passes through a heat treatment furnace which is 6 m in length. The ingot must reach a temperature of 850°C before it comes out of the furnace. The furnace gas is at 1280°C and ingot initial temperature is 100°C. What is the maximum speed with which the ingot should move in the furnace to attain the required temperature? The combined radiative and convective surface heat transfer coefficient is 100 W/m^2 °C. Take k (steel) = 45 W/m°C and α (thermal diffusivity) = 0.46 × 10^{-5} m^2/s.(**Ans.** 1.619 × 10^{-3} m/s)

Let, ρ = Mass density of the fluid at a particular instant, and

u, v, w = Components of velocity of flow entering the three faces of the parallelopiped.

Rate of mass of fluid entering the face ABCD (*i.e.* fluid influx).

$$= \rho \times \text{velocity in } X\text{-direction} \times \text{area of ABCD}$$

$$= \rho\, u\, dy\, dz \qquad ...(i)$$

Rate of mass of fluid leaving the face EFGH (*i.e.* fluid efflux)

$$= \rho\, u\, dy\, dz + \frac{\partial}{\partial x}(\rho\, u\, dy\, .\, dz)\, dx \qquad ...(ii)$$

The gain in mass per unit time due to flow in the X-direction is given by the difference between the fluid influx and fluid efflux.

$\therefore$ Mass accumulated per unit time due to flow in X-direction

$$= \rho u\, dy\, dz - \left[\rho u + \frac{\partial}{\partial x}(\rho u) dx\right] dy\, dz$$

$$= -\frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz \qquad ...(iii)$$

Similarly, the gain in fluid mass per unit time in the parallelopiped due to flow in Y and Z-direction

$$= -\frac{\partial}{\partial y}(\rho v)\, dy\, dz \text{ (in } Y\text{–direction)} \qquad ...(iv)$$

$$= -\frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz \text{ (in Z–direction)} \qquad ...(v)$$

The total (or net) gain in fluid mass per unit time for fluid flow along three coordinate axes

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\ dy\ dz \qquad ...(vi)$$

Rate of change of mass of the parallelopiped (control volume)

$$= \frac{\partial}{\partial t}(\rho\ dx\ dy\ dz) \qquad ...(vii)$$

From eqns. (*vi*) and (*vii*), we get

$$= -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial w}(\rho w)\right] dx\ dy\ dz = \frac{\partial}{\partial t}(\rho\ dx\ dy\ dz)$$

Simplification and rearrangement of terms would reduce the above expression to

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0 \qquad ...(5.5)$$

This equation (Eqn. 5.5) is the general equation of continuity in three-dimensions and is applicable to *any type of flow* and for any fluid whether *compressible or incompressible.*

For steady flow $\left(\frac{\partial \rho}{dt} = 0\right)$ *incompressible fluids* (ρ = constant) the equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad ...(5.6)$$

For two dimensional flow, eqn. (5.6) reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (\because w = 0)$$

For one dimensional flow, say in X-direction, eqn. (5.6) takes the form

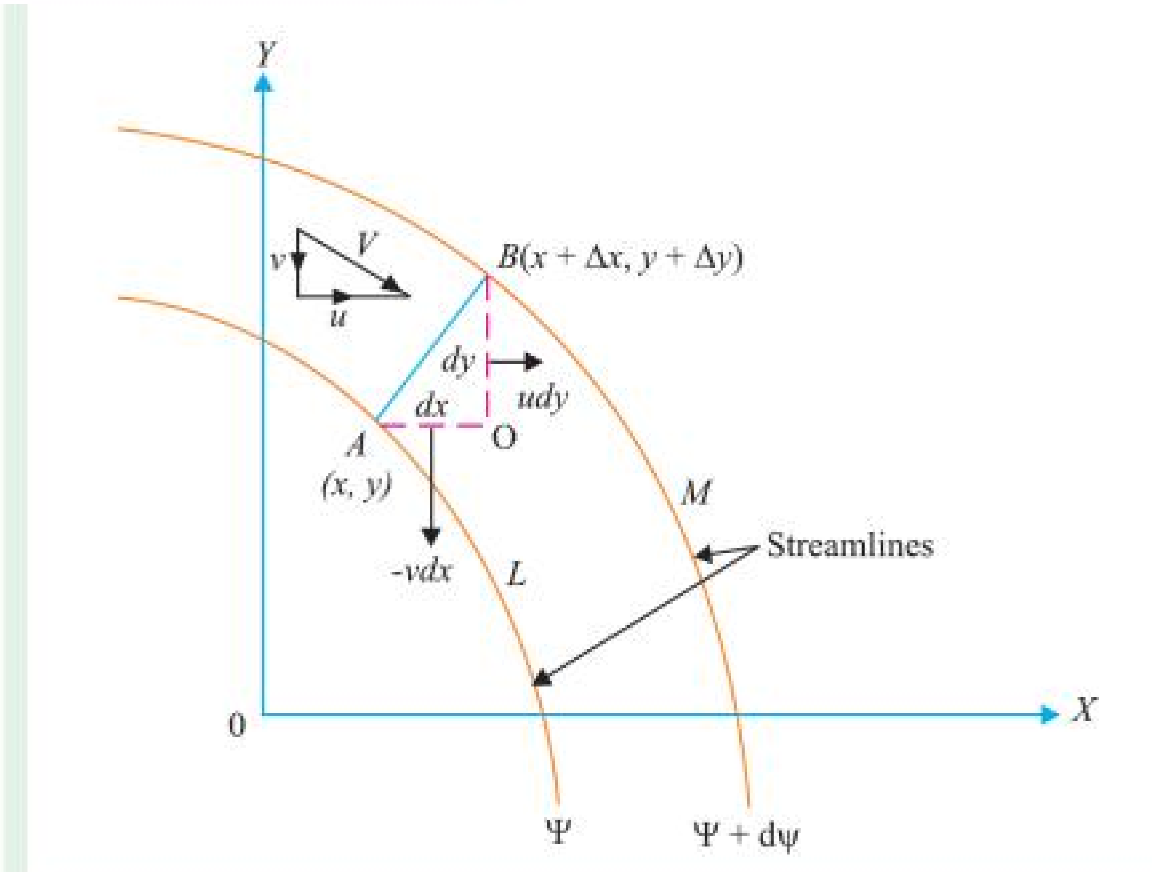

Fig. 5.3. Flow between two points and its relation to stream function.

Flow across AB = Flow across AO + flow across OB

$$Vds = -v\,dx + u\,dy$$

(The minus sign indicates that the velocity v is acting in the downward direction.)

$$Vds = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = d\psi \quad ...(5.15)$$

i.e.

$$dq = d\psi \quad ...(5.16)$$

Obviously, the *stream function* can also be defined as the *flux or flow rate between two streamlines*. The units of ψ are m^3/s; discharge per unit thickness of flow.

Properties of stream function :

The properties of stream function are:

1. On any streamline, ψ is constant everywhere.

$$\begin{bmatrix} \psi = \text{constant, represents the family of streamlines} \\ \psi = \text{constant, is a streamline equation} \end{bmatrix}$$

2. If the flow is continuous, the flow around any path in the fluid is zero.
3. The rate of change of ψ with distance in arbitrary direction is proportional to the component of velocity normal to that direction.
4. The algebraic sum of stream functions for two incompressible flow patterns is the stream function for the flow resulting from the super-imposition of these patterns,

i.e.,
$$\frac{\partial \psi_1}{\partial s} + \frac{\partial \psi_2}{\partial s} = \frac{\partial(\psi_1 + \psi_2)}{\partial s} \quad ...(5.17)$$

Cauchy Riemann equations :

From the above discussion of velocity potential function and stream function we arrive at the following conclusions:

1. *Potential function* (ϕ) *exists only for irrotational flow.*
2. *Stream function* (ψ) *applies to both the rotational and irrotational flows* (which are steady and incompressible).
3. In case of *irrotational flow, both the stream function and velocity function satisfy Laplace*

when, $Re < 2000$ ... the flow is *laminar* (or viscous)

$Re > 4000$ ... the flow is *turbulent.*

Re between 2000 and 4000 ... the flow is *unpredictable*.

Critical Reynolds number :

—All experiments agree that a lower limit of critical value of $(Re)_{cr}$ exists (though there appears to be no definite upper limit of the critical value of $(Re)_{cr}$ which characterizes full attainment of turbulence) and its value is approximately, 2000 (for circular pipe). This lower critical Reynolds number is of greater engineering importance as it defines the *limit below which all turbulence, no matter how severe, entering the flow from any source will eventually be damped out by viscous action.*

— It has been observed that the upper limit of critical Reynolds number $(Re)_{cr}$ depends upon the following factors:

(*i*) Initial turbulence in the flow (approach), (*ii*) Shape of the pipe entrance, and

(*iii*) Roughness of pipe.

Reynolds found the upper limit of $(Re)_{cr}$ to lie between $12000 < (Re)_{cr} < 14000$; these values are of little practical interest and we may consider the upper limit of $(Re)_{cr}$ to be defined by $2700 < (Re)_{cr} < 4000$.

— For demarcating the regimes of laminar and turbulent flows, the concept of critical Reynolds number proves quite useful.

The *lower* critical Reynolds number for some important cases are as under:

(*i*) $(Re)_{cr} = 1$ ... for sphere

(*ii*) $(Re)_{cr} = 50$ ... for open channels

(*iii*) $(Re)_{cr} = 1000$ ... for parallel plates.

HIGHLIGHTS

1. An *ideal fluid* is one which has no viscosity and surface tension and is incompressible. A *real practical fluid* is one which has viscosity, surface tension and compressibility in addition to the density.
2. *Viscosity* may be defined as the property of a fluid which determines its resistance to shearing stresses. The viscosity may also be defined as the shear stress required to produce unit rate of shear strain.
3. Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid.
4. *Newton's law of viscosity* states that the shear stress (τ) on a fluid element layers is directly proportional to the rate of shear strain.

   Mathematically, $$\tau = \mu \frac{du}{dy}$$
5. *Continuity equation in cartesian coordinates* :

Viscosity tester.

11.	Discharge	Q	m^3/s	L^3T^{-1}
12.	Kinematic viscosity	ν	m^2/s	L^2T^{-1}
	(*d*) Dynamic Quantities			
13.	Force or resistance	F, R	N	MLT^{-2}
14.	Density	ρ	kg/m^3	ML^{-3}
15.	Specific weight	*w*	N/m^3	$ML^{-2}T^{-2}$
16.	Dynamic viscosity	μ	kg/ms	$ML^{-1}T^{-1}$
17.	Work, energy	W, E	Nm	ML^2T^{-2}
18.	Power	P	Nm/s or J/s or W	ML^2T^{-3}
	(*e*) Thermodynamic quantities			
19.	Heat	Q, H	J	ML^2T^{-2}
20.	Thermal conductivity	k	W/m°C	$MLT^{-3}\theta^{-1}$
21.	Specific heat	c_p, c_v	kJ/kg°C	$L^2T^{-2}\theta^{-1}$
22.	Heat transfer coefficient	h, U	$W/m^2°C$	$MT^{-3}\theta^{-1}$
23.	Gas constant	R	kJ/kg°C	$L^2T^{-2}\theta^{-1}$
24.	Thermal diffusivity	α	m^2/s	L^2T^{-1}

6.4. METHODS OF DIMENSIONAL ANALYSIS

With the help of dimensional analysis the equation of a physical phenomenon can be developed in terms of dimensionless groups or parameters and thus reducing the number of variables. The methods of dimensional analysis are based on the Fourier's principle of homogeneity. Out of several methods of dimensional analysis, the following two methods will be discussed.

1. Rayleigh's method
2. Buckingham's π-method/theorem

6.4.1. RAYLEIGH'S METHOD

This method gives a special form of relationship among the dimensionless group, and has the *inherent drawback* that it *does not provide any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis*. Due to this reason this method has become *obsolete* and is *not favoured for use*.

Rayleigh's method is used for determining the expression for a variable which depends upon maximum three or four variables only. In case the number of independent variables become more than four, then it is very difficult to find the expression for the dependent variable.

In this method a functional relationship of some variables is expressed in the form of an exponential equation which must be dimensionally homogeneous. Thus if X is a variable which depends on $X_1, X_2, X_3, \ldots X_n$; the functional equation can be written as :

$$X = f(X_1, X_2, X_3, \ldots X_n) \qquad \ldots(6.1)$$

In the above equation X is a *dependent variable*, while $X_1, X_2, X_3, \ldots X_n$ are *independent variables*. A dependent variable is the one about which information is required while independent variables are those which govern the variation of dependent variable.

Equation (6.1) can also be written as :

$$X = C\,(X_1^a, X_2^b, X_3^c, \ldots X_b^n) \qquad \ldots(6.2)$$

where C is a constant and $a, b, c, \ldots n$ are the arbitrary powers. The values of $a, b, c, \ldots n$ are obtained by comparing the powers of the fundamental dimensions of both sides. Thus the expression is obtained for dependent variable.

Number of dimension π-terms = $n - m = 6 - 3 = 3$

Thus three π–terms say π_1, π_2, and π_3 are formed.

The eqn. (*ii*) may be written as

$$f_1(\pi_1, \pi_2, \pi_3) = 0 \qquad ...(iii)$$

Step 2. *Selection of repeating variables*: Out of six variables R, V, l, μ, ρ, g three variables (as $m = 3$) are to be selected as *repeating variables*. R is a dependent variable and should *not* be selected as a repeating variable. Out of the remaining five variables one variable should have *geometric property*, second should have *flow property* and third one should have *fluid property*; these requirements are met by selecting l, V and ρ as *repeating variables*. The repeating variables themselves should not form a dimensionless term and must contain *jointly all fundamental dimensions equal to m i.e.*, 3 here. Dimensions of l, V and ρ are L, LT^{-1}, ML^{-3} and hence the three fundamental dimensions exist in l, V and ρ and also *no dimensionless group is formed by them*.

Step 3. Each π–term (= $m + 1$ variables) is written as given in eqn. (6.6), *i.e.*

$$\left.\begin{aligned} \pi_1 &= l^{a_1} . V^{b_1} . \rho^{c_1} . R \\ \pi_2 &= l^{a_2} . V^{b_2} . \rho^{c_2} . \mu \\ \pi_3 &= l^{a_3} . V^{b_3} . \rho^{c_3} . g \end{aligned}\right\} \qquad ...(iv)$$

Step 4. Each π-term is solved by the *principle of dimensional homogeneity*, as follows:

π_1–term:

$$\pi_1 = l^{a_1} . V^{b_1} . \pi^{c_1} . R$$

$$M^0 L^0 T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-3})^{c_1} . (MLT^{-2})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_1 + 1$

For L : $0 = a_1 + b_1 - 3c_1 + 1$

For T : $0 = -b_1 - 2$

∴ $c_1 = -1; b_1 = -2$

and $a_1 = -b_1 + 3c_1 - 1 = 2 - 3 - 1 = -2$

Substituting the values of a_1, b_1 and c_1 in π_1, we get

$$\therefore \quad \pi_1 = l^{-2} . V^{-2} . \rho^{-1} . R = \frac{R}{l^2 V^2 \rho} \qquad ...(v)$$

π_2–term:

$$\pi_2 = l^{a_2} . V^{b_2} . \rho^{c_2} . \mu$$

$$M^0 L^0 T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-3})^{c_2} . (ML^{-1}T^{-1})$$

Equating the exponents of M, L and T respectively, we get

For M : $0 = c_2 + 1$

For L : $0 = a_2 + b_2 - 3c_2 - 1$

For T : $0 = -b_2 - 1$

∴ $c_2 = -1; b_2 = -1$

and, $a_2 = -b_2 + 3c_2 + 1 = 1 - 3 + 1 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2, we get

$$\therefore \quad \pi_2 = l^{-1} . V^{-1} . \rho^{-1} . \mu = \frac{\mu}{l V \rho}$$

6.5. DIMENSIONAL ANALYSIS APPLIED TO FORCED CONVECTION HEAT TRANSFER

Let us assume that the heat transfer coefficient in a fully developed forced convection in a tube is a function of the following variables:

$$h = f(\rho, D, V, \mu, c_p, k) \quad ...(i)$$

or, $$f_1(h, \rho, D, V, \mu, c_p, k) \quad ...(ii)$$

The physical quantities with their dimensions are as under:

S.No.	*Variables*	*Symbols*	*Dimensions*
1	Heat transfer coefficient	h	$MT^{-3}\,\theta^{-1}$
2	Fluid density	ρ	ML^{-3}
3	Tube diameter	D	L
4	Fluid velocity	V	LT^{-1}
5	Fluid viscosity	μ	$ML^{-1}\,T^{-1}$
6	Specific heat	c_p	$L^2\,T^{-2}\,\theta^{-1}$
7	Thermal conductivity	k	$MLT^{-3}\,\theta^{-1}$

Total number of variables, $n = 7$

Fundamental dimensions in the problem are M, L, T, θ and hence $m = 4$

Number of dimensionless π-terms $= (n - m) = 7 - 4 = 3$

The eqn. (*ii*) may be written as:

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

We choose h, ρ, D, V as the core group (repeating variables) with unknown exponents. The groups to be formed are now represented as the following π groups.

$$\pi_1 = h^{a_1} . \rho^{b_1} . D^{c_1} . V^{d_1} . \mu$$

$$\pi_2 = h^{a_2} . \rho^{b_2} . D^{c_2} . V^{d_2} . c_p$$

$$\pi_3 = h^{a_3} . \rho^{b_3} . D^{c_3} . V^{d_3} . k$$

π_1–term:

$$M^0L^0T^0 = (MT^{-3}\,\theta^{-1})^{a_1} . (ML^{-3})^{b_1} . (L)^{c_1} . (LT^{-1})^{d_1} . (ML^{-1}\,T^{-1})$$

Equating the exponents of M, L, T and θ respectively, we get

For M : $0 = a_1 + b_1 + 1$

For L : $0 = -3b_1 + c_1 + d_1 - 1$

For T : $0 = -3a_1 - d_1 - 1$

For θ : $0 = -a_1$

Solving the above equations, we have

$$a_1 = 0, b_1 = -1, c_1 = -1, d_1 = -1$$

$\therefore$ $$\pi_1 = \rho^{-1} . D^{-1} . V^{-1} . \mu$$

or, $$\pi_1 = \frac{\mu}{\rho DV}$$

π_2–term:

$$M^0L^0T^0 = (MT^{-3}\,\theta^{-1})^{a_2} . (ML^{-3})^{b_2} . (L)^{c_2} . (LT^{-1})^{d_2} . (L^2\,T^{-2}\,\theta^{-1})$$

For M : $0 = a_2 + b_2$

For L : $0 = -3b_2 + c_2 + d_2 + 2$

4. It enables getting up a theoretical solution in a simplified dimensionless form.
5. Dimensionless analysis provides partial solutions to the problems that are too complex to be dealt with mathematically.
6. Dimensional analysis is a useful tool in the analysis and correlation of experimental data, in the planning of experiments and in the formulation of empirical correlation describing a particular phenomenon.
7. The results of one series of tests can be applied, with the help of dimensional analysis, to a large number of other similar problems.

Limitations :

1. Dimensional analysis does not give any clue regarding the selection of variables. If the variables are wrongly taken, the resulting functional relationship is erroneous. It provides the information about the grouping of variables. In order to decide whether selected variables are pertinent or superfluous experiments have to be performed.
2. The complete information is not provided by dimensional analysis; it only indicates that there is some relationship between the parameters. It does not give the values of coefficients in the functional relationship. The values of coefficients and hence the nature of functions can be obtained only from experiments or from mathematical analysis.
3. No information is given about the internal mechanism of the physical phenomenon.
4. Generally it is desired to find the effect of one physical quantity upon a number of other physical quantities that are supposed to enter into a problem. It is not possible to get such information with the help of dimensional analysis.

6.8. DIMENSIONLESS NUMBERS AND THEIR PHYSICAL SIGNIFICANCE

1. Reynolds number (*Re*) :

It is defined as the *ratio of the inertia force to the viscous force*.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho U^2 L^2}{\mu UL} = \frac{\rho UL}{\mu} = \frac{UL}{\nu} \qquad ...(6.12)$$

- Reynolds number signifies the relative predominance of the inertia to the viscous forces occuring in the flow systems.
- The higher the value of *Re* the greater will be the relative contribution of inertia effect. The smaller the value of *Re*, the greater will be the relative magnitude of the viscous stresses.
- Reynolds number is taken as an important criterion of kinematic and dynamic similarities in forced convection heat transfer.

2. Prandtl number (Pr) :

It is the *ratio of kinematic viscosity* (ν) *to thermal diffusivity* (α).

$$Pr = \frac{\mu c_p}{k} = \frac{\rho \nu c_p}{k} = \frac{\nu}{(k/\rho c_p)} = \frac{\nu}{\alpha} \qquad ...(6.13)$$

Kinematic viscosity indicates the impulse transport through molecular friction whereas thermal diffusivity indicates the heat energy transport by conduction process.

- Prandtl number provides a measure of the relative effectiveness of the momentum and energy transport by diffusion.
- Prandtl number is a connecting link between the velocity field and temperature field, and its value strongly influences relative growth of velocity and thermal boundary layers.

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This primarily holds when it is *required to find the pressure drop*. However, it is also used in *problems of heat transfer by convection*, because of the existing similarity between momentum transfer and heat transfer.

The equivalent diameter or characteristic length of few geometries are given below:

1. *For rectangular duct* [Fig. 6.1 (*i*)]:

$$D_e = \frac{4A_c}{P} = \frac{4lb}{2\,(l+b)} = \frac{2lb}{l+b} \quad \text{...(6.24)}$$

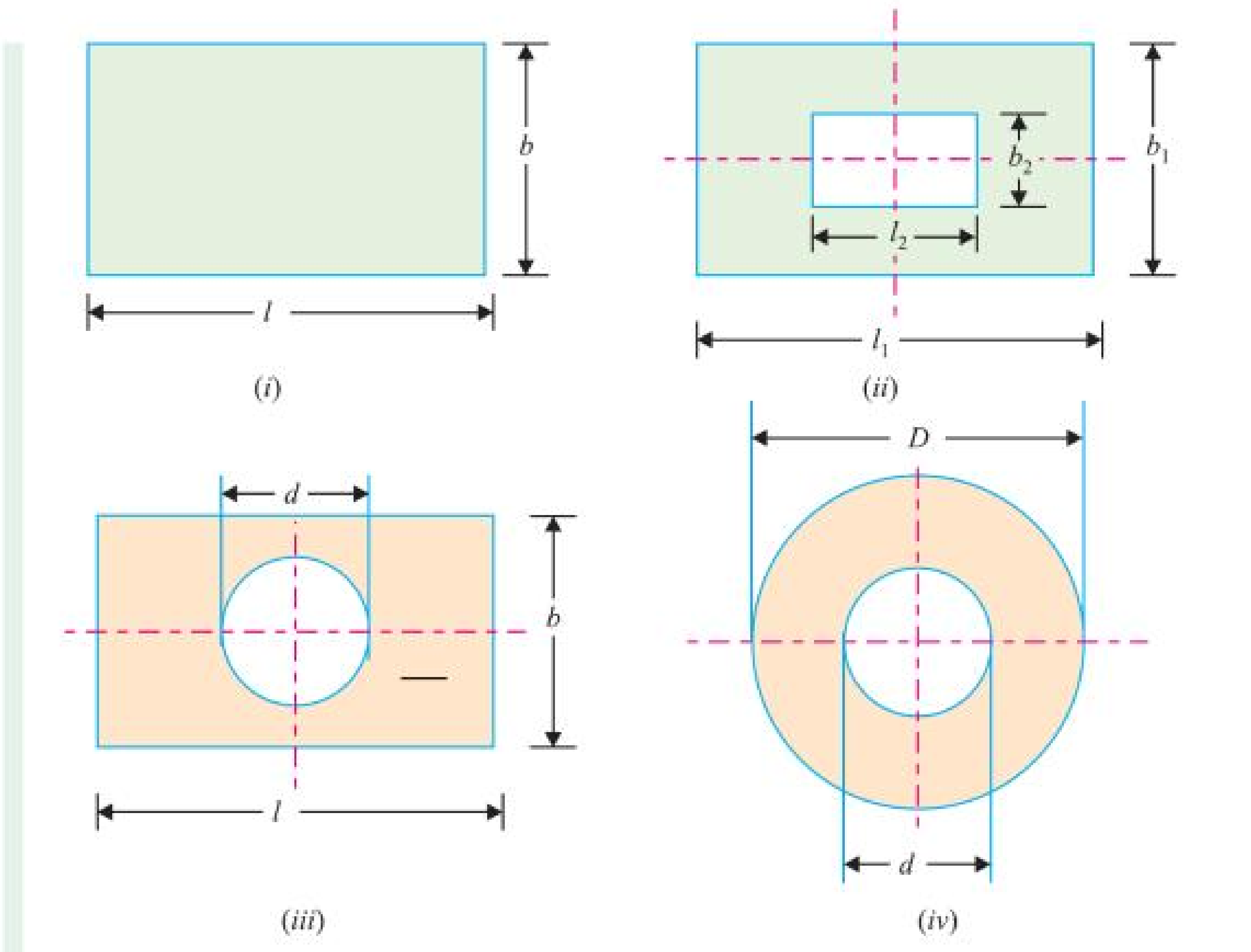

Fig. 6.1. Equivalent diameter (D_e) for various geometries.

2. *For rectangular annulus* [Fig. 6.1 (*ii*)] :

$$D_e = \frac{4A_c}{P} = \frac{4 \times (l_1 b_1 - l_2 b_2)}{2\,[(l_1 + b_1) + (l_2 + b_2)]} = \frac{2\,(l_1 b_1 - l_2 b_2)}{[(l_1 + b_1) + (l_2 + b_2)]} \quad \text{...(6.25)}$$

When $l_1 = b_1$ and $l_2 = b_2$,

$$D_e = \frac{2\,(l_1^2 - l_2^2)}{2l_1 + 2l_2} = (l_1 - l_2) \quad \text{...(6.26)}$$

3. *For annulus* [Fig. 6.1 (*iii*)]

$$D_e = \frac{4A_c}{P} = \frac{4\left(lb - \frac{\pi}{4} d^2\right)}{[2\,(l+b) + \pi d]} \quad \text{...(6.27)}$$

4. *For annulus* [Fig. 6.1 (*iv*)]

$$D_e = \frac{4A_c}{P} = \frac{4 \times \frac{\pi}{4}\,(D^2 - d^2)}{\pi\,(D+d)} = (D - d) \quad \text{...(6.28)}$$

(*i*) A thin layer adjoining the boundary called the *boundary layer where the viscous shear takes place.*

(*ii*) A region outside the boundary layer where the flow behaviour is quite like that of an *ideal fluid and the potential flow theory is applicable.*

7.1.1.1 Boundary Layer Definitions and Characteristics

Consider the boundary layer formed on a flat plate kept parallel to flow of fluid of velocity U (Fig. 7.1) (Though the growth of a boundary layer depends upon the *body shape*, flow over a flat plate aligned in the direction of flow is considered, since most of the flow surface can be *approximated to a flat plate and for simplicity*).

- The edge facing the direction of flow is called *leading edge*.
- The rear edge is called the *trailing edge*.
- Near the leading edge of a flat plate, the boundary layer is *wholly laminar*. For a laminar boundary layer the velocity distribution is *parabolic*.
- The thickness of the boundary layer (δ) increases with distance from the leading edge x, as more and more fluid is slowed down by the viscous boundary, becomes unstable and breaks into turbulent boundary layer over a transition region.

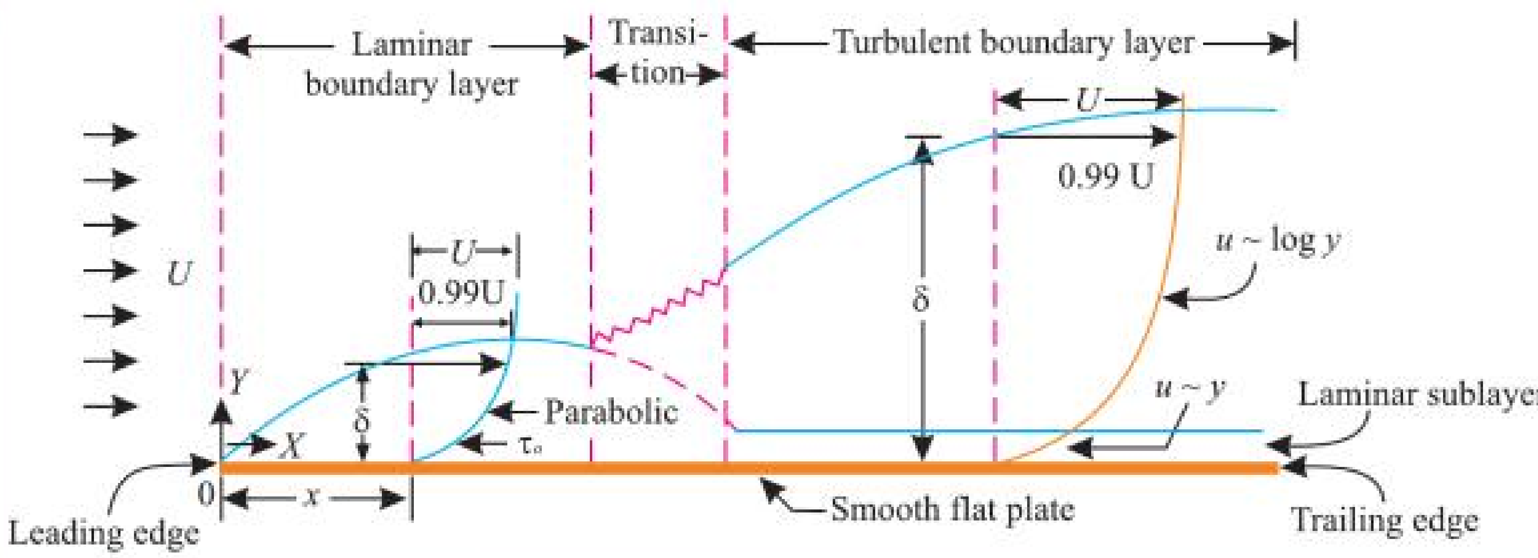

Fig. 7.1. Boundary layer on a flat plate.

For a turbulent boundary layer, if the boundary is smooth, the roughness projections are covered by a very thin layer which remains laminar, called *laminar sublayer*. The velocity distribution in the turbulent boundary layer is given *by Log law of Prandtl's one-seventh power law*.

The *characteristics* of a boundary layer may be summarised as follows :

(*i*) δ (thickness of boundary layer) increases as distance from leading edge x increases.

(*ii*) δ decreases as U increases.

(*iii*) δ increases as kinematic viscosity (ν) increases.

(*iv*) $\tau_0 = \mu\left(\frac{U}{\delta}\right)$; hence τ_0 decreases as x increases. However, when boundary layer becomes turbulent, it shows a sudden increase and then decreases with increasing x.

(*v*) When U decreases in the downward direction, boundary layer growth is reduced.

(*vi*) When U decreases in the downward direction, flow near the boundary is further retarded, boundary layer growth is faster and is susceptible to separation.

(*vii*) The various characteristics of the boundary layer on flat plate (*e.g.*, variation of δ, τ_0 or force F) are governed by inertial and viscous forces ; hence they are functions of either $\frac{Ux}{\nu}$ or $\frac{UL}{\nu}$.

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The change of momentum of the mass m_y along Y-direction is given by

$$dM_y = m_y\left[\left(u + \frac{\partial u}{\partial y}\,.\,dy\right) - u\right] = m_y\left(\frac{\partial u}{\partial y}\,.\,dy\right)$$

$$= \rho \upsilon \frac{\partial u}{\partial y}\,.\,dx.dy \qquad ...(7.7)$$

Total viscous force along the X-direction is given by

$$F_x = [(\tau + \delta\tau) - \tau] \times \text{area}$$

$$= \left[\left\{\mu\,.\,\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(u\,.\,\frac{\partial u}{\partial y}\right).\,dy\right\} - \mu\frac{\partial u}{\partial y}\right](dx \times 1)$$

$$= \mu\frac{\partial^2 u}{\partial y^2}\,.\,dx\,.\,dy \qquad ...(7.8)$$

Assuming the gravitational forces are balanced by buoyancy forces for equilibrium of the element, we have

Inertia forces = Viscous forces

$$\therefore \quad \rho u\frac{\partial u}{\partial x}\,.dx.dy + \rho\upsilon\frac{\partial u}{\partial y}\,.dx.dy = \mu\frac{\partial^2 u}{\partial y^2}\,.dx.dy$$

or,
$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\,.\,\frac{\partial^2 u}{\partial y^2}$$

or,
$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad ...(7.9)$$

(substituting $\nu = \frac{\mu}{\rho}$)

Equation (7.9) is known as **equation of motion or momentum equation** *for hydrodynamic boundary layer.*

7.1.3. BLASIUS EXACT SOLUTION FOR LAMINAR BOUNDARY LAYER FLOWS

The velocity distribution in the boundary layer can be obtained by solving the equation of motion for hydrodynamic boundary layer [Eqn. (7.9)]. The following boundary conditions should be satisfied.

(*i*) At $y = 0$, $u = 0$ (*ii*) At $y = 0$, $\upsilon = 0$ (*iii*) At $y = \infty$, $u = U$

The Blasius technique for an exact solution of the hydrodynamic boundary layer lies in the conversion of the following differential equations into a single differential equation.

The hydrodynamic equation for boundary layer : $u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$ ...(Eqn. 7.9)

Continuity equation : $\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0$ ...(7.10)

Prandtl suggested that the solution of Eqn. (7.9) can be obtained by reducing the number of variables with the help of magnitude analysis of the boundary layer thickness and transforming the partial differential equation into ordinary differentials.

The *inertia forces* represented by the left terms, in the Eqn. (7.9), must be balanced by the viscous forces represented by the right terms.

As $u \geq \upsilon$, therefore, we may write as

$$\rho u \frac{\partial u}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \qquad \left(\because v = \frac{\mu}{\rho} \right)$$

Also as $u \propto U$ and $\frac{\partial u}{\partial x} \propto \frac{U}{L}$, along a plate length L, therefore, we have

$$\frac{\rho U^2}{L} = \mu \frac{U}{\delta^2}$$

$$\therefore \qquad \delta = \sqrt{\frac{\mu L}{\rho U}} = \sqrt{\frac{vL}{U}} = \sqrt{\frac{vx}{U}} \qquad ...(7.11)$$

From experiments it has been observed that velocity profiles at different locations along the plate are geometrically similar, *i.e.*, they differ only by a **stretching factor** in the Y-direction. This implies that the dimensionless velocity $\frac{u}{U}$ can be expressed at any location x as a function of the dimensionless distance from the wall $\frac{y}{\delta}$.

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \qquad ...(7.12)$$

Substituting the value of δ from eqn. (7.11) in eqn. (7.12), we obtain,

$$\frac{u}{U} = f\left[\frac{y}{\sqrt{x}}\sqrt{\frac{U}{v}}\right] = f(\eta) \qquad ...(7.13)$$

where, $\eta = y\sqrt{\frac{U}{vx}}$ denotes the *stretching factor*.

In order to account for the fact that the vertical component of velocity occurs in the boundary layer equation of motion (7.9), it is essential to define a **stream function** ψ such that,

$$\frac{\psi}{U} = \left[\sqrt{\frac{vx}{U}}\right] f(\eta) \qquad ...(7.14)$$

$$\text{or,} \qquad \psi = \sqrt{vxU}\, f(\eta) \qquad ...[7.14\ (a)]$$

The continuous stream function ψ is the mathematical postulation such that its partial differential with respect to x gives the velocity in the Y-direction (generally taken as negative) and its partial differential with respect to y gives the velocity in the X-direction :

$$u = \frac{\partial \psi}{\partial y}; \qquad \upsilon = \frac{\partial \psi}{\partial x}$$

$$\therefore \qquad u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta} \times \frac{\partial \eta}{\partial y} = \frac{\partial}{\partial \eta}\left[U\sqrt{\frac{vx}{U}} f(\eta)\right] \times \frac{\partial}{\partial y}\left[y\sqrt{\frac{U}{vx}}\right]$$

$$\text{or,} \qquad u = U\sqrt{\frac{vx}{U}} \frac{df}{d\eta}\left[\sqrt{\frac{U}{vx}}\right] = U\frac{df}{d\eta} \qquad ...(7.15)$$

Here f is abbreviated as $f(\eta)$

$$\therefore \qquad \frac{\partial u}{\partial x} = U\frac{\partial}{\partial x}\left(\frac{df}{d\eta}\right) = U\frac{\partial}{\partial \eta}\left(\frac{df}{d\eta}\right)\frac{\partial \eta}{\partial x} = -U\frac{d^2 f}{d\eta^2}.\frac{1}{2x}.\, y\sqrt{\frac{U}{vx}}$$

$$\text{or,} \qquad \frac{\partial u}{\partial x} = -U\frac{\eta}{2x}\frac{d^2 f}{d\eta^2} \qquad ...(7.16)$$

3.6	1.9294	0.9233	0.0981	0.6972
4.0	2.3058	0.9555	0.0642	0.7582
4.4	2.6924	0.9759	0.0390	0.8007
4.8	3.0853	0.9878	0.0219	0.8280
5.0	3.2833	0.9915	0.0159	0.8372
5.2	3.4820	0.9942	0.0113	0.8441
5.6	3.8803	0.9975	0.0054	0.8528
6.0	4.2796	0.9990	0.0024	0.8571
6.4	4.6794	0.9996	0.0010	0.8591
6.8	5.0793	0.9999	0.0003	0.8599
7.2	5.4792	1.0000	0.0001	0.8602
7.6	5.8792	1.0000	0.0000	0.8603
8.0	6.2792	1.0000	0.0000	0.8604
8.4	6.6792	1.0000	0.0000	0.8604

2. The graph/curve I (*i.e.*, the velocity distribution parallel to the surface) enables us to calculate the parameters : (*i*) Boundary layer thickness, δ and (*ii*) skin friction coefficient, C_f.

(*i*) *Boundary layer thickness*, δ :

The boundary layer thickness δ is taken to be the distance from the plate surface to a point at which the velocity is within 1% of the asymptotic limit, *i.e.*, $\frac{u}{U} = 0.99$; it occurs at $\eta = 5.0$ (Fig. 7.4). Therefore, the value of η at the edge of boundary layer ($y = \delta$) is given by

$$\eta = y\sqrt{\frac{U}{\nu x}} = \delta\sqrt{\frac{U}{\nu x}} = 5$$

or,
$$\frac{\delta}{x} = 5\sqrt{\frac{\nu}{Ux}} = \frac{5}{\sqrt{Re_x}} \qquad \text{...(7.22)}$$

where $Re_x = \frac{Ux}{\nu}$ is the *local* Reynolds number based on distance x from the leading edge of the plate.

(*ii*) *Skin friction coefficient* ; C_f:

The skin friction coefficient (C_f) is defined *as the ratio of shear stress* τ_0 *at the plate to the dynamic head* $\frac{1}{2}\rho U^2$ *caused by free stream velocity*. Thus the local skin friction coefficient C_{fx} at any value of x is

$$C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho U^2} = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{1}{2}\rho U^2} \qquad \text{...(7.23)}$$

From Fig. 7.4, the gradient at $\eta = 0$ is

$$\left[\frac{\partial(u/U)}{d\eta}\right]_{\eta=0} = 0.332$$

or,
$$\frac{1}{U}\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0.332$$

Solution. Velocity distribution : $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Velocity of air, $U = 12$ m/s

Length of plate, $L = 1.1$ m

Width of plate, $B = 0.9$ m

Reynolds number upto which laminar boundary exists, $Re = 2 \times 10^5$

Kinematic viscosity of air, $\nu = 0.15$ stokes $= 0.15 \times 10^{-4}$ m²/s

(*i*) The maximum distance from the leading edge upto which laminar boundary layer exists, x:

$$Re_x = \frac{Ux}{\nu} \quad \text{or} \quad 2 \times 10^5 = \frac{12 \times x}{0.15 \times 10^{-4}}$$

or,
$$x = \frac{2 \times 10^5 \times 0.15 \times 10^{-4}}{12} = \mathbf{0.25\,m} \ \textbf{(Ans.)}$$

(*ii*) The maximum thickness of boundary layer, δ :

For the given velocity profile, the maximum thickness of boundary layer is given by

$$\delta = \frac{5.48\ x}{\sqrt{Re_x}}$$

$$= \frac{5.48 \times 0.25}{\sqrt{2 \times 10^5}} = 0.00306\,\text{m} \ \text{ or } \ \mathbf{3.06\,mm} \ \textbf{(Ans.)}$$

Example 7.6. *A plate of length 750 mm and width 250 mm has been placed longitudinally in a stream of crude oil which flows with a velocity of 5 m/s. If the oil has a specific gravity of 0.8 and kinematic viscosity of 1 stoke, calculate :*

(*i*) *Boundary layer thickness at the middle of plate,*

(*ii*) *Shear stress at the middle of plate, and*

(*iii*) *Friction drag on one side of the plate.*

Solution. Length of the plate, $L = 750$ mm $= 0.75$ m

A cooling system shows an example of air filters used to create a laminar air flow.

But, $$C_{fx} = \frac{0.654}{Re_x} = \frac{0.654}{\sqrt{\frac{Ux}{\nu}}} = \frac{0.654}{\sqrt{\frac{4 \times 0.25}{0.15 \times 10^{-4}}}} = 0.002533$$

∴ $$(\tau_0)_{x = 0.25\text{m}} = 0.002533 \times \frac{1.24 \times 4^2}{2} = \mathbf{0.025\ N/m^2\ (Ans.)}$$

(*iii*) Drag force on one side of the plate, F_D :

$$F_D = \overline{C}_f \times \frac{1}{2} \rho A U^2$$

where, $$\overline{C}_f = \frac{1.31}{\sqrt{Re_L}} = \frac{1.31}{\sqrt{1.33 \times 10^5}} = 0.003592$$

and A = area of the plate = $L \times B = 0.5 \times 0.6 = 0.3\ \text{m}^2$

∴ $$F_D = 0.003592 \times \frac{1}{2} \times 1.24 \times 0.3 \times 4^2 = \mathbf{0.01069\ N\ (Ans.)}$$

7.1.5. THERMAL BOUNDARY LAYER

Whenever a flow of fluid takes place past a heated or cold surface, a temperature field is set up in the field next to the surface. If the surface of the plate is hotter than fluid, the temperature distribution will be as shown in the Fig. 7.6. *The zone or this layer wherein the temperature field exists is called the* **thermal boundary layer**. Due to the exchange of heat between the plate and the fluid, temperature gradient occurs/results.

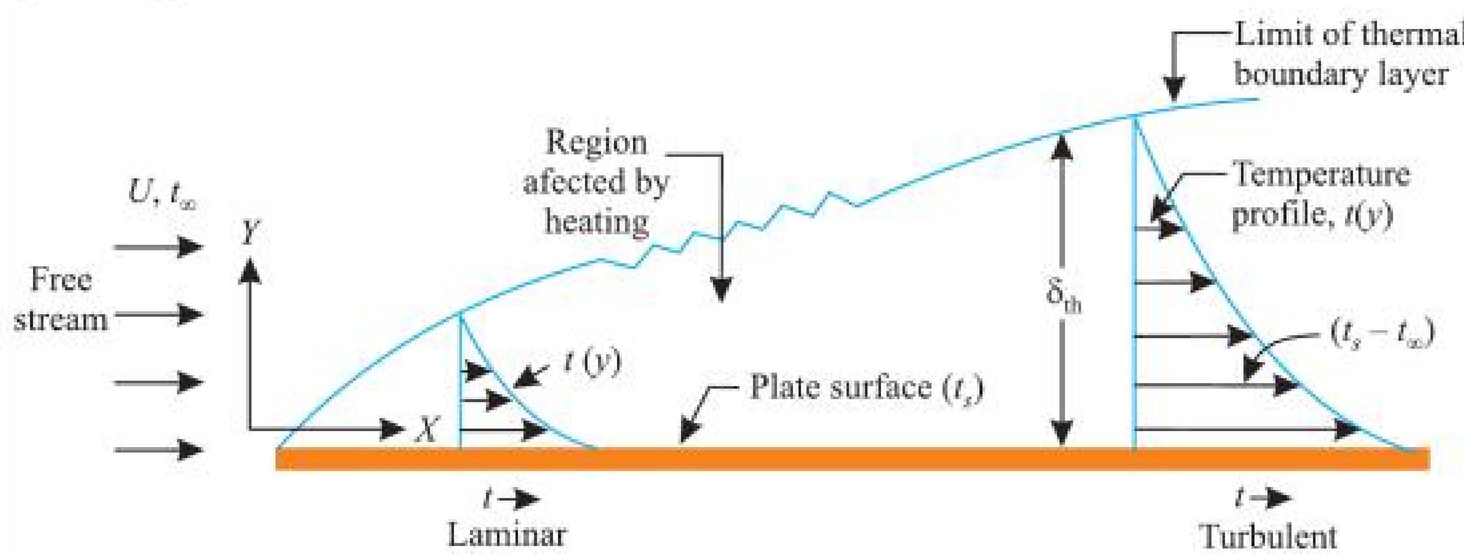

Fig. 7.6. Thermal boundary layer formed during flow of cool fluid over a warm plate.

The thermal boundary layer thickness (δ_{th}) is arbitrarily defined as the distance y from the plate surface at which

$$\frac{t_s - t}{t_s - t_\infty} = 0.99 \qquad ...(7.40)$$

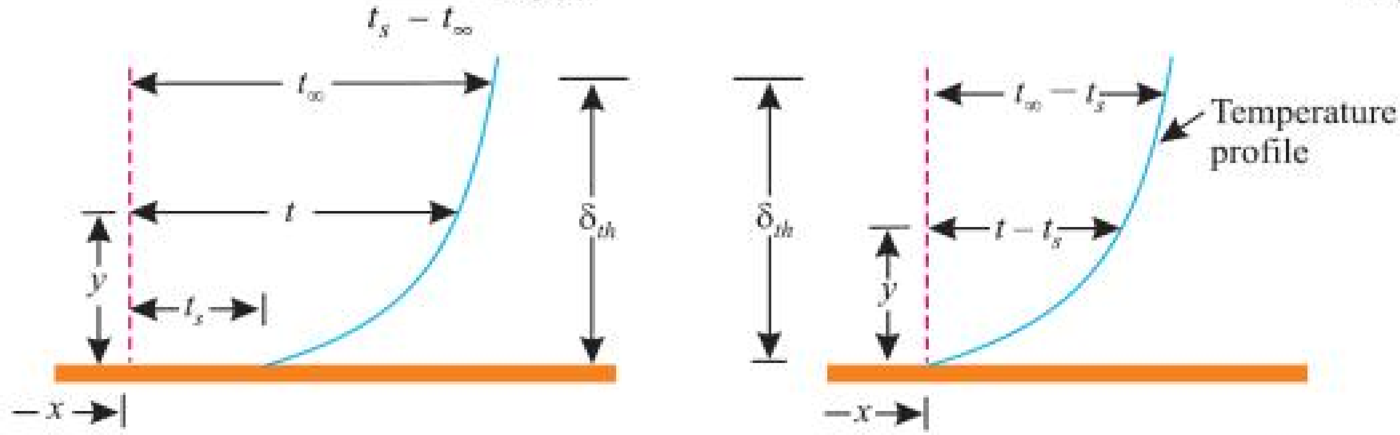

Fig. 7.7. Thermal boundary layer formed flow of warm fluid over a cool plate.

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Viscous heating of fluid dampers.

$$\theta = \frac{t_s - t}{t_s - t_\infty} = f(\eta) = f\left[y\sqrt{\frac{U}{vx}} \right] \quad ...(7.49)$$

Also, the values of the velocity components u and υ already calculated earlier are :

$$u = U\frac{df}{d\eta} \quad ...[\text{Eqn. } (7.15)]$$

$$\upsilon = \left[\frac{y}{2x} U \frac{df}{d\eta} - \frac{1}{2}\sqrt{\frac{Uv}{x}} f(\eta) \right] \quad ...[\text{Eqn. } (7.19)]$$

Further, from temperature parameter θ (non-dimensional) defined above, we have

$$t = t_s + (t_\infty - t_s)\theta$$

$$\frac{\partial t}{\partial x} = (t_\infty - t_s)\frac{\partial \theta}{\partial x} = (t_\infty - t_s)\frac{\partial \theta}{\partial \eta} \times \frac{\partial \eta}{\partial x}$$

or,

$$\frac{\partial t}{\partial x} = (t_\infty - t_s)\left[-\frac{y}{2x^{3/2}}\sqrt{\frac{U}{v}} \right]\frac{\partial \theta}{\partial \eta} \quad ...(7.50)$$

and,

$$\frac{\partial t}{\partial y} = (t_\infty - t_s)\frac{\partial \theta}{\partial y} = (t_\infty - t_s)\frac{\partial \theta}{\partial \eta} \times \frac{\partial \eta}{\partial y}$$

or,

$$\frac{\partial t}{\partial y} = (t_\infty - t_s)\sqrt{\frac{U}{vx}}\frac{d\theta}{d\eta} \quad ...(7.51)$$

Also,

$$\frac{\partial^2 t}{\partial y^2} = \frac{\partial}{\partial y}\left[(t_\infty - t_s)\sqrt{\frac{U}{vx}}\frac{d\theta}{d\eta} \right]$$

$$= (t_\infty - t_s)\sqrt{\frac{U}{vx}}\frac{d}{d\eta}\left(\frac{d\theta}{d\eta} \right)\frac{d\eta}{dy}$$

$$= (t_\infty - t_s)\sqrt{\frac{U}{vx}}\frac{d^2\theta}{d\eta^2}\sqrt{\frac{U}{vx}}$$

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... (In non-dimensional form)

[where, h_x = Local convective heat transfer coefficient, and

Nu_x = Local value of Nusselt number (at a distance x from the leading edge of the plate,].

The *average* heat transfer coefficient is given by

$$\bar{h} = \frac{1}{L}\int_0^L h_x \, . \, dx = \frac{1}{L}\int_0^L 0.332\frac{k}{x}(Re_x)^{1/2}(Pr)^{1/3}\, dx$$

$$= \frac{1}{L}\int_0^L 0.332\, k\,(Pr)^{1/3}\sqrt{\left(\frac{U}{\nu}\right)}\, x^{-1/2}\, dx$$

or, $$\bar{h} = 0.664\left(\frac{k}{L}\right)(Re_L)^{1/2}(Pr)^{1/3} \quad ...(7.66)$$

If we compare the eqns. (7.64) and (7.66), we find that

$$\bar{h} = 2h_x \quad ...(7.67)$$

and $\overline{Nu}$ (average value of Nusselt number) $= \frac{\bar{h}L}{k} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3}$ ...(7.68)

All the results in eqns. (7.64), (7.65) and (7.68) are valid for $Pr > 0.5$.

7.1.7. INTEGRAL ENERGY EQUATION (APPROXIMATE SOLUTION OF ENERGY EQUATION)

Consider a control volume shown in Fig. 7.10. *Assume* that ρ, c_p and k (thermo-plastic properties) of fluid remain constant within the operating range of the temperature, and the heating of the plate commences at a distance x_0 from the leading edge of the plate (so that the boundary layer initiates at $x = x_0$ and develops and grows beyond that). For *unit width* of the plate we have :

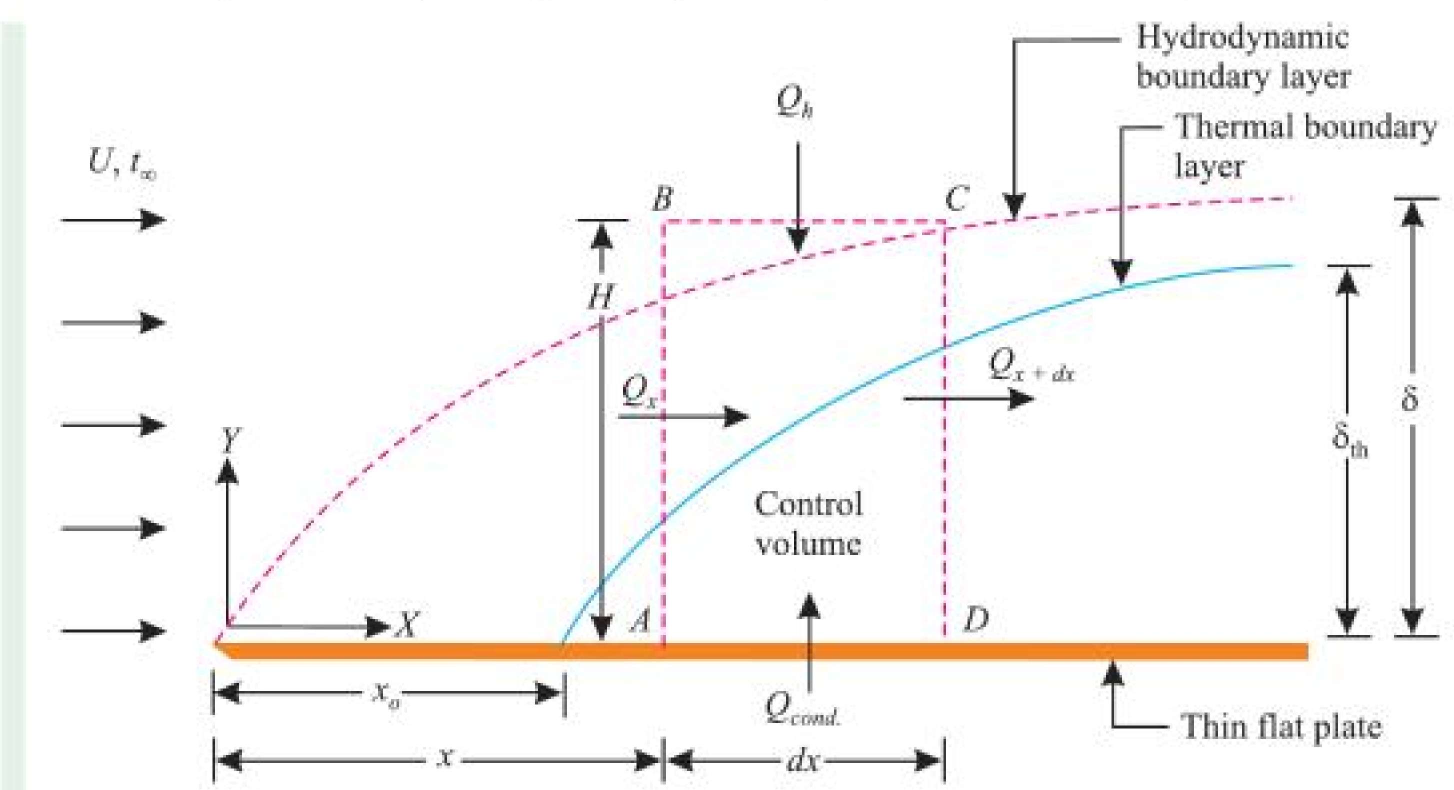

Fig. 7.10. Integral energy equation – control volume.

Mass of fluid entering through face $AB = \int_0^H \rho u\, dy$ ...(7.69)

Mass of fluid leaving through face $CD = \int_0^H \rho u\, dy + \frac{\partial}{\partial x}\left[\int_0^H \rho u\, dy\right]dx$ ...(7.70)

∴ Mass of fluid entering the control volume through face BC

Local heat transfer coefficient h_x :

We know that, $$\frac{Q}{A} = h_x (t_s - t_\infty) = -k\left(\frac{dt}{dy}\right)_{y=0}$$

or, $$h_x = \frac{-k(dt/dy)_{y=0}}{t_s - t_\infty}$$

But, $$\left(\frac{dt}{dy}\right)_{y=0} = \frac{3}{2}\left(\frac{t_s - t_\infty}{\delta_{th}}\right) \quad \text{...[Eqn. (7.83)]}$$

$\therefore$ $$h_x = \frac{-k \times \frac{3}{2}\left(\frac{t_s - t_\infty}{\delta_{th}}\right)}{(t_s - t_\infty)} = \frac{3k}{2\delta_{th}} = \frac{3k}{2} \times \frac{1}{r\delta} \quad \text{...(7.90)}$$

Substituting $r = \frac{0.975}{(Pr)^{1/3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$ and $\delta = \frac{4.64\,x}{\sqrt{Re_x}}$ in eqn. (7.90), we get

$$h_x = \frac{3k}{2} \times \frac{(Pr)^{1/3}}{0.975\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \times \frac{\sqrt{Re_x}}{4.64\,x}$$

or, $$h_x = 0.332\,\frac{k}{x}(Pr)^{1/3}\,(Re)^{1/2} \times \frac{1}{\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \quad \text{...(7.91)}$$

or, $$Nu_x = \frac{h_x x}{k} = \frac{0.332\,(Pr)^{1/3}\,(Re_x)^{1/2}}{\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}} \quad \text{...(7.92)}$$

When the plate is heated over the whole length *i.e.*, $x_0 = 0$, we have

$$h_x = 0.332\frac{k}{x}(Pr)^{1/3}\,(Re_x)^{1/2} \quad \text{...(7.93)}$$

and, $$Nu_x = 0.332\,(Pr)^{1/3}\,(Re_x)^{1/2} \quad \text{...(7.94)}$$

The above results are applicable for *laminar conditions only*.

Example 7.10. *Air at 20°C and at a pressure of 1 bar is flowing over a flat plate at a velocity of 3 m/s. If the plate is 280 mm wide and at 56°C, calculate the following quantities at x = 280 mm, given that properties of air at the bulk mean temperature* $\left(\frac{20 + 56}{2}\right)$ *= 38°C are :*

ρ = 1.1374 kg/m^3; k = 0.02732 W/m°C; c_p = 1.005 kJ/kgK; ν = 16.768 × 10^{-6} m^2/s; Pr = 0.7.

(*i*) *Boundary layer thickness,*
(*ii*) *Local friction coefficient,*
(*iii*) *Average friction coefficient,*
(*iv*) *Shearing stress due to friction,*
(*v*) *Thickness of the boundary layer,*
(*vi*) *Local convective heat transfer coefficient,*
(*vii*) *Average convective heat transfer coefficient,*
(*viii*) *Rate of heat transfer by convection,*
(*ix*) *Total drag force on the plate, and*
(*x*) *Total mass flow rate through the boundary.*

$$m_x = \int_0^{\delta} (dy \times 1) u \,.\, \rho = \int_0^{\delta} \rho u \; dy$$

Now, $$u = U \left[\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right]$$ ...*assumed*

$\therefore$ $$m_x = \int_0^{\delta} \rho U \left[\frac{3}{2} \frac{y}{\delta} - \frac{1}{2} \left(\frac{y}{\delta} \right)^3 \right] dy$$

$$= \rho U \left[\frac{3}{4} \left(\frac{y^2}{\delta} \right) - \frac{1}{8} \left(\frac{y^4}{\delta^3} \right) \right]_0^{\delta} = \frac{5}{8} \rho U \delta$$

$$= \frac{5}{8} \times 1.16 \times 2 \times 0.00857 = \mathbf{0.01242\,kg/s} \quad \textbf{(Ans.)}$$

Note. If the mass added in the boundary is to be calculated when the fluid moves from x_1 to x_2 along the main flow direction, then it is given by

$$\Delta m = \frac{5}{8} \rho U \; (\delta_2 - \delta_1)$$

where δ_1 and δ_2 are the boundary layer thicknesses at x_1 and x_2 .

(*ii*) Heat transferred per hour, *Q* :

$$\overline{Nu} = \frac{\bar{h} L}{k} = 0.664 \, Re^{1/2} \, Pr^{1/3}$$

$$\bar{h} = \frac{k}{L} = 0.664 \, Re^{1/2} \, Pr^{1/3}$$

$$= \frac{0.02749}{0.4} \times 0.664 \times (46869)^{1/2} \times (0.7)^{1/3} = 8.77 \, W/\text{m}^2\,°\text{C}$$

$$Q = \bar{h} A (t_s - t_\infty)$$

$$= 8.77 \times (0.4 \times 1) \, (60 - 27) = 115\,.76 \text{ J/s}$$

$$= \frac{115.76 \times 3600}{1000} = \mathbf{416.74\,kJ/h} \quad \textbf{(Ans.)}$$

Example 7.14. *Air at 1 bar and at a temperature of 30°C (μ = 0.06717 kg/hm) flows at a speed of 1.2 m/s over a flat plate. Determine the boundary layer thickness at distance of 250 mm and 500 mm from the leading edge of the plate. Also, calculate the mass entrainment between these two sections. Assume the parabolic velocity distribution as :* $\frac{u}{U} = \frac{3}{2} \left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^3$.

Solution. *Given :* $t_\infty = 30°C$, μ = 0.06717 kg/hm, U = 1.2 m/s

Boundary layer thicknesses :

The density of air, $$\rho = \frac{p}{RT} = \frac{1 \times 10^5}{287 \times (30 + 273)} = 1.15 \text{ kg/m}^3$$

At x = 250 *mm* = 0.25 *m*

Reynolds number $$Re_x = \frac{\rho U x}{\mu} = \frac{1.15 \times 1.2 \times 0.25 \times 3600}{0.06717} = 18490$$

$\therefore$ Boundary layer thickness, $$\delta_1 = \frac{4.64 \, x}{\sqrt{Re_x}}$$ ...[Eqn. (7.36)]

or, $$\delta_1 = \frac{4.64 \times 0.25}{\sqrt{18490}} = 0.00853 \text{ m or } \mathbf{8.53\ mm} \quad \textbf{(Ans.)}$$

(i) Using exact solution :

The average Nusselt number is given by

$$\overline{Nu} = 0.664\,(Re_L)^{1/2}\,(Pr)^{1/3} \qquad \text{...[Eqn. (7.68)]}$$

or, $$\frac{\bar{h}L}{k} = 0.664\,(2.087 \times 10^5)^{1/2}\,(0.696)^{1/3} = 268.82$$

or, $$\bar{h} = \frac{268.82\,k}{L} = \frac{268.82 \times 0.02894}{2.2} = 3.536 \text{ W/m}^2{}^\circ\text{C}$$

∴ Heat transfer rate from the plate,

$$Q = \bar{h}\,A_s\,(t_s - t_\infty) = 3.536 \times (2.2 \times 1)\,(100 - 20) = \mathbf{622.34\ W\ (Ans.)}$$

(ii) Using approximate solution :

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.646\,(Re_L)^{1/2}\,(Pr)^{1/3}$$

or, $$\frac{\bar{h}\,L}{k} = 0.646\,(2.087 \times 10^5)^{1/2}\,(0.696)^{1/3} = 261.53$$

or, $$\bar{h} = \frac{261.53\,k}{L} = \frac{261.53 \times 0.02894}{2.2} = 3.44 \text{ W/m}^2\,{}^\circ C$$

∴ Heat transfer rate from the plate,

$$Q = \bar{h}\,A_s\,(t_s - t_\infty) = 3.44 \times (2.2 \times 1) \times (100 - 20) = \mathbf{605.44\ W}\quad \mathbf{(Ans.)}$$

Example 7.19. *Air at a temperature of 30°C flows past a flat plate at a velocity of 1.8 m/s. The flat surface has a sharp leading edge and its total length equals 750 mm. Calculate :*

(i) *The average skin friction or drag coefficient,*

(ii) *The average shear stress, and*

(iii) *The ratio of the average shear stress to the shear stress at the trailing edge.*

Properties of air at 30°C are : $\rho = 1.165$ *kg/m³*, $\mu = 6.717 \times 10^{-2}$ *kg/hm*, $\nu = 16 \times 10^{-6}$ *m²/s.*

Solution. *Given :* $t_\infty = 30$°C, $U = 1.8$ m/s, $L = 750$ mm $= 0.75$ m, $\rho = 1.165$ kg/m³, $\mu = 6.717 \times 10^{-2}$ kg/hm, $\nu = 16 \times 10^{-6}$ m²/s.

(i) The average skin friction, $\bar{C}_f$:

Reynolds number, $$Re_L = \frac{UL}{\nu} = \frac{1.8 \times 0.75}{1.6 \times 10^{-6}} = 84375$$

Since the Reynolds number is less than 5×10^5, the boundary layer over the entire plate is laminar in nature.

∴ $$\bar{C}_f = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{84375}} = \mathbf{0.004572\ \ (Ans.)} \qquad \text{...[Eqn. (7.25)]}$$

(ii) The average shear stress, τ_w :

$$\tau_w = \frac{1}{2}\rho U^2 \times \bar{C}_f$$

$$= \frac{1}{2} \times 1.165 \times 1.8^2 \times 0.004572 = \mathbf{0.008629\ N/m^2}\quad \mathbf{(Ans.)}$$

(iii) The ratio of average shear stress to the shear stress at the trailing edge :

The skin friction coefficient at the trailing edge ($x = L$)

$$C_f = \frac{0.664}{\sqrt{Re_x}} \qquad \text{...[Eqn. (7.24)]}$$

or, $$C_{fx} = \frac{0.664}{\sqrt{84375}} = 0.002286$$

or, $F_D = 0.01596 \times \frac{1}{2} \times 956.8 \times 0.1^2 \times (4.5 \times 1)$ = **0.3436 N per meter width. (Ans.)**

(*iii*) The local heat transfer coefficient at the trailing edge, h_x (at $x = L$) :

$$Nu_x = \frac{h_x x}{k} = 0.332\,(Re_x)^{1/2}\,(Pr)^{1/3}$$

$$= 0.332 \times (6923)^{1/2}\,(902.77)^{1/3} = 266.98 \qquad \text{...[Eqn. (7.65)]}$$

or, $$h_x = \frac{266.98 \times k}{x} = \frac{266.98 \times 0.213}{4.5} = \mathbf{12.64\ W/m^2\,°C} \quad \textbf{(Ans.)}$$

(*iv*) The heat transfer rate, Q :

$$Q = \bar{h}\,A_s\,(t_s - t_\infty)$$

where, $$\bar{h} = 2h_x = 2 \times 12.64 = 25.28\text{W/m}^2\text{°C} \qquad \text{...[Eqn. (7.67)]}$$

∴ $$Q = 25.28 \times (4.5 \times 1)\,(95 - 25) = \mathbf{7963.2\ W} \quad \textbf{(Ans.)}$$

Example 7.23. *Air at 20°C and at atmospheric pressure flows at a velocity of 4.5 m/s past a flat plate with a sharp leading edge. The entire plate surface is maintained at a temperature of 60°C. Assuming that the transition occurs at a critical Reynolds number of 5×10^5, find the distance from the leading edge at which the flow in the boundary layer changes from laminar to turbulent. At the location, calculate the following* :

(*i*) *Thickness of hydrodynamic layer,*

(*ii*) *Thickness of therml boundary layer,*

(*iii*) *Local and average convective heat transfer coefficients,*

(*iv*) *Heat transfer rate from both sides for, unit width of the plate,*

(*v*) *Mass entrainment in the boundary layer, and*

(*vi*) *The skin friction coefficient.*

Assume cubic velocity profile and approximate method.

The thermo-physical properties of air at mean film temperature (60 + 20)/2 = 40°C are :

$\rho = 1.128$ kg/m^3, $\nu = 16.96 \times 10^{-6}$ m^2/s, $k = 0.02755$ W/m°C, $Pr = 0.699$.

Solution. *Given*: $t_\infty = 20$ °C, $t_s = 60$°C, $U = 4.5$ m/s.

At the transition point, $Re_c = (Re)_{trans.} = \frac{Ux_c}{\nu}$

or, $$x_c = \frac{Re_c \times \nu}{U} = \frac{5 \times 10^5 \times 16.96 \times 10^{-6}}{4.5} = \mathbf{1.88\ m} \quad \textbf{(Ans.)}$$

(where, x_c = Distance from the leading edge at which the flow in the boundary layer changes *from laminar to turbulent*).

(*i*) Thickness of hydrodynamic layer, δ :

The thickness of hydrodynamic layer for, cubic velocity profile is given by,

$$\delta = \frac{4.64\,x_c}{\sqrt{Re_c}} \qquad \text{...[Eqn. (7.36)]}$$

$$= \frac{4.64 \times 1.88}{\sqrt{5 \times 10^5}} = 0.01234 \text{ m or, } \mathbf{12.34\ mm} \quad \textbf{(Ans.)}$$

(*ii*) Thickness of thermal boundary layer, δ_{th} :

The thermal boundary layer is given by,

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or, $$\tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2} \quad \text{...(7.95)}$$

– Equation (7.95) shows that flow will occur only if *pressure gradient exists in the direction of flow.*

  The *negative sign shows that pressure decreases in the direction of flow.*

– Equation (7.95) indicates that the shear stress varies linearly across the section (see Fig. 7.14). Its value is zero at the centre of pipe ($r = 0$) and maximum at the pipe wall given by

$$\tau_0 = -\frac{\partial p}{\partial x}\left(\frac{R}{2}\right) \quad \text{...[7.95 (a)]}$$

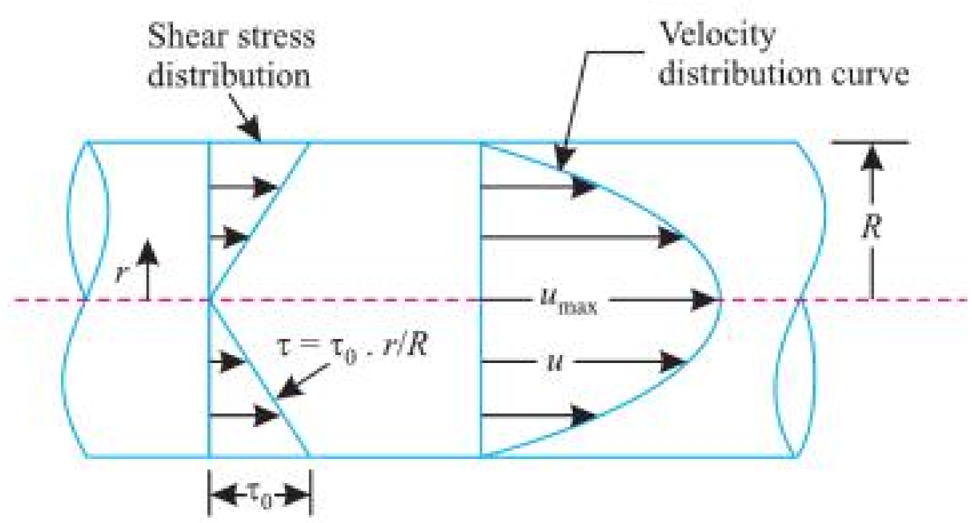

Fig. 7.14. Shear stress and velocity distribution across a section.

From Newton's law of viscosity,

$$\tau = \mu \cdot \frac{du}{dy} \quad \text{...}(i)$$

In this equation, the distance y is measured from the boundary. The radial distance r is related to distance y by the relation

$$y = R - r \text{ or, } dy = -dr$$

The eqn. (i) becomes

$$\tau = -\mu \frac{du}{dr} \quad \text{...(7.96)}$$

Comparing two values of τ from eqns. 7.95 and 7.96, we have

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$$

or, $$du = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) r \cdot dr$$

Integrating the above equation *w.r.t.* 'r', we get

$$u = \frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} r^2 + C \quad \text{...(7.97)}$$

where, C is the constant of integration and its value is obtained from the boundary condition :

At $r = R$, $u = 0$

or, $$\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)=\frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}.\left(r-\frac{r^3}{R^2}\right) \quad ...(7.108)$$

Let us consider the case of uniform heat flux along the wall, where we can take $\frac{\partial t}{\partial x}$ as a constant. Integrating eqn. (7.108) we have

$$r\frac{\partial t}{\partial r}=\frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}.\left(\frac{r^2}{2}-\frac{r^4}{4R^2}\right)+C_1$$

or, $$\frac{\partial t}{\partial r}=\frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}.\left(\frac{r}{2}-\frac{r^3}{4R^2}\right)+\frac{C_1}{r}$$

Integrating again, we have

$$t=\frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)+C_1\ ln\ (r)+C_2 \quad ...(7.109)$$

(where C_1 and C_2 are the constants of integration).

The boundary conditions are :

At $r = 0$, $\frac{\partial t}{\partial r}=0$

At $r = R$, $t = t_s$

Applying the above boundary conditons, we get

$C_1 = 0$, $C_2 = t_s - \frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\frac{3R^2}{16}$

Substituting the values of C_1 and C_2 in eqn. (7.109), we have

$$t=\frac{1}{\alpha}\frac{\partial t}{\partial x}u_{max}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)+\left[t_s-\frac{1}{\alpha}\frac{\partial t}{\partial r}u_{max}\frac{3R^2}{16}\right]$$

or, $$t_s-t=\frac{u_{max}}{\alpha}.\frac{\partial t}{\partial x}\left[\frac{3R^2}{16}-\frac{r^2}{4}+\frac{r^4}{16R^2}\right] \quad ...(7.110)$$

For determining the *heat transfer coefficient* for *fully developed pipe flow*, it is imperative to define a characteristic temperature of the fluid. It is the *bulk temperature* (t_b) or the *mixing up temperature* of the fluid which is an average taken so as to yield the total energy carried by the fluid and is defined as the *ratio of flux of enthalpy at a cross-section to the product of the mass flow rate and the specific heat of the fluid.* Thus,

$$t_b=\frac{\int_0^R \rho\,(2\pi r.dr)\,u\,c_p\,t}{\int_0^R \rho\,(2\pi r.dr)\,u\,c_p} \quad ...(7.111)$$

For an incompressible fluid having constant density and specific heat

$$t_b=\frac{\int_0^R u\,t\,r\,dr}{\int_0^R u\,r\,dr} \quad ...(7.112)$$

The average/mean velocity ($\bar{u}$) also known as the *bulk mean velocity* is calculated from the following definition :

$$\bar{u}=\frac{1}{\pi R^2}\int_0^R 2\pi r.dr.u$$

or, $$\bar{u}=\frac{2}{R^2}\int_0^R ur.dr$$

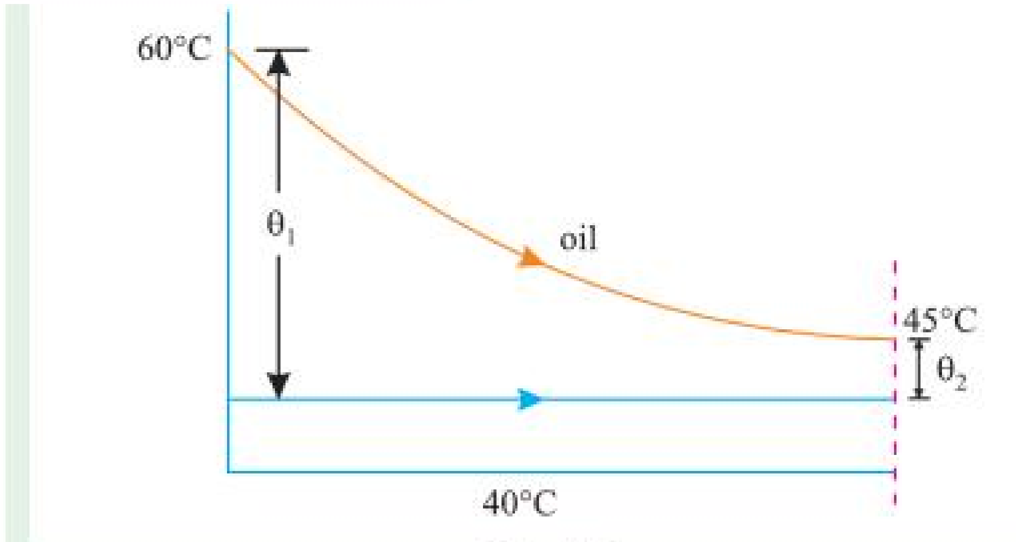

Fig. 7.16.

Length required, *L* :

$$Q = m\, c_p\,(t_i - t_0)$$
$$= (\rho A_f U)\, c_p\,(t_i - t_0)$$

(where U = average velocity, A_f = flow area)

$$= \left(\rho \frac{\pi}{4} D^2 U\right) c_p\,(t_i - t_0)$$

$$= (865 \times \frac{\pi}{4} \times 0.01^2 \times 3) \times 1.78 \times 10^3\,(60 - 45) = 5441.7 \text{ W}$$

Also, $Q = \bar{h}\, A\,\theta_m$

where, A = heat transfer area $= \pi DL$, and

$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)} = \frac{(60-40)-(45-40)}{ln\left[\frac{(60-40)}{(45-40)}\right]} = \frac{15}{1.386} = 10.82°C$$

$$\overline{Nu} = \frac{\bar{h}\,D}{k} = 3.657 \qquad \text{...(Given)}$$

$$\bar{h} = \frac{3.657\,k}{D} = \frac{3.657 \times 0.140}{0.01} = 51.2 \text{ W/m}^2\text{K}$$

Now, $Q = 5441.7 = 51.2 \times \pi DL \times 10.82$

$$\therefore \quad L = \frac{5441.7}{51.2 \times \pi \times 0.01 \times 10.82} = \mathbf{312.7\ m} \quad \textbf{(Ans.)}$$

Example 7.27. *When 0.5 kg of water per minute is passed through a tube of 20 mm diameter, it is found to be heated from 20°C to 50°C. The heating is accomplished by condensing steam on the surface of the tube and subsequently the surface temperature of the tube is maintained at 85°C.Determine the length of the tube required for fully developed flow.*

Take the thermo-physical properties of water at 60°C as :

$\rho = 983.2 \text{ kg/m}^3$, $c_p = 4.178 \text{ kJ/kgK}$, $k = 0.659 \text{ W/m°C}$, $\nu = 0.478 \times 10^{-6} \text{ m}^2\text{/s}$

Solution. *Given :* $m = 0.5$ kg/min, $D = 20$ mm $= 0.02$ m, $t_i = 20°C$, $t_o = 50°C$

Length of the tube required for fully developed flow, *L* :

The mean film temperature, $t_f = \frac{1}{2}\left(85 + \frac{20+50}{2}\right) = 60°C$

Let us first determine the type of the flow.

$$m = \rho A \bar{u} = 983.2 \times \frac{\pi}{4} \times (0.02)^2 \times \bar{u} = \frac{0.5}{60}\ (\text{kg/s})$$

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(ii) Shear stress, τ_0 :

$$\tau_0 = 0.0225\rho U^2 \left(\frac{\mu}{\rho U \delta}\right)^{1/4} \qquad \text{...[Eqn. (7.121)]}$$

Substituting the value of δ from eqn. (7.123), we get

$$\tau_0 = 0.0225\rho U^2 \left(\frac{\mu}{\rho U \times \dfrac{0.371x}{(Re_x)^{1/5}}}\right)^{1/4}$$

$$= \frac{0.00225}{(0.371)^{1/4}}\rho U^2 \left[\frac{\mu}{\rho Ux} \times (Re_x)^{1/5}\right]^{1/4} = 0.0288\rho U^2 \left[\frac{(Re_x)^{1/5}}{Re_x}\right]^{1/4}$$

$$\left(\because Re_x = \frac{\mu}{\rho Ux}\right)$$

or, $$\tau_0 = \frac{\rho U^2}{2} \times \frac{0.0576}{(Re_x)^{1/5}} \left[= \frac{0.0288\rho U^2}{(Re_x)^{1/5}}\right] \qquad \text{...(7.124)}$$

(iii) Local skin friction (drag) coefficient, C_{fx} :

We know $$\tau_0 = \frac{\rho U^2}{2} \times \frac{0.0576}{(Re_x)^{1/5}} \qquad \text{[Eqn. (7.124)]}$$

Also $$\tau_0 = C_{fx} \times \frac{1}{2}\rho U^2 \qquad \text{[Eqn. (7.30)]}$$

Now equating the eqns. (7.124) and (7.30), we have

$$C_{fx} \times \frac{1}{2}\rho U^2 = \frac{\rho U^2}{2} \times \frac{0.0576}{(Re_x)^{1/5}}$$

or, $$C_{fx} = \frac{0.0576}{(Re_x)^{1/5}} \qquad \text{...(7.125)}$$

(iv) Average value of skin friction (drag) coefficient, $\bar{C}_f$:

$$\bar{C}_f = \frac{1}{L}\int_0^L C_{fx}\, dx$$

$$= \frac{1}{L}\int_0^L \frac{0.0576}{(Re_x)^{1/5}}\, dx$$

$$= \frac{1}{L}\int_0^L \frac{0.0576}{\left(\dfrac{\rho Ux}{\mu}\right)^{1/5}}\, dx$$

$$= \frac{1}{L}\int_0^L 0.0576\left(\frac{\mu}{\rho U}\right)^{1/5} (x)^{-1/5}\, dx$$

$$= 0.0576\left(\frac{\mu}{\rho U}\right)^{1/5} \frac{1}{L}\int_0^L (x)^{-1/5}\, dx$$

$$= 0.0576\left(\frac{\mu}{\rho U}\right)^{1/5} \frac{1}{L}\left[5/4 \times x^{4/5}\right]_0^L$$

Forced air convection bottom heater provides sufficient energy for very large boards.

$$= 2 \times 2.642 \times 10^{-3} \times \left[\frac{1}{2} \times 1000 \times (5 \times 0.75) \times 5^2\right] = \mathbf{247.68\ N} \quad \textbf{(Ans.)}$$

Example 7.29. *A submarine can be assumed to have cylindrical shape with rounded nose. Assuming its length to be 50 m and diameter 5.0 m, determine the total power required to overcome boundary friction if it cruises at 8 m/s velocity in sea water at 20°C (ρ = 1030 kg/m^3), $\nu = 1 \times 10^{-6}$ m^2/s.*

Solution. Length of submarine, $L = 50$ m

Diameter of submarine, $D = 50$ m

Velocity of submarine, $U = 8$ m/s

Density of sea water, $\rho = 1030$ kg/m^3

Kinematic viscosity of sea water, $\nu = 1 \times 10^{-6}$ m^2/s

Total power required to overcome boundary friction, *P* :

Reynolds number, $$R_{eL} = \frac{UL}{\nu} = \frac{8 \times 50}{1 \times 10^{-6}} = 4 \times 10^8$$

The length over which boundary layer will be laminar is given by

$$\frac{Ux}{\nu} = 5 \times 10^5 \text{ or } x = \frac{5 \times 10^5 \times \nu}{U}$$

or, $$x = \frac{5 \times 10^5 \times 1 \times 10^{-6}}{8} = 0.0625 \text{ m}$$

This being very small contribution to total drag from laminar boundary layer is negligible; hence $\bar{C}_f$ is given by

$$\bar{C}_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} = \frac{0.455}{\left[\log_{10}(4 \times 10^8)\right]^{2.58}} = 0.001765$$

Area, $$A = \pi DL = \pi \times 5 \times 50 = 785.4 \text{ m}^2$$

∴ Drag force, $$F_D = \bar{C}_f \times \frac{1}{2}\rho A U^2 = 0.001765 \times \frac{1}{2} \times 1030 \times 785.4 \times 8^2$$

$$= 45690.2 \text{ N}$$

Hence total power required to overcome boundary friction,

$$P = \frac{F_D U}{1000} kW = \frac{45690.2 \times 8}{1000} = \mathbf{365.52\ kW} \quad \textbf{(Ans.)}$$

Example 7.30. *Find the ratio of friction drag on the front half and rear half of the flat plate kept at zero incidence in a stream of uniform velocity, if the boundary layer is turbulent over the whole plate.*

Solution. The average coefficient of drag $(\bar{C}_f)$ for turbulent boundary layer is given by

$$\bar{C}_f = \frac{0.072}{(Re_L)^{1/5}} \quad \text{...[Eqn. (7.126)]}$$

For the entire plate, $$Re_L = \frac{UL}{\nu}$$

For the first half of the plate, $Re_x = \frac{Ux}{\nu} = \frac{UL}{2\nu}$

Drag force per unit width for the entire plate is,

$$F_D = \bar{C}_f \times \frac{\rho U^2}{2} \times \text{area per unit width}$$

$$\bar{h} = \frac{k}{L}(P_r)^{1/3}\,[0.036\,(R_{e_L})^{0.8} - 836] \quad ...(7.143)$$

and $$\overline{Nu} = \frac{\bar{h}L}{k} = (Pr)^{1/3}\,[0.036\,(Re_L)^{0.8} - 836] \quad ...(7.144)$$

Example 7.34. *Air flows over a heated plate at a velocity of 50 m/s. The local skin friction co-efficient at a point on a plate is 0.004. Estimate the local heat transfer coefficient at this point. The following property data for air are given :*

Density = 0.88 kg/m³; viscosity = 2.286 × 10⁻⁵ kg m/s;

specific heat, c_p = 1.001 kJ/kg K; conductivity = 0.035 W/m K.

Use $St\,Pr^{1/3} = \frac{C_{fx}}{2}$ **(U.P.S.C., 1993)**

Solution. *Given :* U = 50 m/s; C_{fx} = 0.004; ρ = 0.88 kg/m³; $\mu = 2.286 \times 10^{-5}$ kg m/s;

c_p = 1.001 kJ/kg K; k = 0.035 W/m K.

Local heat transfer coefficient, h_x :

Prandtl number, $$Pr = \frac{\mu \cdot c_p}{k} = \frac{2.286 \times 10^{-5} \times (1.001 \times 1000)}{0.035} = 0.654$$

Stanton number, $$St = \frac{h_x}{\rho \cdot c_p \cdot U} = \frac{h_x}{0.88 \times (1.001 \times 1000) \times 50} = \frac{h_x}{44044}$$

Now, $$St \cdot (Pr)^{2/3} = \frac{c_{fx}}{2} \quad ...(\text{Given})$$

$$\frac{h_x}{44044}(0.654)^{2/3} = \frac{0.004}{2}$$

or, $$h_x = \frac{0.004}{2} \times \frac{44044}{(0.654)^{2/3}} = \mathbf{116.9\ W/m^2\,K} \quad \textbf{(Ans.)}$$

Example 7.35. *The crankcase of an I.C. engine measuring 80 cm × 20 cm may be idealised as a flat plate. The engine runs at 90 km/h and the crankcase is cooled by the air flowing past it at the same speed. Calculate the heat loss from the crank surface maintained at 85°C, to the ambient air at 15°C. Due to road induced vibration, the boundary layer becomes turbulent from the leading edge itself.*

Solution. *Given :* $U = 90\text{ km/h} = \frac{90 \times 1000}{3600} = 25$ m/s; t_s = 85°C; t_∞ = 15°C; L = 80 cm = 0.8 m; B = 20 cm = 0.2m.

The properties of air at $t_f = \frac{85 + 15}{2} = 50$°C are :

k = 0.02824 W/m°C, $\nu = 17.95 \times 10^{-6}$ m²/s, Pr = 0.698 ... (From tables)

Heat loss from the crankcase, Q :

The Reynolds number, $$Re_L = \frac{UL}{\nu} = \frac{25 \times 0.8}{17.95 \times 10^{-6}} = 1.114 \times 10^6$$

Since $Re_L > 5 \times 10^5$, the nature of flow is *turbulent*.

For turbulent boundary layer,

$$\overline{Nu} = \frac{\bar{h}L}{k} = 0.036\,(Re_L)^{0.8}\,(Pr)^{0.333} = 0.036\,(1.114 \times 10^6)^{0.8}\,(0.698)^{0.333} = 2196.92$$

or, $$\bar{h} = \frac{k}{L} \times 2196.92 = \frac{0.02824}{0.8} \times 2196.92 = 77.55\ \text{W/m}^2\,°\text{C}$$

Example 7.40. *An aeroplane flies with a speed of 450 km/h at a height where the surrounding air has a temperature of 1°C and pressure of 65 cm of Hg. The aeroplane wing idealised as a flat plate 6m long, 1.2 m wide is maintained at 19°C. If the flow is made parallel to the 1.2 m width calculate :*

(*i*) *Heat loss from the wing;*

(*ii*) *Drag force on the wing.*

The properties of air at 10°C $\left(t_f = \frac{19+1}{2} = 10°C\right)$ *are :*

$k = 0.02511$ W/m°C, $\nu = 14.16 \times 10^{-6}$ m²/s, $Pr = 0.705$.

Solution. *Given :* $U = \frac{450 \times 1000}{3600} = 125$ m/s, $L = 6$ m, $B = 1.2$ m

(*i*) Heat loss from the wing :

The pressure of the air at flight altitude $= p = \frac{65}{76} \times 1.013 = 0.866$ bar

From the characteristic gas equation, we have

$$\rho = \frac{p}{RT} = \frac{0.866 \times 10^5}{287 \times (10 + 273)} = 1.066 \text{ kg/m}^3$$

The Reynolds number for the entire wing is

$$Re_L = \frac{UL}{\nu} = \frac{125 \times 1.2}{14.16 \times 10^{-6}} = 10.59 \times 10^6$$

Assuming that the critical Reynolds number is 5×10^5, we have

$$Re_c = 5 \times 10^5 = \frac{U\, x_c}{\nu} = \frac{125 \times x_c}{14.16 \times 10^{-6}}$$

or,
$$x_c = \frac{5 \times 10^5 \times 14.16 \times 10^{-6}}{125} = 0.0566 \text{ m}$$

Propane fired heater longwave infrared, supplemented by forced convection system.

Length of the tube required, L :

$$Q = hA_s\, \theta_m \qquad ...(i)$$

where $A_s = \pi D_i L = \pi \times 0.025 \times L = 0.0785\, L\, m^2$

$$\theta_m = \frac{(\theta_1 - \theta_2)}{ln\,(\theta_1/\theta_2)} = \frac{(100-25)-(100-55)}{ln\left(\frac{100-25}{100-55}\right)} = \frac{75-45}{ln\left(\frac{75}{45}\right)} = 58.7°C$$

Also, $Q = \dot{m}\, c_p\, (t_o - t_i)$

The properties of water should be taken at

$$\left[\frac{100 + \left(\frac{55+25}{2}\right)}{2}\right] = 70°C$$

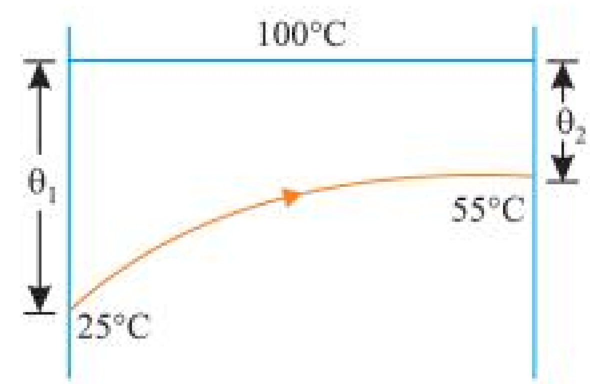

Fig. 7.22.

$\therefore \quad Q = 0.8333 \times 4187 \times (55 - 25) = 104671$ W

To find h we have to use the given empirical relation.

The mass flow rate is given by,

$$\dot{m} = 0.8333 = \frac{\pi}{4} D_i^2 \times V \times \rho = \frac{\pi}{4} \times 0.025^2 \times V \times 977.8$$

$$\therefore \quad U = \frac{0.8333 \times 4}{\pi \times (0.025)^2 \times 977.8} = 1.74 \text{ m/s}$$

$$Re = \frac{\rho V D_i}{\mu} = \frac{977.8 \times 1.74 \times 0.025}{405 \times 10^{-6}} = 1.05 \times 10^5$$

$$Pr = \frac{\mu c_p}{k} = \frac{405 \times 10^{-6} \times 4187}{66.72 \times 10^{-2}} = 2.54$$

Now substituting the values in the given empirical formula, we get

$$Nu = \frac{hD_i}{k} = 0.023\, (Re)^{0.8}\, (Pr)^{0.4}$$

$$= \frac{h \times 0.025}{66.72 \times 10^{-2}} = 0.023\, (1.05 \times 10^5)^{0.8}\, (2.54)^{0.4} = 347.23$$

$$\therefore \quad h = \frac{66.72 \times 10^{-2} \times 347.23}{0.025} = 9268.2 \text{ W/m}^2\text{°C}$$

Now substituting the values in (i), we get

$$104671 = 9268.2 \times 0.0785\, L \times 58.7$$

$$\therefore \quad L = \frac{104671}{9268.2 \times 0.0785 \times 58.7} = \mathbf{2.45\ m} \qquad \textbf{(Ans.)}$$

Example 7.46. *Water at 25°C flows across a horizontal copper tube 1.5 cm OD with a velocity of 2 m/s. Calculate the heat transfer rate per unit length if the wall temperature is maintained at 75°C. Given properties of water :*

$$\rho = 988 \text{ kg/m}^3$$

$$k = 0.648 \text{ W/m K}$$

$$\mu = 549.2 \times 10^{-6} \text{ N s/m}^2$$

$$c_p = 4.174 \text{ kJ/kg K}$$

7.5. EMPIRICAL CORRELATIONS FOR FORCED CONVECTION

The following dimensionless numbers are used for the usual forced convection problems :

(*i*) *Nusselt number,* $Nu = \frac{hL}{k}$; (*ii*) *Reynolds number,* $Re = \frac{\rho LU}{\mu}$

(*iii*) *Prandtl number,* $Pr = \frac{\mu c_p}{k}$; (*iv*) *Stanton number,* $St = \frac{h}{\rho c_p U}$

In order to determine the value of convection coefficient h, the following conventional generalised basic equations are used :

$$Nu = f_1 (Re, Pr) = C_1 (Re)^m (Pr)^n \; ; \; St = f_2 (Re, Pr) = C_2 (Re)^a (Pr)^b$$

The values (numerical) of the constants and exponents are determined through experiments. The properties of the fluid are evaluated on the basis of bulk temperature (unless stated otherwise).

A. *Laminar Flow*

7.5.1. LAMINAR FLOW OVER FLAT PLATES AND WALLS

(*a*) The *local value of heat transfer coefficient* is given by

$$Nu_x = \frac{h_x x}{k} = 0.332 (Re_x)^{0.5} (Pr)^{0.333} \quad \text{...Blasius equation} \quad ..(7.153)$$

The *average value of heat transfer coefficient* is given by

$$Nu = \frac{\bar{h}L}{k} = 0.664 (Re_L)^{0.5} (Pr)^{0.333} \quad ...(7.154)$$

where, $Re_x = \frac{Ux}{\nu}$, $Re_L = \frac{UL}{\nu}$ and $Pr = \frac{\mu c_p}{k}$

The above equations are valid for the following :

(*i*) All fluids ($Pr \geq 0.6$) except liquid metals

(*ii*) Reynolds number $Re \geq 40000$

(*iii*) The fluid properties are evaluated at the mean film temperature, $t_f = \frac{t_s + t_\infty}{2}$.

(*b*) For *liquid metals*, the following correlation has been proposed

$$Nu_x = 0.565 (Pe_x) \quad ...(7.155)$$

where $Pe_x = Re_x \cdot Pr$

The above equation is valid for the following :

(*i*) $Pr \leq 0.05$

(*ii*) The fluid properties are evaluated at the film temperature.

Example 7.50. *Air at a temperature of 15°C flows at a velocity of 6.5 m/s across a flat plate maintained at a temperature of 605°C. Calculate the amount of heat transferred per metre width from both sides of the plate over a distance of 350 mm from the leading edge. The following relation holds good in the case of large temperature difference between the plate and the fluid :*

$$Nu_x = 0.332 (Pr)^{1/3} (Re)^{1/2} \left(\frac{T_s}{T_\infty}\right)^{0.117}$$

where T_s and T_∞ are the absolute temperatures of the plate surface and free stream respectively and all fluid properties are evaluated at the mean film temperature.

Solution. *Given :* $T_\infty = 15 + 273 = 288$ K, $T_s = 605 + 273 = 878$ K, $x = 350$ mm $= 0.35$ m.

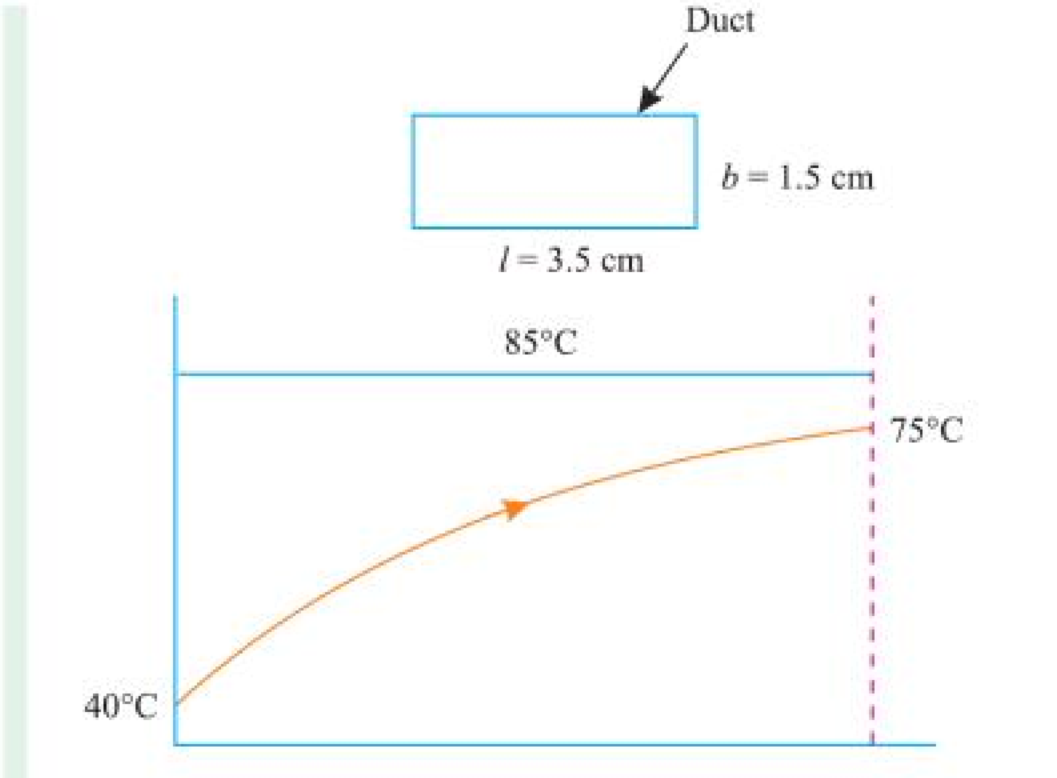

Fig. 7.25.

$$= 0.035 \times 0.015 \times 1.2 \times 985.5 = 0.621 \text{ kg/s}$$

$$\theta_m = \frac{(85-40)-(85-75)}{ln\left[\frac{85-40}{85-75}\right]} = \frac{45-10}{ln\left(\frac{45}{10}\right)} = 23.3°C$$

The heat transfer co-efficient h for the given configuration is given by :

$$Nu = \frac{hL_c}{k} = 0.023\,(Re)^{0.8}\,(Pr)^{0.33} \qquad \text{[Eqn. (7.151)]} \qquad ...(ii)$$

Characteristic length, $$L_c = \frac{4A_c}{P} = \frac{4\,(l \times b)}{2\,(l+b)} = \frac{2lb}{(l+b)} \qquad ...\text{[Eqn. (6.24)]}$$

$$= \frac{2 \times 0.035 \times 0.015}{(0.035 + 0.015)} = 0.021 \text{ m}$$

$$Re = \frac{L_c U}{\nu} = \frac{0.021 \times 1.2}{0.517 \times 10^{-6}} = 0.487 \times 10^5$$

$$Pr = \frac{\mu\, c_p}{k} = \frac{\rho \nu c_p}{k} = \frac{985.5 \times 0.517 \times 10^{-6} \times (4.19 \times 10^3)}{0.653} = 3.27$$

Substituting the values in (*ii*), we get

$$\frac{h \times 0.021}{0.653} = 0.023\,(0.487 \times 10^5)^{0.8}\,(3.27)^{0.33} = 191.23$$

$$\therefore \qquad h = \frac{0.653 \times 191.23}{0.021} = 5946.3 \text{ W/m}^2\text{°C}$$

Now substituting the values in (*i*), we get

$$5946.3 \times 0.1\,L \times 23.3 = 0.621 \times (4.19 \times 10^3) \times (75-40)$$

$$\therefore \qquad L = \frac{0.621 \times (4.19 \times 10^3) \times 35}{5946.3 \times 0.1 \times 23.3} = \mathbf{6.57\ m\ (Ans.)}$$

In the Eqns (7.189) and (7.190), the Reynolds member is:

$$40 < Re = \frac{U_s.D_p}{\nu} < 2000 \qquad \text{...(7.191)}$$

(where D = diameter of sphere or cylinder)

and, properties of fluid are evaluated at the *film temperature*.

In case of other types of packings, whitaker's definition of D_p may be used.

The correlation for *friction factor*, as given by *Beck*, is:

$$f = \frac{D_p}{L}\cdot\frac{\Delta_p}{\rho.U_s^2} = \frac{1-\varepsilon}{\varepsilon^3}\left[1.75 + 150\left(\frac{1-\varepsilon}{Re}\right)\right] \qquad \text{...(7.192)}$$

Where Δ_p is pressure drop over a length of the packed bed.

Example 7.68. *Air is heated from 50°C to 350°C by passing it through 100 mm diameter pipe of a packed bed heat exchanger, packed wih 8 mm diameter spheres. If the flow rate is 18 kg/h and pipe surface temperature is maintained at 400°C, calculate the length of bed required.*

Solution. *Given*: $t_i = 50°C$; $t_0 = 350°C$; $t_s = 400°C$; D = 8mm = 0.008 m; d_{pipe} = 100 mm = 0.1 m;

$$\dot{m}_{air} = 18\text{kg/h} = \frac{18}{3600} = 0.005 \text{ kg/s}$$

Length of the bed required *L*:

$$\text{Average air temperature} = \frac{50+350}{2} = 200°C$$

$$\therefore \quad \text{Average air temperature, } t_f = \frac{200+400}{2} = 300°C$$

Taking the properties of air at 300°C:

$\rho = 0.596$ kg/m³; $c_p = 1047$J/kg K; $k = 0.0429$ W/mK; $\nu = 49.2 \times 10^{-6}$ m²/s; $Pr = 0.71$.

Equivalent particle diameter = 6 × volume/surface = D for sphere

i.e. $D_p = 0.008$ m

$$\therefore \text{ Superficial velocity, } U_s = \frac{4\dot{m}_{air}}{\rho \times \pi d_{pipe}^2}$$

$$= \frac{4\times0.005}{0.596\times\pi\times(0.1)^2} = 1.068 \text{ m/s}$$

Now,
$$Re = \frac{U_s.D_p}{\nu} = \frac{1.068\times0.008}{49.2\times10^{-6}} = 173.66$$

and,
$$\overline{Nu} = \frac{\bar{h}D_p}{k} = 0.203(Re)^{1/3}(Pr)^{1/3} + 0.220(Re)^{0.8}(Pr)^{0.4} \qquad \text{...[Eqn. (7.190)]}$$

$$\left(\text{for } 40 < \text{Re} = \frac{U_s.D_p}{\nu} < 2000\right)$$

or,
$$\frac{\bar{h}\times0.008}{0.0429} = 0.203(173.66)^{1/3}(0.71)^{1/3} + 0.220(173.66)^{0.8}(0.71)^{0.4}$$

$$= 1.01 + 11.876 = 12.886$$

$$\therefore \text{ Average heat transfer coefficient, } \bar{h} = \frac{12.886\times0.0429}{0.008} = 69.1 \text{ W/m}^2\text{K}$$

Now, heat gaind by air, Q = Heat transfer between the wall surface and air

i.e.
$$Q = \dot{m}_{air}\, c_p(t_0 - t_i)$$

$$= 0.005\times1047(350-50) = 1570.5 \text{ W}$$

$$Q = \dot{m}\, c_p\, \Delta t = 2.25 \times 1.3565 \times 19 = 57.99 \text{ kW}$$

But, $\dot{m} = \rho A U$

∴ Velocity of flow,
$$U = \frac{\dot{m}}{\rho A} = \frac{2.25}{916 \times \left(\frac{\pi}{4} \times 0.025^2\right)} = 5.0 \text{ m/s}$$

and,
$$Re = \frac{UD}{\nu} = \frac{5.0 \times 0.025}{0.594 \times 10^{-6}} = 0.21 \times 10^6$$

and, Peclet number $Pe = Re.Pr = 0.21 \times 10^6 \times 0.0087 = 1827$

Using the following relation, we have:

$$\overline{Nu} = 4.82 + 0.0185\,(\text{Pe})^{0.827} \quad \text{...[Eqn. (7.208)]}$$

$$= 4.82 + 0.0185\,(1827)^{0.827} = 14.038$$

Also,
$$\overline{Nu} = \frac{\bar{h}D}{k}$$

∴
$$\bar{h} = \frac{\overline{Nu}.k}{D} = \frac{14.038 \times 84.90}{0.025} = 47673 \text{ W/m}^2\text{k}$$

Now by energy balance, we get:

$$Q = \bar{h}A(t_w - t_b)$$

$$= \bar{h}\,\pi DL(t_w - t_b)$$

or,
$$57.99 \times 1000 = 47673 \times \pi \times 0.025 \times L \times (195 - 134.5)$$

∴
$$L = \frac{57.99 \times 1000}{47673 \times \pi \times 0.025(195 - 134.5)} = \mathbf{0.256\ m\ (Ans.)}$$

HIGHLIGHTS

Important Formulae

A. Laminar Flow

I. ***Flow over flat plate :***

If $\frac{Ux}{\nu} < 5 \times 10^5$ ...boundary layer is *laminar* (velocity distribution is *parabolic*)

If $\frac{Ux}{\nu} > 5 \times 10^5$ ...boundary layer is *turbulent* on that portion (velocity distribution follows *log law or, a power law*)

(*i*) Displacement thickness,
$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

(*ii*) Momentum thickness,
$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

(*iii*) Energy thickenss,
$$\delta_e = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

(iv) $\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}}$ (Blasius)

(v) $\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$ (Von-Karman)

(vi) $\frac{\delta_{th}}{x} = \frac{5}{\sqrt{Re_x}} = \frac{\delta}{x}$ for $Pr = 1$

(vii) $\frac{\delta_{th}}{\delta} = \frac{1}{(Pr)^{1/3}}$ (Pohlhausen)

(viii) $C_{fx} = \frac{0.664}{\sqrt{Re_x}}$ (Blasius)

(ix) $C_{fx} = \frac{0.646}{\sqrt{Re_x}}$ (Von-Karman)

Forced convection bench oven.

(x) $\bar{C}_f = \frac{1.328}{\sqrt{Re_L}}$ (Blasius)

(xi) $h_x = 0.332 \frac{k}{x} (Re_x)^{1/2} (Pr)^{1/3}$

(xii) $Nu_x = \frac{h_x x}{k} = 0.332 (Re_x)^{1/2} (Pr)^{1/3}$

(xiii) $\bar{h} = 2h_x$

(xiv) $\bar{Nu} = \frac{\bar{h}L}{k} = 0.664 (Re_L)^{1/2} (Pr)^{1/3}$

II. ***Laminar tube flow :***

(i) $u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$ ... Most commonly used equation for the velocity distribution for laminar flow through pipes.

(ii) $h = \frac{48\,k}{11\,D}$

(iii) $Nu = 4.364$

(iv) $Nu = 3.65$ ... For constant wall temperature.

B. Turbulent Flow

I. ***For flat plate :***

(i) $\frac{\delta}{x} = \frac{0.371}{(Re_x)^{1/5}}$

(ii) $\tau_o = \frac{\rho U^2}{2} \times \frac{0.0576}{(Re_x)^{1/5}} \left[= \frac{0.0288\,\rho U^2}{(Re_x)^{1/5}}\right]$

(iii) $C_{fx} = \frac{0.0576}{(Re_x)^{1/5}}$

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which on substitution for C_1 from eqn. (8.23) gives

$$u_{max} = 0.766\, v \left(0.952 + \frac{v}{\alpha}\right)^{-1/2} \left[\frac{g\beta\,(t_s - t_\infty)}{v^2}\right]^{1/2} (x)^{1/2} \qquad ...(8.25)$$

The resultant expression for boundary layer thickness is

$$\frac{\delta}{x} = C_2\, x^{n-1} = C_2\, x^{-3/4}$$

which on substitution for C_2 from eqn. (8.24) yields

$$\frac{\delta}{x} = 3.93 \left(0.952 + \frac{v}{x}\right)^{1/4} \left[\frac{g\beta\,(t_s - t_\infty)\, x^3}{v^2}\right]^{-1/y} \left(\frac{v}{\alpha}\right)^{-1/2} \qquad ...(8.26)$$

or,
$$\frac{\delta}{x} = 3.93\,(0.952 + Pr)^{1/4}\,(Gr_x)^{-1/4}\,(Pr)^{-1/2} \qquad ...(8.27)$$

... in dimensionless numbers

(where $Gr_x = \dfrac{g\beta\,(t_s - t_\infty)\, x^3}{v^2}$ and $Pr = \dfrac{v}{\alpha}$)

8.4.3. FREE CONVECTION HEAT TRANSFER COEFFICIENT FOR A VERTICAL WALL

The heat transfer coefficient may be evaluated from

$$Q_s = -\,kA \left(\frac{dt}{dy}\right)_{y=0} = h_x A\,(t_s - t_\infty) \qquad ...(8.28)$$

Using the temperature distribution of eqn. (8.14), we obtain

$$\left(\frac{dt}{dy}\right)_{y=0} = -\,\frac{2\,(t_s - t_\infty)}{\delta} \qquad ...(8.29)$$

Substituting this value in eqn. (8.28), we get

$$h_x = \frac{2k}{\delta}$$

or,
$$\frac{h_x x}{k} = Nu_x = \frac{2x}{\delta} \qquad ...(8.30)$$

From eqns. (8.27) and (8.29) we obtain the heat transfer correlation for natural convection for a vertical flat plate as

$$Nu_x = 0.508\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr_x)^{1/4} \qquad ...(8.31)$$

For a given Prandtl number, Nu_x varies as $(Gr_x)^{1/4}$ or $h_x \,\alpha\, x^{-1/4}$.

The local film heat transfer coefficient decreases with x. It is inversely proportional to the fourth root of x. By integration over the distance, the average heat transfer coefficient is found to be

$$\bar{h}_L = \frac{1}{L}\int_0^L h_x\, dx = \frac{4}{3}\,(h_x)_{x=L} \qquad ...(8.32)$$

or,
$$\overline{Nu}_L = \frac{\bar{h}\,L}{k} = 0.677\,(Pr)^{1/2}\,(0.952 + Pr)^{-1/4}\,(Gr_x)^{1/4} \qquad ...(8.33)$$

For air with $Pr = 0.7$, eqns. (8.31) and (8.33) simplify to

$$Nu_x = 0.378\,(Gr_x)^{1/4} \qquad ...(8.34)$$

$$\overline{Nu}_L = 0.504\,(Gr_L)^{1/4} \qquad ...(8.35)$$

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